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# ECE 300
# Signals and Systems

## Exam 1
## 20 December, 2007

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This exam is closed-book in nature. You are not to use a calculator or computer during the exam.

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|---|---|---|
| Problem 1 | ________ | / 15 |
| Problem 2 | ________ | / 25 |
| Problem 3 | ________ | / 15 |
| Problem 4 | ________ | / 25 |
| Problem 5 | ________ | / 20 |

Exam 1 Total Score: _______ / 100

Name ____________________ CM__________

**1. Periodicity (15 points)**

**a)** Determine if the following function is periodic. If so, find the fundamental period.

$$x(t) = \frac{e^{j2\pi t} - e^{-j2\pi t}}{j} = 2\sin(2\pi t)$$

$$\omega_0 = 2\pi = \frac{2\pi}{T}$$

$$T = 1\ s.$$

**b)** Two cosine functions are added together. The frequency for the second is larger than the first by a factor of Δf. Derive a relationship between Δf and f that will make the function x(t) periodic.

$$x(t) = \cos(2\pi ft) + \cos(2\pi(f + \Delta f)t)$$

$$x(t+T) = \cos(2\pi f(t+T_1)) + \cos(2\pi(f+\Delta f)(t+T_2))$$

$$2\pi f T_1 = 2\pi r \qquad\qquad 2\pi(f+\Delta f)T_2 = 2\pi g$$

$$T_1 = \frac{r}{f} \qquad\qquad T_2 = \frac{g}{f+\Delta f}$$

$$T_1 = T_2$$

$$\frac{r}{f} = \frac{g}{f+\Delta f}$$

$$r(f+\Delta f) = gf$$

$$\boxed{\Delta f = f\left(\frac{g}{r} - 1\right)}$$

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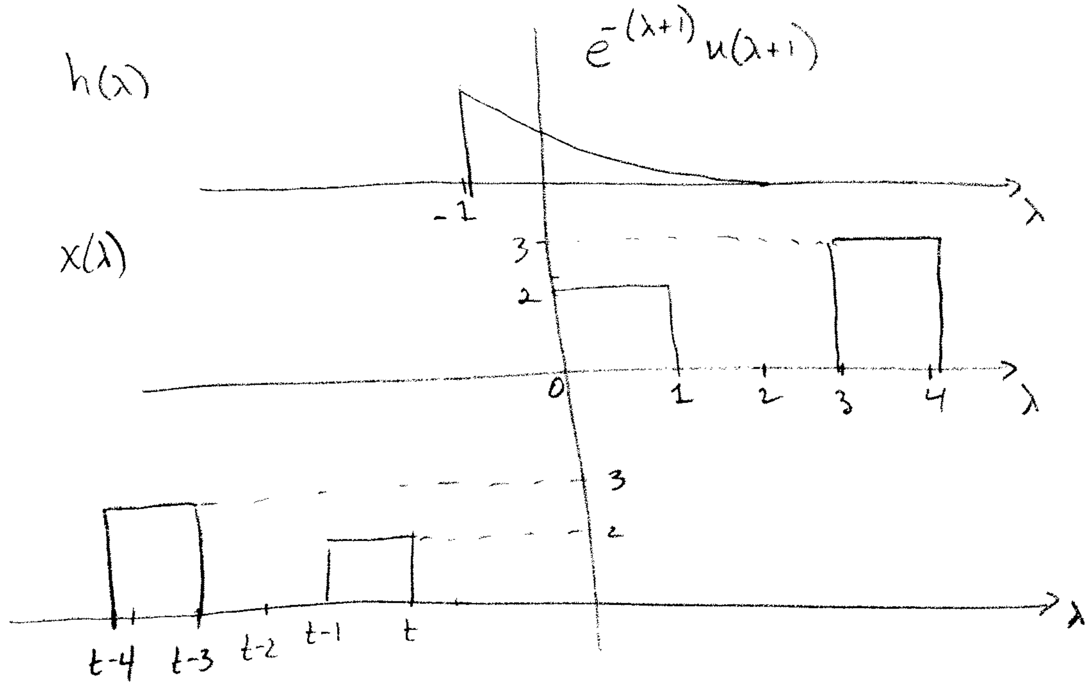
**2. Graphical Convolution (25 points)**
Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-(t+1)}u(t+1)$$

The input to the system is given by

$$x(t) = 2[u(t) - u(t-1)] + 3[u(t-3) - u(t-4)]$$

Use graphical convolution to determine the intervals of integration and their corresponding integrals $y(t) = x(t) * h(t)$. Use x(t) as the signal to "flip and shift" (i.e. x(t-λ)) for the convolution. **DO NOT solve the integrals, just set them up. To get full credit, you must write out the functions for x and h in the integrals.**




Region 1

$t < -1$ $\quad y(t) = 0$

Region 2

$-1 \le t < 0$ $\quad y(t) = \int_{-1}^{t} 2e^{-(\lambda+1)}\,d\lambda$

Region 3

$0 \le t < 2$ $\quad y(t) = \int_{t-1}^{t} 2e^{-(\lambda+1)}\,d\lambda$

Region 4

$2 \le t < 3$ $\quad y(t) = \int_{t-1}^{t} 2e^{-(\lambda+1)}\,d\lambda + \int_{-1}^{t-3} 3e^{-(\lambda+1)}\,d\lambda$

Region 5

$3 \le t$ $\quad y(t) = \int_{t-1}^{t} 2e^{-(\lambda+1)}\,d\lambda + \int_{t-4}^{t-3} 3e^{-(\lambda+1)}\,d\lambda$

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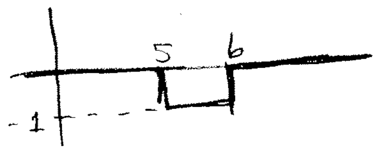
**3. Basic Functions (15 points)**

Simplify or solve the following functions, giving numerical answers whenever possible.

**a)** $\int_{-8}^{10} t\left[u(t-6)-u(t-5)\right]dt$

$u(t-6) - u(t-5)$




$= -\int_5^6 t \, dt$

$= -\left[\frac{t^2}{2}\right]\Big|_5^6 = -\left[18 - 12.5\right] = -5.5$

**b)** $\sin(ty\pi)\delta(t-2)$

$= \sin(2\pi y)\,\delta(t-2)$

**c)** $\int_{-\infty}^{t} e^{-\lambda}\delta(\lambda-5)d\lambda$

$= \int_{-\infty}^{t} e^{-5}\delta(\lambda-5)d\lambda = e^{-5}\int_{-\infty}^{t}\delta(\lambda-5)d\lambda$

$= e^{-5}u(t-5)$

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**4. System Properties (25 points)**

**a)** Using a formal technique, such as the flow graphs used in class, determine whether the system described by the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda) d\lambda$$

is time-invariant.

$$z_1 = \mathcal{H}\{x(t-t_0)\} = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda - t_0) d\lambda$$

$$z_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = \int_{-\infty}^{t-t_0} e^{-(t-t_0-\lambda)} x(\lambda) d\lambda$$

in $z_1$, let $\sigma = \lambda - t_0$ $\quad d\sigma = d\lambda \quad \lambda = \sigma + t_0$

$$z_1 = \int_{-\infty}^{t-t_0} e^{-(t-\sigma-t_0)} x(\sigma) d\sigma = z_2 \quad \text{so TI}$$

**b)** Using a formal technique, such as the flow graphs used in class, determine whether the system described by the equation

$$y(t) = t^2 \sin(t) x(t)$$

Is linear.

$$z_1 = \mathcal{H}\{\alpha_1 x_1(t) + \alpha_2 x_2(t)\} = t^2 \sin(t) [\alpha_1 x_1(t) + \alpha_2 x_2(t)]$$

$$z_2 = \alpha_1 \mathcal{H}\{x_1(t)\} + \alpha_2 \mathcal{H}\{x_2(t)\} = \alpha_1 [t^2 \sin(t) x_1(t)] + \alpha_2 [t^2 \sin(t) x_2(t)]$$

$$= t^2 \sin(t) [\alpha_1 x_1(t) + \alpha_2 x_2(t)] = z_1 \quad \text{so Linear}$$

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**5. Impulse Response (20 points)**

**a)** Determine the impulse response for the system modeled by the differential equation

$$\dot{y}(t) - 2y(t) = x(t+3)$$

$$\dot{h}(t) - 2h(t) = \delta(t+3)$$

$$\frac{d}{dt}\left(h(t)e^{-2t}\right) = e^{-2t}\delta(t+3) = e^{6}\delta(t+3)$$

$$\int_{-\infty}^{t} \frac{d}{d\lambda}\left(h(\lambda)e^{-2\lambda}\right)d\lambda = h(t)e^{-2t} = \int_{-\infty}^{t} e^{6}\delta(\lambda+3) = e^{6}u(t+3)$$

$$\boxed{h(t) = e^{2t}e^{6}u(t+3) = e^{2(t+3)}u(t+3)}$$

note $h(-\infty) = 0$ for impulse response

**b)** If the input to a particular LTI system is $x(t) = u(t)$, and the corresponding output of the system is $y(t) = (1 - e^{-\frac{t}{\tau}})u(t)$ ( $y(t)$ is the step response of the system), determine the output of the system when the input is $x_{new}(t) = A[u(t) - u(t-T)]$

$$\boxed{y_{new}(t) = A\left[1 - e^{-t/\tau}\right]u(t) - A\left[1 - e^{-(t-T)/\tau}\right]u(t-T)}$$