ECE 300
Signals and Systems
Homework 2

Due Date: Tuesday December 12 at 12:40 PM

Reading: K & H, pp. 25-47 (skip the discrete-time stuff).

Problems

1. Consider the following system representations:

a) $y(t) = e^{x(t)}$ **b)** $y(t) = \sin(t)x(t)$ **c)** $\dot{y}(t) + 2y(t) = 3x(t)$ **d)** $y(t) = x(t-1)$

e) $y(t) = x\left(\frac{t}{3}\right) + 2$ **f)** $y(t) = e^{t}\int_{-\infty}^{t} e^{-\lambda}x(\lambda - c)d\lambda,\ c > 0$ **g)** $y(t) = \int_{0}^{t} \lambda x(\lambda)d\lambda,\ t > 0$

For each of these models fill in the following table (or a similar table) to summarize your results (put a **Y** or **N** for each question). You need to justify your answers. For part **c** you should show $y(t) = x(t_0)e^{-2(t-t_0)} + \int_{t_0}^{t} 3e^{2(t-\lambda)}x(\lambda)d\lambda$. If you have trouble with the differential equation, read the handout. *You are responsible for being able to solve a first order differential equation such as that in part* **c** *using an integrating factor. These will show up throughout this course, so learn to solve them now!*

Part	Causal?	Memoryless?	Linear?	Time Invariant?
a				
c				
d				
e				
f				
g				

2. For each of the following signals, determine E_∞ and P_∞ into 1 Ω . (Express your answer in Joules or Watts.) Classify each of the following signals as energy or power signals (or neither).

a) $v(t) = 4$ **b)** $v(t) = 3\cos(2\pi 10t + 15°)$ **c)** $i(t) = 4\exp(-2|t|)$ **d)** $x(t) = 4rect\left(\frac{t-2}{3}\right)$

3. K & H Problem 1.15. *Hint: draw pictures and think about when the delta function is included in your interval. The answer to **d** is l/2.*

4. An LTI system responds to the following inputs with the corresponding outputs:

If (input) $x(t) = u(t)$ then (output) $y(t) = (1 - e^{-2t})u(t)$
If (input) $x(t) = \cos(2t)$ then (output) $y(t) = 0.3953\cos(2t - 71.56^{\circ})$

Find (the output) $y(t)$ for the following inputs:

a) $x(t) = 2u(t) - 2u(t-1)$ **b)** $x(t) = 4\cos(2(t-2))$ **c)** $x(t) = 5u(t) + 10\cos(2t-6)$

d) $x(t) = \delta(t)$ **e)** $x(t) = tu(t)$

*Hint: Use the results from problem k & H 1.23 for parts **d** and **e**.*

5. (Matlab Problem) The ***average value*** of a function x(t) is defined as

$$\overline{x} = \frac{1}{b-a}\int_a^b x(t)dt$$

and the ***root-mean-square (rms)*** value of a function is defined as

$$x_{rms} = \sqrt{\frac{1}{b-a}\int_a^b x^2(t)dt}$$

Read the **Appendix**, then

a) use Matlab to find the average and rms values of the function $x(t) = t^2$ for $-1 < t < 1$

b) use Matlab to find the average and rms values of the following functions

$$\begin{array}{ll} x(t) = \cos(t) & 0 < t < \pi \\ x(t) = \cos(t) & 0 < t < 2\pi \\ x(t) = |t| & -1 < t < 1 \\ x(t) = t\cos(t) & -2 < t < 4 \end{array}$$

Hint: You will probably find the sqrt function useful.

For this problem you can just copy down the answers from the Matlab screen. We will assume you are doing this in Matlab because if you are not it will soon become obvious….

Appendix

Maple is often used for symbolically integrating a function. Sometimes, though, what we really care about is the numerical value of the integral. Rather than integrating symbolically, we might want to just use numerical integration to evaluate the integral. Since we are going to be using Matlab a great deal in this course, in this appendix we will learn to use one of Matlab's built-in functions for numerical integration. In order to efficiently use this function, we will learn how to construct what are called *anonymous* functions. We will then use this information to determine the average and rms value of a function. Some of this is going to seem a bit strange at first, so just try and learn from the examples.

Numerical Integration in Matlab Let's assume we want to numerically integrate the following:

$$I = \int_0^{2\pi} (t^2 + 2)dt$$

In order to do numerical integration in Matlab, we will use the built-in command **quadl.** The ***arguments*** to quadl, e.g., the information passed to quadl, are

- A function which represents the integrand (the function which is being integrated). Let's call the integrand $x(t)$. This function must be written in such a way that it returns the value of $x(t)$ at each time t. Clearly here $x(t) = t^2 + 2$
- The lower limit of integration, here that would be 0
- The upper limit of integration, here that would be 2π

Note that an optional fourth argument is the tolerance, which defaults to 10^{-6}. When the function value is very small, or the integration time is very small, you will have to change this.

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#1

(a) $y(t) = e^{x(t)}$

The system is clearly causal and memoryless since y at time t only depends on x at time t

Linear? $y_1 = \mathcal{H}\{ax_1 + bx_2\} = e^{ax_1(t) + bx_2(t)}$

$y_2 = a\mathcal{H}\{x_1\} + b\mathcal{H}\{x_2\} = ae^{x_1(t)} + be^{x_2(t)}$

$y_1 \neq y_2$ for all a and b, so not linear

as an alternative, if we double x(t) we clearly do not double y(t), so the system is non-linear

Time-Invariant? $y_1 = \mathcal{H}\{x(t-t_0)\} = e^{x(t-t_0)}$

$$y_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = e^{x(t)}\Big|_{t=t-t_0} = e^{x(t-t_0)}$$

$y_1 = y_2$ so time-invariant

(b) $y(t) = \sin(t)\, x(t)$

The system is clearly causal and memoryless

Linear? $y_1 = \mathcal{H}\{ax_1 + bx_2\} = \sin(t)(ax_1(t) + bx_2(t))$

$y_2 = a\mathcal{H}\{x_1\} + b\mathcal{H}\{x_2\} = a(\sin(t)\, x_1(t)) + b(\sin(t)\, x_2(t))$

$= \sin(t)(ax_1(t) + bx_2(t)) = y_1$ linear

Time-Invariant? $y_1 = \mathcal{H}\{x(t-t_0)\} = \sin(t)\, x(t-t_0)$

$$y_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = \sin(t)\, x(t)\Big|_{t=t-t_0} = \sin(t-t_0)\, x(t-t_0)$$

$y_2 \neq y_1$ not time invariant

#1 (continued)

ⓒ $\dot{y}(t) + 2y(t) = 3x(t)$ integrating factor e^{2t}

$$\frac{d}{dt}\left[y(t)e^{2t}\right] = \dot{y}(t)e^{2t} + y(t)2e^{2t} = \left(\dot{y}(t) + 2y(t)\right)e^{2t} = \left(3x(t)\right)e^{2t}$$

so $\frac{d}{dt}\left[y(t)e^{2t}\right] = 3e^{2t}x(t)$

$$\int_{t_0}^{t} \frac{d}{dt}\left[y(t)e^{2t}\right]dt = y(t)e^{2t} - y(t_0)e^{2t_0} = \int_{t_0}^{t} 3e^{2\lambda}x(\lambda)d\lambda$$

$$y(t)e^{2t} = y(t_0)e^{2t_0} + \int_{t_0}^{t} 3e^{2\lambda}x(\lambda)d\lambda$$

$$y(t) = y(t_0)e^{-2(t-t_0)} + \int_{t_0}^{t} 3e^{-2(t-\lambda)}x(\lambda)d\lambda$$

looking at the integral, $y(t)$ depends on previous values of x, so <u>not memoryless</u>. y at time t depends only on values of x up to time t, so <u>causal</u>

Linear? $\dot{y}_1 + 2y_1 = 3x_1 \longrightarrow a\dot{y}_1 + 2ay_1 = 3ax_1$

$\dot{y}_2 + 2y_2 = 3x_2 \longrightarrow b\dot{y}_1 + 2by_2 = 3bx_2$

adding $(a\dot{y}_1 + b\dot{y}_2) + 2(ay_1 + by_2) = 3(ax_1 + bx_2)$

$Y = ay_1 + by_2 \quad X = ax_1 + bx_2 \quad \dot{Y} + 2Y = 3X$ same equation so <u>linear</u>

Time-Invariant?

$$\mathcal{H}\{x(t-t_0)\} \xrightarrow[\text{replace } y(t) \text{ by } y(t-t_0)]{\text{replace } x(t) \text{ by } x(t-t_0)} \dot{y}(t-t_0) + 2y(t-t_0) = 3x(t-t_0)$$

$$\mathcal{H}\{x(t)\}\Big|_{t=t-t_0} \xrightarrow[\text{in entire equation}]{\text{replace } t \text{ by } t-t_0} \left[\dot{y}(t) + 2y(t) = 3x(t)\right]_{t=t-t_0}$$

$$\dot{y}(t-t_0) + 2y(t-t_0) = 3x(t-t_0)$$

same equation so <u>time invariant</u>

#1 (continued)

(d) $y(t) = x(t-1)$

The system is causal and has memory

Linear? $y_1 = \mathcal{H}\{a x_1 + b x_2\} = a x_1(t-1) + b x_2(t-1)$

$y_2 = a\mathcal{H}\{x_1\} + b\mathcal{H}\{x_2\} = a x_1(t-1) + b x_2(t-1)$

$y_1 = y_2$ so linear

Time-Invariant?

$y_1 = \mathcal{H}\{x(t-t_0)\} = x(t-t_0-1)$

$y_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = x(t-1)\Big|_{t=t-t_0} = x(t-t_0-1)$

$y_1 = y_2$ so time-invariant

(e) $y(t) = x\left(\frac{t}{3}\right) + 2$

The system is not causal $\left(y(-1) = x\left(\frac{-1}{3}\right) + 2\right)$, and has memory

Linear? $y_1 = \mathcal{H}\{a x_1 + b x_2\} = a x_1\left(\frac{t}{3}\right) + b x_2\left(\frac{t}{3}\right) + 2$

$y_2 = a\mathcal{H}\{x_1\} + b\mathcal{H}\{x_2\} = a\left[x_1\left(\frac{t}{3}\right) + 2\right] + b\left[x_2\left(\frac{t}{3}\right) + 2\right]$

$= a x_1\left(\frac{t}{3}\right) + b x_2\left(\frac{t}{3}\right) + 2(a+b) \neq y_1$ For all a, b not linear

as an alternative, if we double x we do not double y, so not linear

Time-Invariant?

$y_1 = \mathcal{H}\{x(t-t_0)\} = x\left(\frac{t}{3} - t_0\right) + 2$

$y_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = x\left(\frac{t-t_0}{3}\right) + 2$

$y_1 \neq y_2$ not time invariant

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#1 (continued)

(F) $y(t) = e^{t}\int_{-\infty}^{t} e^{-\lambda} x(\lambda - c)\, d\lambda \quad c > 0$

The system is <u>causal</u>, since $y(t)$ only depends on x up to time $t-c$

The system <u>has memory</u>

Linear? $y_1 = \mathcal{H}\{a x_1 + b x_2\} = e^{t}\int_{-\infty}^{t} e^{-\lambda}\left[a x_1(\lambda - c) + b x_2(\lambda - c)\right] d\lambda$

$$y_2 = a\mathcal{H}\{x_1\} + b\mathcal{H}\{x_2\} = a\left[e^{t}\int_{-\infty}^{t} e^{-\lambda} x_1(\lambda - c)\, d\lambda\right] + b\left[e^{t}\int_{-\infty}^{t} e^{-\lambda} x_2(\lambda - c)\, d\lambda\right]$$

$$= e^{t}\int_{-\infty}^{t} e^{-\lambda}\left[a x_1(\lambda - c) + b x_2(\lambda - c)\right] d\lambda = y_1 \quad \underline{\text{linear}}$$

Time-Invariant?

$$y_1 = \mathcal{H}\{x(t - t_0)\} = e^{t}\int_{-\infty}^{t} e^{-\lambda} x(\lambda - t_0 - c)\, d\lambda$$

$$y_2 = \mathcal{H}\{x(t)\}\Big|_{t = t - t_0} = e^{t - t_0}\int_{-\infty}^{t - t_0} e^{-\lambda} x(\lambda - c)\, d\lambda$$

To show these are equal (or not), make the integrands equal (if possible)

in y_1, let $\sigma = \lambda - t_0 \quad d\lambda = d\sigma \quad \lambda = \sigma + t_0$

so $$y_1 = e^{t}\int_{-\infty}^{t - t_0} e^{-(\sigma + t_0)} x(\sigma - c)\, d\sigma = e^{t - t_0}\int_{-\infty}^{t - t_0} e^{-\sigma} x(\sigma - c)\, d\sigma$$

$$= y_2$$

so <u>time-invariant</u>

#1 (continued)

(g) $y(t) = \int_0^t \lambda x(\lambda)\, d\lambda \quad t>0$

The system is <u>causal</u> and has <u>memory</u>

Linear? $y_1 = \mathcal{H}\{a x_1 + b x_2\} = \int_0^t \lambda (a x_1(\lambda) + b x_2(\lambda))\, d\lambda$

$y_2 = a\mathcal{H}\{x_1\} + b\mathcal{H}\{x_2\} = a\int_0^t \lambda x_1(\lambda) d\lambda + b\int_0^t \lambda x_2(\lambda) d\lambda$

$= \int_0^t \lambda (a x_1(\lambda) + b x_2(\lambda))\, d\lambda = y_1$ <u>linear</u>

Time-Invariant?

$y_1 = \mathcal{H}\{x(t-t_0)\} = \int_0^t \lambda x(\lambda - t_0)\, d\lambda$

$y_2 = \mathcal{H}\{x(t)\}\Big|_{t=t-t_0} = \int_0^{t-t_0} \lambda\, x(\lambda)\, d\lambda$

To show these are equal (or not) make the integrands equal (if possible)

in y_1, let $\sigma = \lambda - t_0$ $\quad d\sigma = d\lambda \quad \lambda = \sigma + t_0$

$y_1 = \int_{-t_0}^{t-t_0} (\sigma + t_0)\, x(\sigma)\, d\sigma \neq y_2$ not time invariant

#2

a) $v(t) = 4$ $\quad E_\infty = \int_{-\infty}^{\infty} |v(t)|^2 dt = \int_{-\infty}^{\infty} 4^2 dt = \infty$ not an energy signal

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 4^2 dt = \frac{16(2T)}{2T} = \boxed{16 \text{ watts} = P_\infty}$$

b) $v(t) = 3\cos(2\pi 10 t + 15^\circ)$

from class $E_\infty = \infty$ $\quad P_\infty = \frac{A^2}{2} = \boxed{\frac{9}{2} \text{ watts} = P_\infty}$

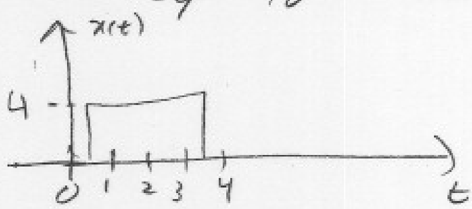
c) $i(t) = 4e^{-2|t|}$

$$E_\infty = \int_{-\infty}^{\infty} |i(t)|^2 dt = \int_{-\infty}^{\infty} 16 e^{-4|t|} dt$$

$$= 2\int_0^{\infty} 16 e^{-4t} dt = 32 \frac{e^{-4t}}{-4}\Big|_0^{\infty} = \boxed{8 \text{ joules} = E_\infty}$$

$P_\infty = 0$

d) $x(t) = 4\,\mathrm{rect}\left(\frac{t-2}{3}\right)$



$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_1^4 16 dt = 3 \cdot 16 = \boxed{48 \text{ joules} = E_\infty}$$

$P_\infty = 0$

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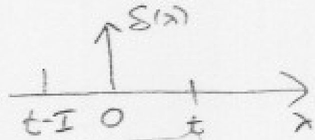
#3

K+H 1.15 $\quad y(t) = \frac{1}{I}\int_{t-I}^{t} x(\lambda)\,d\lambda \quad I>0$ continuous-time moving average filter

(a) assume $x(t) = \delta(t)$ so $y(t) = \frac{1}{I}\int_{t-I}^{t} \delta(\lambda)\,d\lambda$

draw a picture!

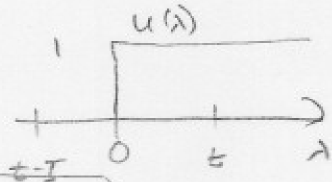


$y(t) = \frac{1}{I}$ if $\delta(\lambda)$ is in the region of integration

$$\boxed{y(t) = \begin{cases} \frac{1}{I} & 0<t<I \\ 0 & \text{else} \end{cases}}$$

(b) assume $x(t) = u(t)$ so $y(t) = \frac{1}{I}\int_{t-I}^{t} u(\lambda)\,d\lambda$

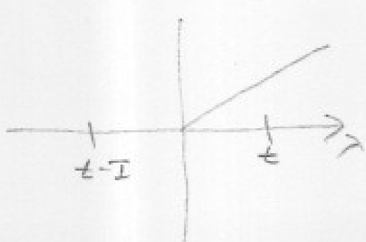
draw a picture!



$$\boxed{y(t) = \begin{cases} \frac{t}{I} & 0<t<I \\ 1 & t\geq I \\ 0 & \text{else} \end{cases}}$$

(c) assume $x(t) = t\,u(t)$ so $y(t) = \frac{1}{I}\int_{t-I}^{t} \lambda\, u(\lambda)\,d\lambda$

draw a picture!



for $0<t<I$ $\quad y(t) = \frac{1}{I}\int_{0}^{t} \lambda^2 d\lambda = \frac{t^2}{2I}$

for $t>I$ $\quad y(t) = \frac{1}{I}\int_{t-I}^{t} \lambda^2 d\lambda = \frac{t^2-(t-I)^2}{2I}$

$$\boxed{y(t) = \begin{cases} \frac{t^2}{2I} & 0<t\leq I \\ \frac{t^2-(t-I)^2}{2I} & t\geq I \\ 0 & \text{else} \end{cases}}$$

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(d) for $t > T$ $\quad y(t) = \dfrac{t^2 - (t-T)^2}{2T} = \dfrac{t^2 - [t^2 - 2tT + T^2]}{2T}$

$$= \frac{2tT - T^2}{2T} = \frac{2Tt}{2T} - \frac{T^2}{2T} = t - \frac{T}{2}$$

input $\quad$ output

$x(t) = t$ $\quad$ $y(t) = t - T/2$ $\quad$ $\boxed{\text{delay is } T/2}$

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD

#4 LTI system

$x_1(t) = u(t) \longrightarrow y_1(t) = (1-e^{-2t})u(t)$

$x(t)_2 = \cos(2t) \longrightarrow y_2(t) = 0.3953\cos(2t - 71.56°)$

a) $x_{new}(t) = 2u(t) - 2u(t-1) \rightarrow y_{new}(t) = 2y_1(t) - 2y_1(t-1)$

$$\boxed{y_{new}(t) = 2(1-e^{-2t})u(t) - 2(1-e^{-2(t-1)})u(t-1)}$$

b) $x_{new}(t) = 4\cos(2(t-2)) \longrightarrow 4y_2(t-2) = \boxed{1.5812\cos(2(t-2) - 71.56°) = y_{new}(t)}$

c) $x_{new}(t) = 5x_1(t) + 10\,x_2(t-3)$

$$\boxed{y_{new}(t) = 5(1-e^{-2t})u(t) + 3.953\cos(2(t-3) - 71.56°)}$$

d) for $x_{new}(t) = \delta(t) = \frac{d}{dt}x_1(t)$ $\quad y_{new}(t) = \frac{d}{dt}y_1(t)$

$$\frac{d}{dt}y_1(t) = 2e^{-2t}u(t) - (1-e^{-2t})\delta(t)$$
$$= 2e^{-2t}u(t) - (1-e^{-2t})\Big|_{t=0}\delta(t) = \boxed{2e^{-2t}u(t) = y_{new}(t)}$$

e) for $x_{new}(t) = tu(t) = r(t) = \int_{-\infty}^{t} x_1(\lambda)d\lambda$

$$y_{new}(t) = \int_{-\infty}^{t} y_1(\lambda)d\lambda = \int_{-\infty}^{t}(1-e^{-2\lambda})u(\lambda)d\lambda = \int_{0}^{t}(1-e^{-2\lambda})d\lambda$$

$$= \left(\lambda + \tfrac{1}{2}e^{-2\lambda}\right)\Big|_0^t = \boxed{\left(t + \tfrac{1}{2}e^{-2t} - \tfrac{1}{2}\right)u(t) = y_{new}(t)}$$

only valid if $t>0$

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To get started, select MATLAB Help or Demos from the Help menu.

ave =

0.3333 $\bar{x}$ for $x(t) = t^2$

rms =

0.4472 x_{rms}

ave =

4.9335e-017 $\bar{x}$ for $x(t) = \cos(t)$ $0 < t < \pi$

rms =

0.7071 x_{rms}

ave =

7.8024e-013 $\bar{x}$ for $x(t) = \cos(t)$ $0 < t < 2\pi$

rms =

x_{rms}

0.7071

ave = $\bar{x}$ for $x(t) = |t|$ $-1 < t < 1$

0.5000

rms =

x_{rms}

0.5774

ave =

-0.8472

$\bar{x}$ for $x(t) = t\cos(t)$ $-2 < t < 4$

ms =

1.5652