ECE 300
Signals and Systems
Homework 4

Due Date: Tuesday January 9 at 2:40 PM

Problems:

1. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of ω_0 and the c_k. *Hints:* (1) *Draw the signal, and then use the sifting property to calculate the* c_k. (2) *If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t-3p)$$

2. For the periodic square wave $x(t)$ with period $T_o = 0.5$ and

$$x(t) \begin{cases} 1 & 0 \le t < 0.25 \\ -1 & 0.25 \le t < 0.5 \end{cases}$$

show that the Fourier series coefficients are given by

$$c_k = \begin{cases} \dfrac{-2j}{k\pi} & k \quad odd \\ 0 & k \quad even \end{cases}$$

where $x(t) = \sum_k c_k e^{jk4\pi t}$

Special Note: We will be using the code you write in the next part for the next few homeworks and labs, so be sure you do this and understand what is going on!

3. Read the **Appendix** and then do the following:

a) Copy the file **Trigonometric_Fourier_Series.m** to file **Complex_Fourier_Series.m**.

b) Modify **Complex_Fourier_Series.m** so it computes the average value c_o

c) Modify **Complex_Fourier_Series.m** so it also computes c_k for $k=1$ to $k=N$

d) Modify **Complex_Fourier_Series.m** so it also computes the Fourier series estimate using the formula

$$x(t) \approx c_o + \sum_{k=1}^{N} 2\,|c_k|\cos(k\omega_o t + \measuredangle c_k)$$

You will probably need to use the Matlab functions **abs** and **angle** for this.

e) Using the code you wrote in part **d,** find the complex Fourier series representation for the following functions (defined over a single period)

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

These are the same functions you used for the trigonometric Fourier series. Use N = 10 and turn in your plots for each of these functions. Also, turn in your Matlab program for one of these. *Note that the values of **low** and **high** will be different for each of these functions!*

Appendix

In the majority of this course we will be using the complex (or exponential) form of the Fourier series, since it is really easier to do various mathematical things with it once you get used to it.

Exponential Fourier Series If $x(t)$ is a periodic function with fundamental period T, then we can represent $x(t)$ as a Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}$$

where $\omega_o = \frac{2\pi}{T}$ is the fundamental period, $\mathrm{c_o}$ is the average (or DC, i.e. zero frequency) value, and

$$\mathrm{c_o} = \frac{1}{T}\int_0^T x(t)dt$$

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt$$

If $x(t)$ is a real function, then we have the relationships $|c_k| = |c_{-k}|$ (the magnitude is even) and $\measuredangle c_{-k} = -\measuredangle c_k$ (the phase is odd). Using these relationships we can then write

$$x(t) = c_o + \sum_{k=1}^{\infty} 2|c_k|\cos(k\omega_o t + \measuredangle c_k)$$

This is usually a much easier form to deal with, since it lends itself easily to thinking of a phasor representation of $x(t)$. This will be particularly useful when we starting filtering periodic signals.