Names _____ CM_____

ECE-300, **Solutions**

Fill in the following table using only the Fourier transform tables.
You can work with one other person if you wish.

	$x(t)$	$\Leftrightarrow$	$X(\omega)$
a.)	$\frac{2}{\pi}\text{sinc}\left(\frac{2t}{\pi}\right)$	$\Leftrightarrow$	$rect\left(\frac{\omega}{4}\right)$
b.)	$\text{sinc}\left(\frac{3t}{\pi}\right)$	$\Leftrightarrow$	$\frac{\pi}{3}\text{rect}\left(\frac{\omega}{6}\right)$
c.)	$\frac{5}{2\pi}\text{rect}\left(\frac{5t}{2\pi}\right)$	$\Leftrightarrow$	$\text{sinc}\left(\frac{\omega}{5}\right)$
d.)	$\text{sinc}^2\left(\frac{2t-4}{3}\right)$		$\frac{3}{2}\Lambda\left(\frac{3\omega}{8\pi}\right)e^{-j2\omega}$
e.)	$0.090e^{j1000t}$	$\Leftrightarrow$	$\delta(\omega-1000)\cos(\omega)$
f.)	4	$\Leftrightarrow$	$8\pi\delta(\omega)$
g.)	$\frac{3}{\pi}\text{sinc}\left(\frac{3}{\pi}[t-3]\right)-\frac{1}{\pi}\text{sinc}\left(\frac{1}{\pi}[t-3]\right)$	$\Leftrightarrow$	$\left[\text{rect}\left(\frac{\omega}{6}\right)-\text{rect}\left(\frac{\omega}{2}\right)\right]e^{-j3\omega}$
h.)	$2e^{2t}u(-t)-\delta(t)$	$\Leftrightarrow$	$\frac{j\omega}{2-j\omega}$
i.)	$15e^{-10t}e^{-j10t}u(t)$	$\Leftrightarrow$	$\frac{3}{2+j\left(\frac{\omega}{5}+2\right)}$
j.)	$-3t\sqrt{\frac{3}{2\pi}}e^{-\frac{3}{2}t^2}$	$\Leftrightarrow$	$j\omega e^{-\frac{\omega^2}{6}}$
k.)	$\frac{2}{3+(2t+3)^2}$	$\Leftrightarrow$	$\frac{\pi}{\sqrt{3}}e^{j\frac{3}{2}\omega}e^{-\frac{\sqrt{3}}{2}\lvert\omega\rvert}$

Quiz Solutions

a) $G(\omega) = \text{rect}\left(\frac{\omega}{4}\right)$ for $G(\omega) = \frac{1}{W}\text{rect}\left(\frac{\omega}{2\pi W}\right) \leftrightarrow g(t) = \text{sinc}(Wt)$

$2\pi W = 4 \quad W = \frac{4}{2\pi} = \frac{2}{\pi}$

$G(\omega) = \text{rect}\left(\frac{\omega}{4}\right) \leftrightarrow \boxed{\frac{2}{\pi}\text{sinc}\left(\frac{2}{\pi}t\right) = g(t)}$

b) $g(t) = \text{sinc}\left(\frac{3}{\pi}t\right)$ for $g(t) = \text{sinc}(Wt) \leftrightarrow G(\omega) = \frac{1}{W}\text{rect}\left(\frac{\omega}{2\pi W}\right)$

here $W = \frac{3}{\pi}$

$g(t) = \text{sinc}\left(\frac{3}{\pi}t\right) \leftrightarrow \boxed{G(\omega) = \frac{\pi}{3}\text{rect}\left(\frac{\omega}{6}\right)}$

c) $G(\omega) = \text{sinc}\left(\frac{\omega}{5}\right)$ for $G(\omega) = T\text{sinc}\left(\frac{T}{2\pi}\omega\right) \leftrightarrow g(t) = \text{rect}\left(\frac{t}{T}\right)$

$\frac{T}{2\pi} = \frac{1}{5} \quad T = \frac{2\pi}{5}$

$G(\omega) = \text{sinc}\left(\frac{\omega}{5}\right) \leftrightarrow \boxed{g(t) = \frac{5}{2\pi}\text{rect}\left(\frac{t}{\frac{2\pi}{5}}\right)}$

d) $g(t) = \text{sinc}^2\left(\frac{2t-4}{3}\right) = \text{sinc}^2\left(\frac{2}{3}(t-2)\right)$

for $g_1(t) = \text{sinc}^2\left(\frac{2}{3}t\right) \leftrightarrow G_1(\omega) = \frac{3}{2}\Lambda\left(\frac{\omega}{\frac{8\pi}{3}}\right)$

for $g_2(t) = g_1(t-2) \leftrightarrow G_2(\omega) = G_1(\omega)e^{-j2\omega}$

or $\boxed{G(\omega) = \frac{3}{2}\Lambda\left(\frac{3\omega}{8\pi}\right)e^{-j2\omega}}$

e) $G(\omega) = \delta(\omega - 1000)\cos(\omega) = \cos(1000)\,\delta(\omega - 1000)$

for $G_1(\omega) = \delta(\omega) \quad g_1(t) = \frac{1}{2\pi}$

For $G_2(\omega) = G_1(\omega - 1000) \quad g_2(t) = g_1(t)e^{j1000t}$

for $G_3(\omega) = \cos(1000)\,\delta(\omega - 1000) \leftrightarrow g(t) = \frac{\cos(1000)}{2\pi}e^{j1000t}$

$\boxed{g(t) = 0.090e^{j1000t}}$

f) $g(t) = 4 \quad \boxed{G(\omega) = 8\pi\delta(\omega)}$

Quiz Solutions

g) $G(\omega) = \left[\text{rect}\left(\frac{\omega}{6}\right) - \text{rect}\left(\frac{\omega}{2}\right)\right] e^{-j3\omega}$

for $G_1(\omega) = \frac{1}{W}\text{rect}\left(\frac{\omega}{2\pi W}\right) \leftrightarrow g_1(t) = \text{sinc}(Wt)$

$\text{rect}\left(\frac{\omega}{6}\right) \Rightarrow 2\pi W = 6 \quad W = \frac{6}{2\pi} = \frac{3}{\pi}$

$\text{rect}\left(\frac{\omega}{2}\right) \Rightarrow 2\pi W = 2 \quad W = \frac{1}{\pi}$

for $G_2(\omega) = \text{rect}\left(\frac{\omega}{6}\right) - \text{rect}\left(\frac{\omega}{2}\right) \leftrightarrow g_2(t) = \frac{3}{\pi}\text{sinc}\left(\frac{3}{\pi}t\right) - \frac{1}{\pi}\text{sinc}\left(\frac{1}{\pi}t\right)$

for $G_3(\omega) = G_2(\omega)e^{-j3\omega} \leftrightarrow g_3(t) = g_2(t-3)$

$$\boxed{g(t) = \frac{3}{\pi}\text{sinc}\left(\frac{3}{\pi}[t-3]\right) - \frac{1}{\pi}\text{sinc}\left(\frac{1}{\pi}[t-3]\right)}$$

h) $G(\omega) = \frac{j\omega}{2-j\omega}$ For $G_1(\omega) = \frac{1}{2+j\omega}$ $g_1(t) = e^{-2t}u(t)$

For $g_2(t) = g_1(-t) = e^{2t}u(-t) \leftrightarrow G_2(\omega) = G_1(-\omega) = \frac{1}{2-j\omega}$

For $G_3(\omega) = j\omega G_2(\omega) = \frac{j\omega}{2-j\omega} \leftrightarrow g_3(t) = \frac{d}{dt}g_2(t)$

$\frac{d}{dt}g_2(t) = \frac{d}{dt}\left[e^{2t}u(-t)\right] = 2e^{2t}u(-t) + e^{2t}\delta(-t)(-1)$

$$\boxed{g(t) = 2e^{2t}u(-t) - \delta(t)}$$

i) $G(\omega) = \frac{3}{2+j\left(\frac{\omega}{5}+2\right)} = \frac{3}{2+j\left(\frac{1}{5}(\omega+10)\right)}$

For $g_1(t) = 3e^{-2t}u(t)$ $G_1(\omega) = \frac{3}{2+j\omega}$

for $g_2(t) = g_1(5t)$ $G_2(\omega) = \frac{1}{5}G_1\left(\frac{\omega}{5}\right) = \frac{\frac{3}{5}}{2+j\frac{\omega}{5}}$

for $g_3(t) = g_2(t)e^{-j10t} \leftrightarrow G_3(\omega) = G_2(\omega+10) = \frac{\frac{3}{5}}{2+j\left(\frac{\omega+10}{5}\right)}$

for $g_4(t) = 5g_3(t) \leftrightarrow G_4(\omega) = \frac{3}{2+j\left(\frac{\omega+10}{5}\right)}$

$$\boxed{g(t) = 5e^{-j10t}e^{-10t}u(t)}$$

Quiz Solutions

j) $G(\omega) = j\omega e^{-\frac{\omega^2}{6}}$

for $g_1(t) = e^{-t^2/2}$ $\quad G_1(\omega) = \sqrt{2\pi}\, e^{-\omega^2/2}$

for $g_2(t) = g_1(at) = e^{-a^2t^2/2}$ $\quad G_2(\omega) = \frac{\sqrt{2\pi}}{a} e^{-\frac{\omega^2}{2a^2}}$

$a^2 = 3 \quad a = \sqrt{3}$

so $g_2(t) = e^{-3t^2/2} \longleftrightarrow \sqrt{\frac{2\pi}{3}}\, e^{-\omega^2/6}$

$g_3(t) = \sqrt{\frac{3}{2\pi}}\, e^{-3t^2/2} \longleftrightarrow e^{-\omega^2/6} = G_3(\omega)$

for $G_4(\omega) = j\omega G_3(\omega) \longleftrightarrow g_4(t) = \frac{d}{dt} g_3(t)$

$\frac{d}{dt} g_3(t) = \boxed{-3t\sqrt{\frac{3}{2\pi}}\, e^{-3t^2/2} = g(t)}$

k) $g(t) = \frac{2}{3+(2t+3)^2} = \frac{2}{3+\left[2\left(t+\frac{3}{2}\right)\right]^2}$

for $g_1(t) = e^{-\sqrt{3}|t|}$ $\quad G_1(\omega) = \frac{2\sqrt{3}}{3+\omega^2}$

by duality $G_1(t) = \frac{2\sqrt{3}}{3+t^2} \longleftrightarrow 2\pi g_1(-\omega) = 2\pi e^{-\sqrt{3}|\omega|}$

for $g_2(t) = \frac{2}{3+t^2} \longleftrightarrow G_2(\omega) = \frac{2\pi}{\sqrt{3}} e^{-\sqrt{3}|\omega|}$

for $g_3(t) = g_2(2t) = \frac{2}{3+(2t)^2} \longleftrightarrow G_3(\omega) = \frac{1}{2} G_2\left(\frac{\omega}{2}\right) = \frac{\pi}{\sqrt{3}} e^{-\sqrt{3}\left|\frac{\omega}{2}\right|}$

for $g_4(t) = g_3\left(t+\frac{3}{2}\right) = \frac{2}{3+\left[2\left(t+\frac{3}{2}\right)\right]^2} \longleftrightarrow G_4(\omega) = G_3(\omega)\, e^{j\omega 3/2}$

$\boxed{G(\omega) = \frac{\pi}{\sqrt{3}} e^{j\omega 3/2} e^{-\sqrt{3}\left|\frac{\omega}{2}\right|}}$