# Working with dBs

When EEs monitor a signal passing through a system, most often they are concerned with its power level as it propagates through the various components and subsystems. Normally, tracking the power level involves multiplication or division of gains and losses representing the action of the various components of the system on the signal. Many times, the signal power can vary over a wide range of values - several orders of magnitude - which makes representing the signal power at different points a bit inconvenient. The mathematical computations involving multiplication and division of large numbers can be cumbersome.

In order to make the math easier, we often use a logarithmic scale to represent values. There are two chief advantages in working with logarithmic scales:

1. Multiplication becomes addition: log(a **x** b) = log(a) **+** log(b) and log(a **/** b) = log(a) **-** log(b) ).

2. Scales are compressed - if we have values ranging over several orders of magnitude, the plot scale is large for linear representations, while relatively compressed for logarithmic representation of values.

## Definition of deciBel (dB)

The logarithmic scale most often used is one in which the values are represented in *deciBels* (dBs), often written as decibels. Here, the decibel is defined as a logarithmic representation of a unit-less quantity, normally a ratio of two powers. The logarithmic base (or radix) used for dBs is 10:

$$N_{dB} = 10\log_{10}\left(N\right)$$

where N refers to the numerical value being represented and $N_{dB}$ refers to the value in decibels (dBs). Note that other definitions of "dBs" exist, but this is the definition used for representing electrical signal powers, and therefore it is the only *valid* definition as far as we are concerned.

There are a number of things to note about this definition. First, there is a multiplier of 10 in front of the logarithm. That is simply part of the definition which makes the numbers work out more intuitively. However, it is important to remember that the multiplier is 10 only for quantities involving *power*. Next, the logarithm is base 10 only. Other *radixes* may be used to represent values, but not for dBs. Finally, note that the argument of the logarithm is a unit-less value. This last point is very important in our discussion here.

## Representing Values as dBs

There are a number of values for which it will be good to know the dB representation in order to make life easier and to understand the "lingo". Simply plug the values in the dB expression to create this table (verify a few in your head):

| values | |
|---|---|
| lin | dB |
| 1.00E-06 | -60 |
| 0.001 | -30 |
| 0.01 | -20 |
| 0.1 | -10 |
| 0.5 | -3 |
| 1 | 0 |
| 2 | 3 |
| 10 | 10 |
| 100 | 20 |
| 1000 | 30 |
| 1.00E+06 | 60 |

Note that number values less than 1 produce negative dB values, and number values greater than 1 produce positive dB values. The dB value of 1 is zero dB, and the dB value of 0 is undefined (but can be approximated by -99 dB!).

To convert dB values back to linear values, simply invert the definition of decibels as follows:

$$N = 10^{N_{dB}/10}$$

Be sure to verify a few of the table entries using this relationship as well. You must feel very comfortable with the relationship between numerical values and dB representations before moving on.

One of the advantages mentioned above was that multiplication was easier using logarithms. Let's try it using the table above. The value 2 converts to 3 dB, and 100 converts to 20 dB. What should the value 200 convert to? Using multiplication, we see that 200 = 2 x 100. So, when converting to dBs, we find:

$$\begin{aligned} 200 &\rightarrow 10\log_{10}(200) = 10\log_{10}(100\times 2) \\ &= 10\log_{10}(100) + 10\log_{10}(2) = 20 + 3 = 23\text{ dB} \end{aligned}$$

Below are some more examples of using this multiplicative effect to quickly find dB values:

$$50 \rightarrow 10\log_{10}(10) + 10\log_{10}(5) = 10 + 7 = 17\text{ dB}$$

$$1/50 \rightarrow -10\log_{10}(10) - 10\log_{10}(5) = -10 - 7 = -17\text{ dB}$$

$$4.0\text{E}06 \rightarrow 10\log_{10}(10^6) + 10\log_{10}(4) = 60 + 6 = 66\text{ dB}$$

## Representing Powers using dBs

There are two ways we typically use dBs: to represent *powers* (normally average powers), and to represent *power ratios*. We must be fluent in both, and how to combine the two. Lets look first at how we use deciBels to represent power values (or express powers in terms of corresponding voltages or currents). Here, we need to recall one important point from the definition of dBs - the value we represent is to be a unit-less quantity, a power ratio. To create this unit-less ratio, we express the power relative to some standard or *reference power*. For example, suppose we wish to represent 5 W in terms of deciBels. First, consider the use of a power reference of 1W:

$$P = 10\log_{10}\left({}^{5W}\!/_{1W}\right) = 10\log_{10}(5) = 6.99\,dBW \cong 7\,dBW$$

Here, the appended "W" to the dB unit reminds us that this is a decibel representation of a power *relative to 1 W*. A more common measure is dBm, or power relative to 1 mW (note that the "W" is missing in this unit):

$$P = 10\log_{10}\left({}^{5W}\!/_{1mW}\right) = \quad 10\log_{10}(5000) = 37\,dBm$$

and we can relate the power in dBW to power in dBm rather simply:

$$\begin{aligned} P\;dBm &= 10\log_{10}\left(\frac{5W}{1mW}\frac{1000\,mW}{1W}\right) = 10\log_{10}\left(\frac{5W}{1W}\frac{1000\,mW}{1mW}\right) = 10\log_{10}(5\cdot 1000) \\ &= 10\log_{10}(5) + 10\log_{10}(1000) = P\,dBW + 30 \end{aligned}$$

So 5 W may be represented by 7 dBW or 37 dBm, values separated by 30 dB or a factor of 1000. Do not think of dBW and dBm as different units. They both are dBs - the "W" and "m" suffixes are there to remind us of the power reference.

A couple of examples would be nice:

$$\begin{aligned} &1\,\mathrm{W} \rightarrow 10\log_{10}\left({}^{1\,\mathrm{W}}\!/_{1\,\mathrm{W}}\right)\mathrm{dBW} = 10\log_{10}(1)\,\mathrm{dBW} = 0\,\mathrm{dBW} \\ &1\,\mathrm{W} \rightarrow 10\log_{10}\left({}^{1\,\mathrm{W}}\!/_{1\,\mathrm{mW}}\right)\mathrm{dBW} = 10\log_{10}(1000)\,\mathrm{dBm} = 30\,\mathrm{dBm} \\ &20\,\mathrm{mW} \rightarrow 10\log_{10}\left({}^{20\,\mathrm{mW}}\!/_{1\,\mathrm{mW}}\right)\mathrm{dBW} = 10\log_{10}(20)\,\mathrm{dBm} = 13\,\mathrm{dBm} \end{aligned}$$

Another dB unit used to represent a power level is the dBV or dBmV, which is a power level referenced back to an equivalent rms voltage level which would produce that power given a 1 Ω resistance. Suppose we know a signals rms voltage. Given the resistance over which the voltage is developed, the power would be equal to $v_{rms}^2 / R$. It turns out that many times we ignore the resistance value in the calculation, calculating the power developed across a 1 Ω resistance. Thus P = $v_{rms}^2$. We can represent this signal using dB units and a reference of W as follows:

$$P_{dBW} = 10\log_{10}\left( V_{rms}^2 \; W / 1 \; W \right) \quad dBW .$$

Now, we could reason that the 1 W reference is just $(1\ V_{rms})^2$, and rewrite

$$P_{dB\_X} = 10\log_{10}\left( v_{rms}^2 / (1V_{rms})^2 \right) \quad dBW = 20\log_{10}\left( v_{rms} / 1V_{rms} \right) \quad dBV .$$

Here, we have used the fact that $\log(x^2) = 2\log(x)$. The new unit, dBV, is still a dB measure of power, but the suffix "V" reminds us that we have referred the power back to an equivalent rms voltage. A similar power level unit is dBmV, defined as

$$P_{dBmV} = 20\log_{10}\left( v_{rms} / 1\ mV_{rms} \right) \quad dBmV .$$

It is important to remember that dBW, dBm, dBV, and dBmV are all dB units of power. Each has a different calculation method and reference.

Here are some examples of how the powers corresponding to rms voltage amplitudes can be expressed in dBV and dBmV units:

$$5\,\mathrm{V_{rms}} \rightarrow 20\log_{10}\left( 5\,\mathrm{V_{rms}} / 1\,\mathrm{V_{rms}} \right)\mathrm{dBV} = 20\log_{10}(5)\,\mathrm{dBV} = 14\,\mathrm{dBV}$$

$$0.1\mathrm{V_{rms}} \rightarrow 20\log_{10}\left( 0.1\,\mathrm{V_{rms}} / 1\mathrm{mV_{rms}} \right)\mathrm{dBmV} = 20\log_{10}(100)\,\mathrm{dBmV} = 20\ \mathrm{dBmV}$$

$$0.4\,\mu\mathrm{V_{rms}} \rightarrow 20\log_{10}\left( 0.4\mathrm{x}10^{-3}\ \mathrm{mV_{rms}} / 1\mathrm{mV_{rms}} \right) = 20\log_{10}\left(0.4\mathrm{x}10^{-3}\right)\mathrm{dBmV} = -68\ \mathrm{dBmV}$$

## Representing Power Ratios using dBs

When we wish to represent a power ratio, we simply determine the dB value of that ratio. For example, lets look at *power gain (or loss)* for a circuit:

$$G_p = 10\log_{10}\left( Pout / Pin \right)$$

The symbol G is commonly used to represent gain, and here we use the subscript "p" to indicate that we are calculating the power gain, defined as the ration of output power to input power. The power gain of a circuit (or system), a unit-less quantity, is often expressed in dBs. An important convention is that dBs are normally reserved for ratios of powers, although there are ways to represent voltage and current ratios.

Now, how is this related to voltage gain, which is what we spend much of our time calculating in circuits classes? Actually, decibels are defined for power, so we need to modify the calculation slightly to deal with voltages (assuming proper *load matching*):

$$G_p = 10\log_{10}\left(Pout/Pin\right) = 10\log_{10}\left(\frac{Vout^2}{R}\bigg/\frac{Vin^2}{R}\right) = 10\log_{10}\left(Vout^2/Vin^2\right)\ dB$$

$$= 20\log_{10}\left(Vout/Vin\right)\ dB$$

where the fact that $\log(x^2)=2\log(x)$ has been used.

As an example, suppose we measure the voltage at the input to a subsystem as 15 V across 50 ohms. The output voltage, again across a 50 ohm load, is 12 V. Calculate the power gain in dBs.

The power at the input is $V^2/R=4.5$ W, and the power at the output is 2.88 W. This gives a power gain of $G_p=2.88/4.5=0.64$ -> -1.94 dB. This is actually a power loss (negative power gain – tricky!). Let's try it with voltage: $G_p = 20\log_{10}(V_{out}/V_{in}) = -1.94$ dB. Why doesn't the resistance make a difference?

Returning to the power gain computation above, we can see how using logarithms to convert multiplication to addition happens:

$$G_p = 10\log_{10}\left(Pout/Pin\right) = 10\log_{10}\left(\frac{Pout}{P_{ref}}\right) - 10\log_{10}\left(\frac{Pin}{P_{ref}}\right)$$

$$= Pout\,(dBm) - Pin\,(dBm)$$

and multiplication is transformed to addition.

## Signal Propagation

Suppose that we wish to determine how much signal power should arrive at a certain location, given the gains and losses along its path. This is where the use of dBs really shows its advantages. To be continued…

## Rounding Decibel Values

When we first start working with decibels, we are tempted to try to "round off" values as we would with linear values. However, the use of the logarithmic scale distorts the error of that technique. The appropriate number of digits to use to express powers and gains normally varies a bit with application, but a few examples will serve to demonstrate the effect.

Suppose we have a power gain $G_p$ of 3.50 dB. In linear terms, this is

$$G_{p,lin} = 10^{G_{p,dB}/10} = 10^{3.5/10} = 2.238721 \cong 2.24$$

What happens if I represent $G_p$ as 3 dB, or 4 dB? The value 3 dB is equivalent to 1.995262, but that is so close to two that we normally say 3 dB is a factor of 2 (or -3 dB is a factor of 1/2). If we choose to write the gain as 3 dB, the error is about 11%, which is usually not acceptable. If we choose to write the gain as 4 dB, which is equivalent to 2.51, the error is over 12%. So we know that we should at least carry the first digit to the right of the decimal to accurately represent a dB value.

What about the second digit? Let's look at the gain value 3.55 dB (2.265), and compare the errors in writing 3.5 and 3.6 dB. The error writing the gain as 3.5 dB (2.239) is a little over 1%, and the error writing the gain as 3.6 dB (2.291) is about 1.2%. So, carrying the second digit to the right of the decimal can give accuracies on the order of 1%. Normally, dB values are not written with more than 2 values to the right of the decimal in practice.

So, we need at least one digit to the right of the decimal to accurately represent dB values, and 2 digits can give more precision. However, when measuring values in dBs, we often find the accuracy of the measurement equipment is on the order of many tenths of a dB, often 0.5 or 1 dB. So, using the second digit may be a bit silly if we wish to compare our values to measurement. However, certain applications such as communication systems often carry the precision of dB estimates to two digits past the decimal, since in that case performance can vary greatly with a power difference of 0.1 dB. Most classroom applications are fine using one digit to the right of the decimal.

**<u>NOTE:</u>** Percent error is not appropriate for dB measurements – use dB difference instead!

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