Name ________________________________________ CM__________

# ECE 300
# Signals and Systems

## Exam 2
## 24 January, 2007

NAME Solutions

This exam is closed-book in nature. You are not to use a calculator or computer during the exam.

| | | |
|---|---|---|
| Problem 1 | ________ | / 25 |
| Problem 2 | ________ | / 25 |
| Problem 3 | ________ | / 25 |
| Problem 4 | ________ | / 25 |

Exam 2 Total Score: _______ / 100

Name Solution CM

**1. LTI Systems (25 points)**

A periodic function $x(t)$ has the Fourier series representation

$$x(t) = 1 + \sum_{k=-\infty}^{k=\infty} \frac{1}{2+jk} e^{jk2t}$$

$x(t)$ is input to an LTI system with transfer function $H(j\omega)$. The output of the system is determined to be

$$y(t) = -3 + 3\cos(4t - 90^0)$$

In this problem we want to determine as much about the transfer function $H(j\omega)$ as we can. Fill in the following table. If something is unknown or cannot be determined, leave the table entry blank.

| *Parameter* | *Estimated Value* |
|---|---|
| $\lvert H(0) \rvert$ | 2 |
| $\angle H(0)$ *(in degrees)* | $\pm 180$ |
| $\lvert H(j\omega_0) \rvert$ | 0 |
| $\angle H(j\omega_0)$ *(in degrees)* | |
| $\lvert H(j2\omega_0) \rvert$ | $\frac{3\sqrt{8}}{2} = 4.243$ |
| $\angle H(j2\omega_0)$ *(in degrees)* | $-45^0$ |
| $\lvert H(j3\omega_0) \rvert$ | 0 |
| $\angle H(j3\omega_0)$ *(in degrees)* | |

$\omega_0 = 2$ rad/sec

$c_0^x = 1.5 \quad c_0^x H(0) = c_0^y \quad 1.5 H(0) = -3 \quad H(0) = -2 = 2 \angle 180^0$

$|H(j\omega_0)| = 0$ no information about the phase

$|H(j3\omega_0)| = 0$ no information about the phase

$c_2^x = \frac{1}{2+2j} = \frac{1}{\sqrt{4+4}} \angle -45^0 = \frac{1}{\sqrt{8}} \angle 45^0$

$2|c_2^x||H(j2\omega_0)| = 3 \quad |H(j2\omega_0)| = \frac{3}{2|c_2^x|} = \frac{3\sqrt{8}}{2} = 4.243$

$\angle c_2^x + \angle H(j2\omega_0) = -45^0 + \angle H(j2\omega_0) = -90^0 \quad \angle H(j2\omega_0) = -45^0$

Name *Solution* CM

**2. Impulse Response (25 points)**
For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions.*

a) $y(t) = \dfrac{x(t+1) - x(t-1)}{2}$

$$\boxed{h(t) = \frac{\delta(t+1) - \delta(t-1)}{2}}$$

b) $y(t) = \int_{-t+1}^{\infty} e^{-(t-\lambda)} x(\lambda - c)\, d\lambda, \quad c > 0$

$$h(t) = \int_{-t+1}^{\infty} e^{-(t-\lambda)} \delta(\lambda - c)\, d\lambda$$

$$= \begin{cases} e^{-(t-c)} & \text{for } -t+1 < c \\ 0 \end{cases} = \boxed{e^{-(t-c)} u(t-1+c)}$$

c) $2\dot{y}(t) - y(t) = 3x(t+1)$

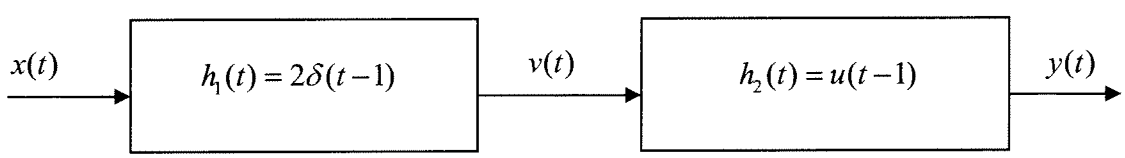
$$\dot{h}(t) - \frac{1}{2} h(t) = \frac{3}{2} \delta(t+1)$$

$$\frac{d}{dt}\left(h(t) e^{-t/2}\right) = \frac{3}{2} e^{-t/2} \delta(t+1) = \frac{3}{2} e^{1/2} \delta(t+1)$$

$$h(t) e^{-t/2} = \frac{3}{2} e^{\frac{1}{2}} \int_{-\infty}^{t} \delta(\lambda+1)\, d\lambda = \frac{3}{2} e^{\frac{1}{2}} u(t+1)$$

$$\boxed{h(t) = e^{\frac{t}{2}} e^{\frac{1}{2}} \frac{3}{2} u(t+1)}$$

d)




(Determine the impulse response of the **system** relating $y(t)$ to $x(t)$).

$$h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(t-\lambda) h_2(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} 2\delta(t-1-\lambda) u(\lambda-1)\, d\lambda = 2u(\lambda-1)\Big|_{\lambda = t-1} = 2u(t-2)$$

$$\boxed{h(t) = 2u(t-2)}$$

Name ______________________________________________ CM____________

**3. Short Answer (25 points)**

(a) The following function was computed for the Fourier Series coefficients of a waveform. Rewrite it in terms of a sinc function and a phase term.

$$c_k = e^{-j5\omega_0 t} - e^{-j3\omega_0 t} = e^{j\left(\frac{-5\omega_0 t - 3\omega_0 t}{2}\right)}\left[e^{j\left(\frac{-5\omega_0 t + 3\omega_0 t}{2}\right)} - e^{-j\left(\frac{-5\omega_0 t + 3\omega_0 t}{2}\right)}\right]$$

$$= e^{-j\frac{8\omega_0 t}{2}}\left[e^{-j\frac{2\omega_0 t}{2}} - e^{j\frac{2\omega_0 t}{2}}\right]$$

$$= -2je^{-j4\omega_0 t}\sin(\omega_0 t) = 2e^{-j(4\omega_0 t + \frac{\pi}{2})}\sin\left(\pi\frac{2t}{T_0}\right)$$

$$= \frac{\frac{\pi 2t}{T_0}}{\frac{\pi 2t}{T_0}}\, 2e^{-j(4\omega_0 t + \frac{\pi}{2})}\sin\left(\pi\frac{2t}{T_0}\right) = \boxed{\frac{4\pi t}{T_0}e^{-j(4\omega_0 t + \frac{\pi}{2})}\operatorname{sinc}\left(\frac{2t}{T_0}\right)}$$

(b) You are given the Fourier Series Representation

$$x(t) = \sum_{k=-\infty}^{k=\infty} 4jk\, e^{-j6k} e^{jk2t}$$

Using the Fourier Series of x(t), derive the Fourier Series coefficients for $y(t) = \dot{x}(t - t_0)$, the derivative of a time-delayed version of x(t).

$$\dot{x}(t-t_0) = \sum_k 4jke^{-j6k}(j2k)e^{jk2(t-t_0)}$$

$$= \sum_k -8k^2 e^{-j6k} e^{jk2(t-t_0)} = \sum_k 8k^2 e^{-j(6k+\pi)} e^{jk2(t)} e^{-jk2t_0}$$

$$= \sum_k 8k^2 e^{-j(6k+\pi+2kt_0)} e^{jk2t}$$

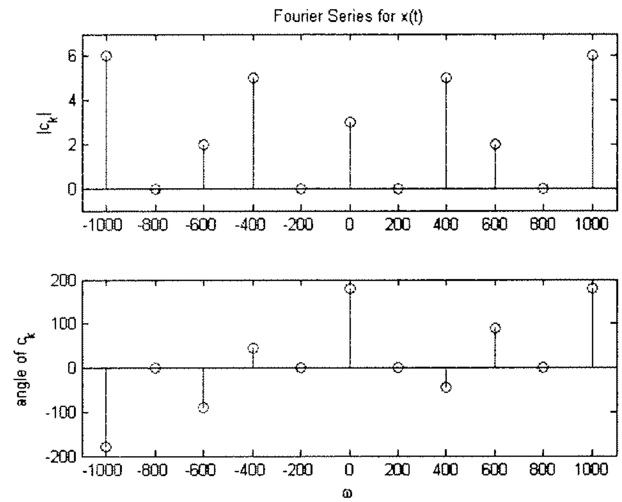
$$Y_k = 8k^2 e^{-j(6k+\pi+2kt_0)}$$

(c) Given only the power spectrum of a signal, x(t), is it possible to write the Fourier Series of x(t)? Explain why or why not.

No, because the power series doesn't

Name ______________________________________ CM__________

**4. System Properties (25 points)**
Use the spectrum of x(t) below to answer the following questions. Note the units of $|c_k|$ are in volts, and all angles are multiples of 45 degrees.




(a) What is the fundamental frequency of x(t) in Hertz?

(b) What is the time average value of x(t)?

(c) What is the average power of x(t) in dBmV?

(d) Express x(t) as a possible DC offset plus a sum of cosine functions.

a) $\omega_0 = 200 = 2\pi f_0$

$f_0 = \frac{100}{\pi}$ Hz $= 31.8$ Hz

b) $c_0 = 3$

(c) $P_{ave} = \sum_k |c_k|^2 = 36 + 4 + 25 + 9 + 25 + 4 + 36$

$= 72 + 8 + 50 + 9 = 139 \text{ W} = \frac{V_{rms}^2}{1\Omega}$

$= 10 \log_{10}\left(\frac{139}{(.001)^2}\right) = \boxed{81.43 \text{ dBmV}}$

(d) $x(t) = -3 + 10\cos(400t - 45^\circ) + 4\cos(600t + 90^\circ) + 12\cos(1000t + 180^\circ)$

$= -3 + 10\cos(400t - 45^\circ) + 4\cos(600t + 90^\circ) - 12\cos(1000t)$