ECE 300
Signals and Systems
Homework 1

Due Date: Tuesday December 5 at 12:40 PM

Reading: K & H, Ch 1.1.1-1.1.7 and your course notes.

Problems

1. K & H, Problem 1.1, figures **a-d**. Use the **rect** function and the **triangle** (Λ) function. See Problem **6** for plotting.

The **rect** function represents a rectangular pulse. To represent a pulse x(t) having amplitude 3, located (centered) at t=5, and width 4, we would write:

$$x(t) = 3\,\text{rect}\left(\frac{t-5}{4}\right)$$

The **triangle** function represents a triangular pulse. To represent a triangle x(t) having amplitude 3, located (centered) at t = 5, and width 4, we would write

$$x(t) = 3\Lambda\left(\frac{t-5}{4}\right)$$

2. Describe a system that you have encountered for which the inputs could be modeled as either a delta, unit-step, ramp function or some combination thereof. Then describe what the output of the system would be in response to the input that you've given. Dr. Simoni's students should submit their answer to the Lessons->Homework->Hw1 Discussion Forum on ANGEL and follow the corresponding directions. Dr. Throne's students should submit a written answer with their homework.

Example:

> When a xylophone is struck with a mallet, the striking of the mallet could be modeled with a delta function. A series of strikes with the mallet could be modeled by a series of delta functions all shifted in time from the first one. The output of the system to each individual delta function is a single musical tone, the amplitude (loudness) of which decays exponentially over time, and the initial amplitude of which is proportional to the amplitude of the delta function.

3. K & H, Problem 1.4, parts **a-c** only. Rewrite each of the functions from parts **a-c** in the form

$$x(t) = x_1(t)[u(t-t_1) - u(t-t_2)] + x_2(t)[u(t-t_2) - u(t-t_3)] + \dots$$

4. Simplify or solve the following as much as possible, giving numerical answers where possible:

a) $\int_{-\infty}^{\infty} e^{-t}u(t-5)dt$

b) $\int_{-8}^{10} t^2\left[u(t-6)-u(t-5)\right]dt$

c) $\int_{-\infty}^{\infty} t^2\delta(t-2)dt$

d) $\sin(ty\pi)\delta(t-2)$

e) $\int_{-\infty}^{\infty} \delta(t-3)\delta(t-4)dt$

f) $\int_{-\infty}^{5} u(t-x+5)\delta(t-4)dt$

g) $H(\omega)\delta(\omega-1)+A(\omega-x+1)\delta(\omega)$

h) $\int_{-9}^{10} u(t+3)u(t-2)dt$

5. For each of the following signals, determine if the signal is periodic and, if so, find the fundamental period. Then plot the signals in Matlab as stated in problem **6**. For part **d**, plot the real and imaginary parts. Plot the signals from 0 to 30 seconds.

a) $x(t)=\sin(2t)+\cos(3t+30^{\circ})$ **b)** $x(t)=\cos(2t)+\cos(\pi t)$

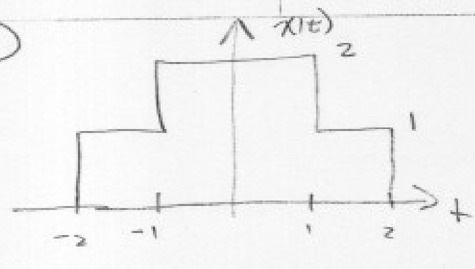
c) $x(t)=e^{-t}\cos(t)$ **d)** $x(t)=2e^{j2t}+3e^{j(3t+2)}$

6. Read the **Appendix** to this assignment, and look at the Matlab tutorials on the class website if you need to. Using the Matlab functions **unit_step.m**, **unit_rect.m**, and **unit_triangle.m** available on the class website, plot your functions for **1.1 a-d, 1.4 a-c, and problem 5**. Note that to generate an exponential, such as e^{-3t} in Matlab we would type exp(-3*t). To generate sin(2πt) it would be sin(2*pi*t). Remember that all arguments to sinusoids must be expressed as radians. Also, for **1.4 b** and **c** you may need to use the **.*** (element by element) multiplication. **Turn in all your plots, neatly labeled.**

(#1) ECE-300 Homework #1 1

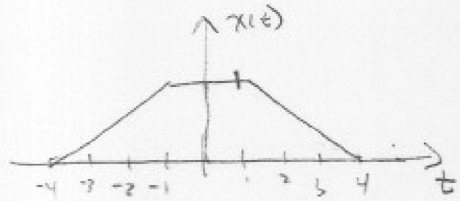
(K+H 1.1)

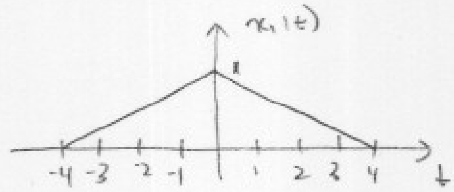
(a)



$$x(t) = \text{rect}\left(\frac{t}{4}\right) + \text{rect}\left(\frac{t}{2}\right)$$

(b)



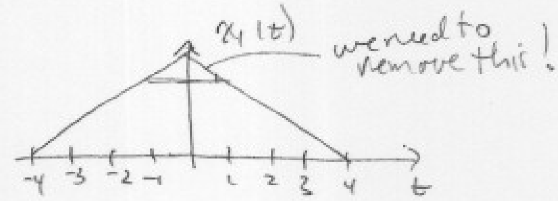


For $x_1(t) = \Lambda\left(\frac{t}{8}\right)$ we have

we need $x_1(t)$ to have a value of 1 when $t = \pm 1$

$0 \le t \le 4, \quad x_1(t) = 1 - \frac{t}{4} \qquad x_1(1) = 1 - 1/4 = 3/4$

so we want $x_1(t) = \frac{4}{3}\Lambda\left(\frac{t}{8}\right)$

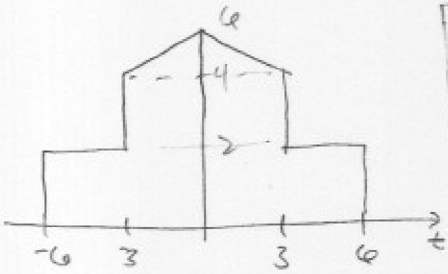


$x_2(t)$ is the small triangle at the top we need to remove.

$x_2(t) = \frac{1}{3}\Lambda\left(\frac{t}{2}\right)$

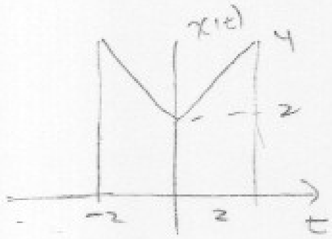
$$x(t) = \frac{4}{3}\Lambda\left(\frac{t}{8}\right) - \frac{1}{3}\Lambda\left(\frac{t}{2}\right)$$

(c)



$$x(t) = 2\,\text{rect}\left(\frac{t}{12}\right) + 2\,\text{rect}\left(\frac{t}{6}\right) + 2\Lambda\left(\frac{t}{6}\right)$$

(d)



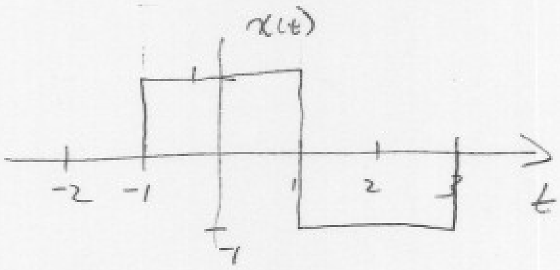
$$x(t) = 4\,\text{rect}\left(\frac{t}{4}\right) - 2\Lambda\left(\frac{t}{4}\right)$$

#3

K+H 1.4

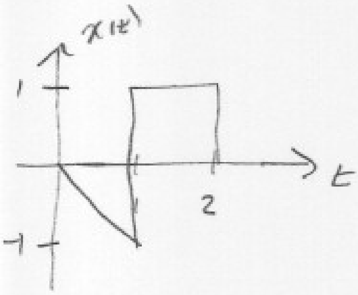
a) $x(t) = u(t+1) - 2u(t-1) + u(t-3)$

$$x(t) = (1)\left[u(t+1) - u(t-1)\right] + (-1)\left[u(t-1) - u(t-3)\right]$$



b) $x(t) = (t+1)u(t-1) - tu(t) - u(t-2)$

$$x(t) = -t\left[u(t) - u(t-1)\right] + \left[u(t-1) - u(t-2)\right]$$



c) $x(t) = 2(t-1)u(t-1) - 2(t-2)u(t-2) + 2(t-3)u(t-3)$

$= 2(t-1)u(t-1) - \left[2(t-1) - 2\right]u(t-2) + (2t-6)u(t-3)$

$$x(t) = 2(t-1)\left[u(t-1) - u(t-2)\right] + 2\left[u(t-2) - u(t-3)\right] + (2t-4)u(t-3)$$

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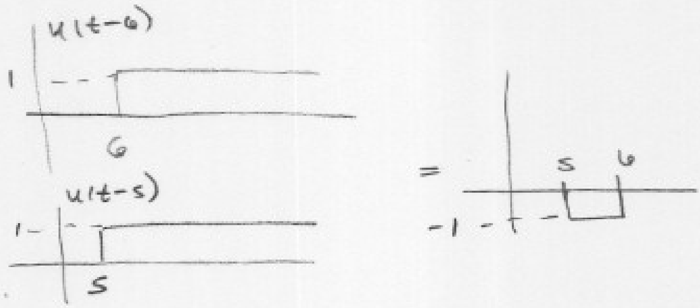
#4

a) $\int_{-\infty}^{\infty} e^{-t} u(t-5)\,dt \qquad u(t-5) = 1$ for $t-5 \geq 0$ or $t \geq 5$

$$= \int_{5}^{\infty} e^{-t}\,dt = -e^{-t}\Big|_{5}^{\infty} = \boxed{e^{-5} = 0.00674}$$

b) $\int_{-8}^{10} t^2 \left[u(t-6) - u(t-5)\right] dt$



$$= \int_{5}^{6} (-1)\, t^2\,dt = -\frac{t^3}{3}\Big|_{5}^{6} = -\left[\frac{6^3 - 5^3}{3}\right] = \boxed{-30.33}$$

c) $\int_{-\infty}^{\infty} t^2 \delta(t-2)\,dt = t^2\Big|_{t=2} = \boxed{4}$

d) $\sin(ty\pi)\,\delta(t-2) = \boxed{\sin(2\pi y)\,\delta(t-2)}$

e) $\int_{-\infty}^{\infty} \delta(t-3)\,\delta(t-4)\,dt = \delta(4-3) = \delta(1) = \boxed{0}$

f) $\int_{-\infty}^{5} u(t-x+5)\,\delta(t-4)\,dt = u(4-x+5) = \boxed{u(9-x)}$

g) $H(\omega)\,\delta(\omega-1) + A(\omega-x+1)\,\delta(\omega) = \boxed{H(1)\,\delta(\omega-1) + A(1-x)\,\delta(\omega)}$

h) $\int_{-9}^{10} u(t+3)\,u(t-2)\,dt$

$u(t+3) = 1$ for $t+3 \geq 0 \quad t \geq -3$
$u(t-2) = 1$ for $t-2 \geq 0 \quad t \geq 2$

$$= \int_{2}^{10} 1\,dt = \boxed{8}$$

#5

a) $x(t) = \sin(2t) + \cos(3t + 30°)$

$x(t+T) = \sin(2t + 2T) + \cos(3t + 3T + 30°)$

for periodic we need $2T = 2\pi g$ g an integer
$3T = 2\pi r$ r an integer

$T = \frac{2\pi g}{2} = \frac{2\pi r}{3} = \frac{g}{2} = \frac{r}{3}$ $g = 2, r = 3$ works

fundamental period is $T = \frac{2\pi(2)}{2} = \boxed{2\pi = T_0}$ $\boxed{\text{periodic}}$

b) $x(t) = \cos(2t) + \cos(\pi t)$

$x(t+T) = \cos(2t + 2T) + \cos(\pi t + \pi T)$

for periodic we need $2T = 2\pi g$ g an integer
$\pi T = 2\pi r$ r an integer

$T = \frac{2\pi g}{2} = \frac{2\pi r}{\pi}$

or $T = \pi g = 2r$ or $\frac{\pi}{2} = \frac{r}{g}$

there are no integers r, g for which this is true

$\boxed{\text{not periodic}}$

c) $x(t) = e^{-t}\cos(t)$

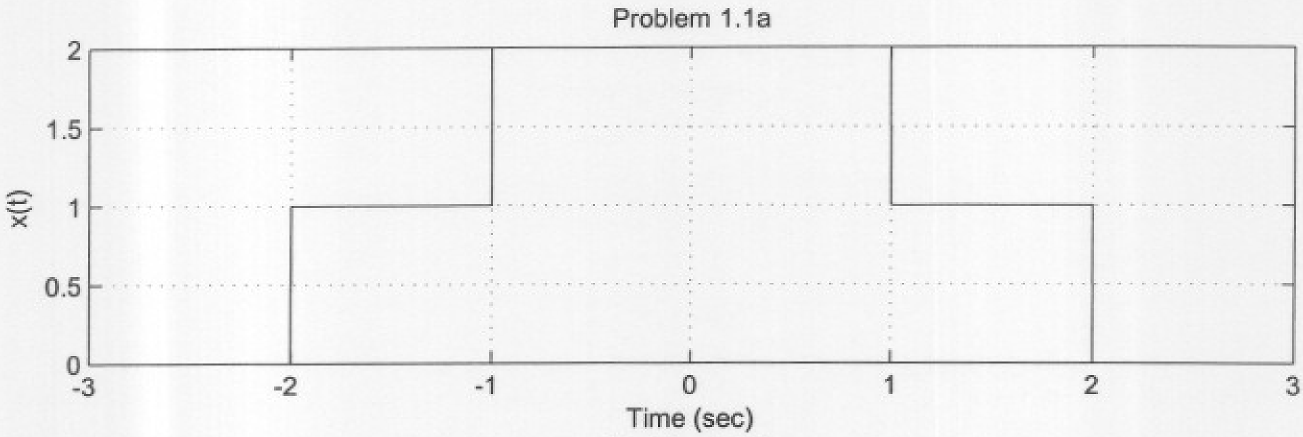
$x(t+T) = e^{-t}e^{-T}\cos(t+T) = x(t) = e^{-t}\cos(t)$

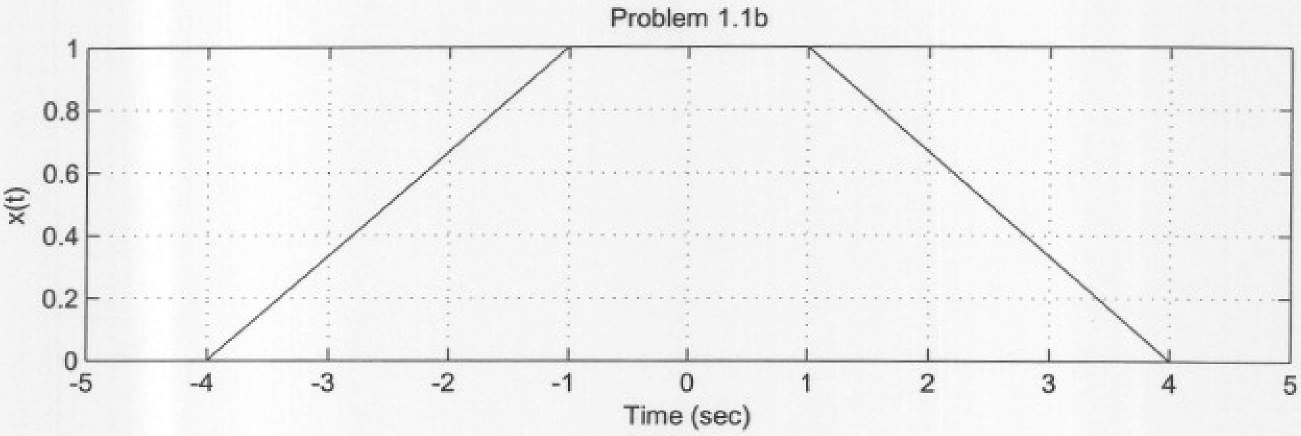
we need $T = 2\pi g$ for g an integer (for the cosine term)
and $T = 0$ (for the exponential term)

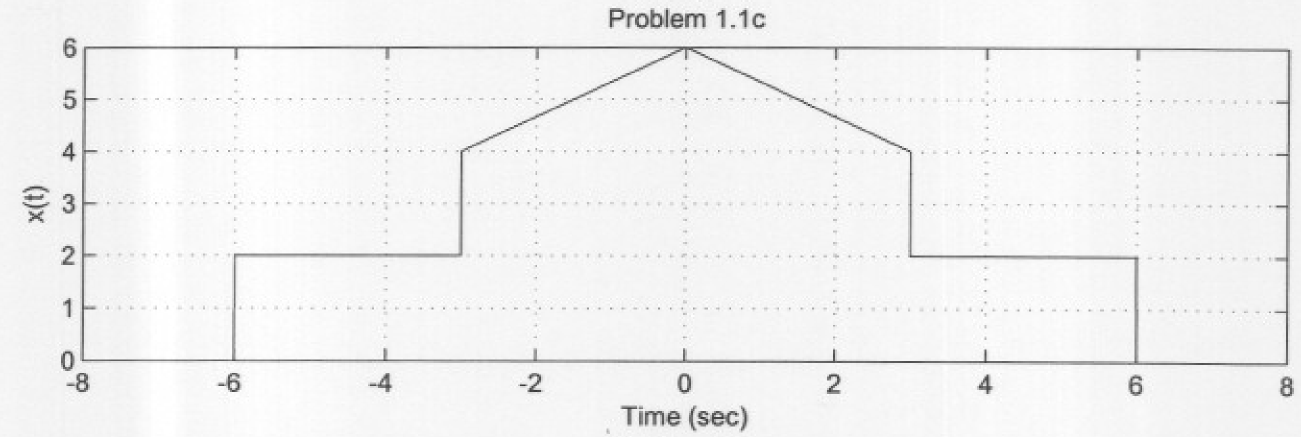
$\boxed{\text{not periodic}}$

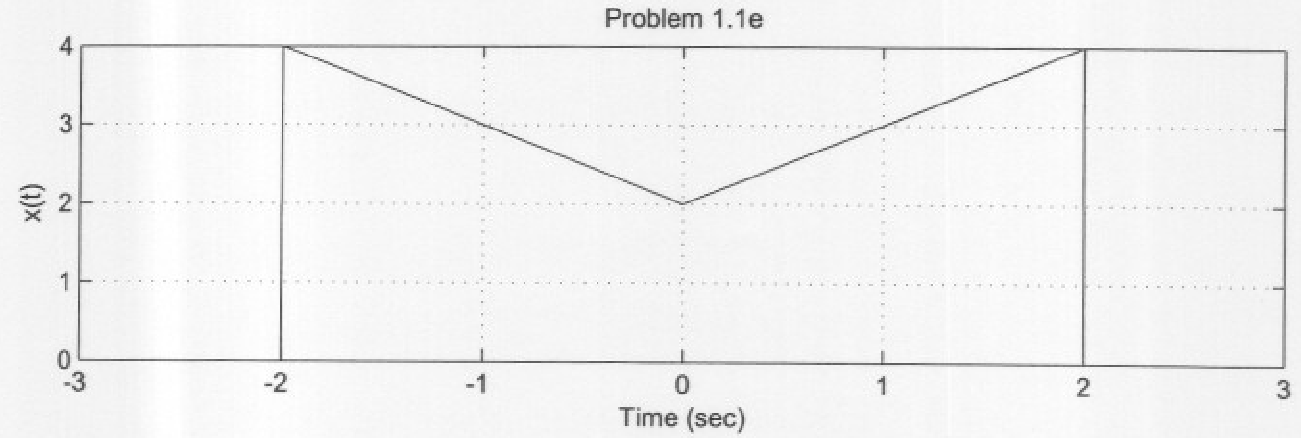
d) $x(t) = 2e^{j2t} + 3e^{j(3t+2)}$ $x(t+T) = 2e^{j(2t+2T)} + 3e^{j(3t+3T+2)}$

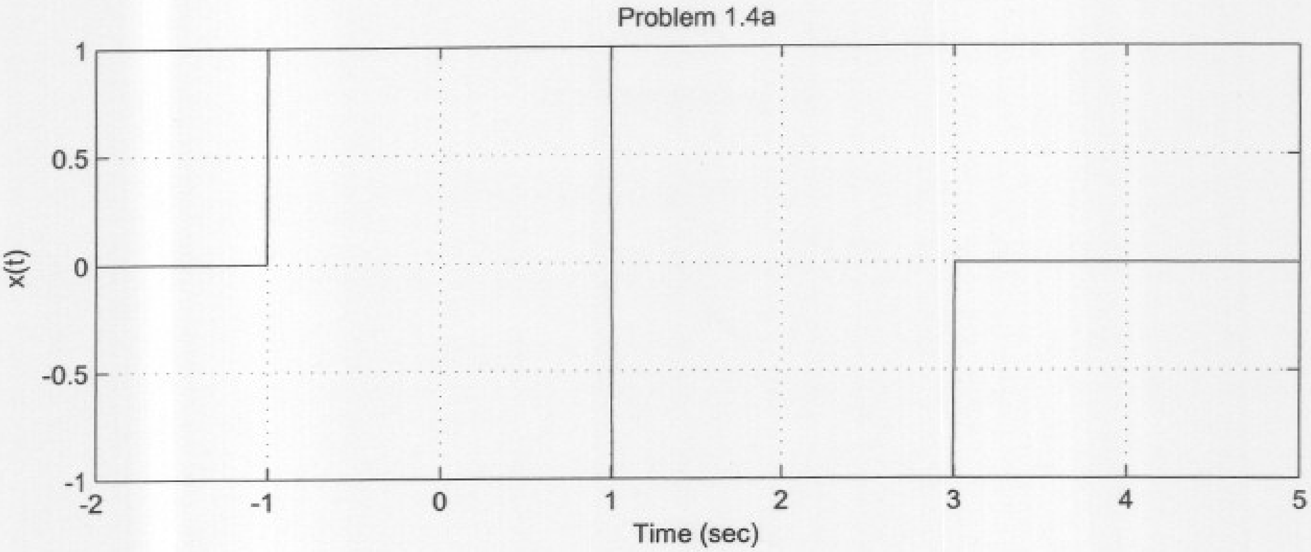
need $2T = 2g\pi$ $T = g\pi$
$3T = 2r\pi$ $T = r\frac{2\pi}{3} = g\pi$ $r = 3$ $g = 2$ $\boxed{\text{periodic } T_0 = 2\pi}$


Problem 1.1a
x(t)
2
1.5
1
0.5
0
-3
-2
-1
0
1
2
3
Time (sec)

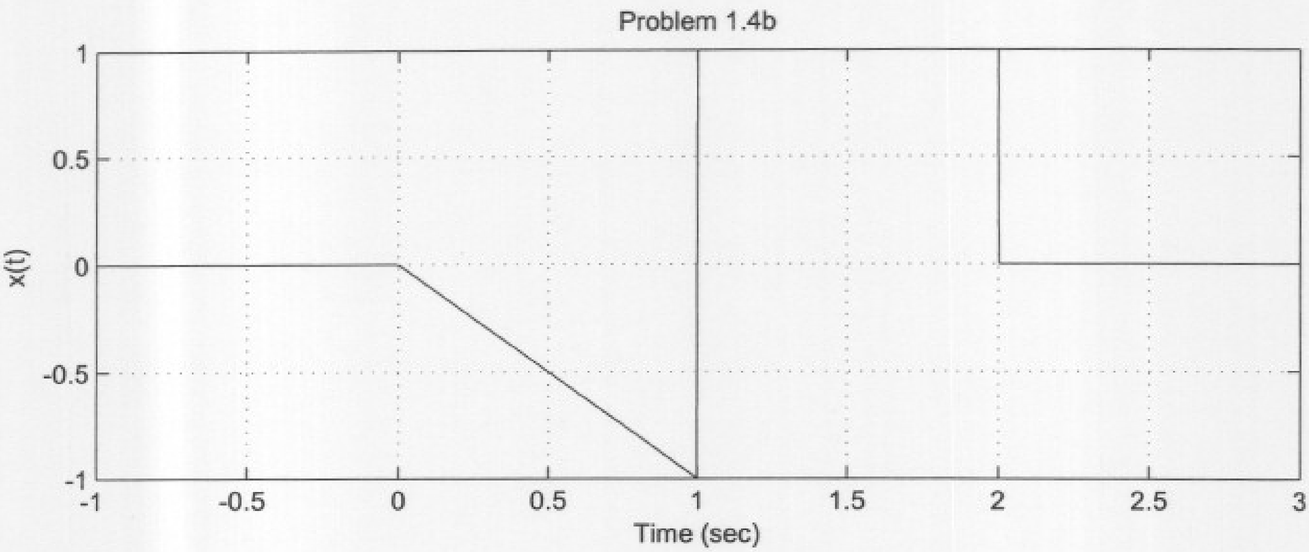

Problem 1.1b
x(t)
1
0.8
0.6
0.4
0.2
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)


Problem 1.1c
x(t)
6
5
4
3
2
1
0
-8
-6
-4
-2
0
2
4
6
8
Time (sec)

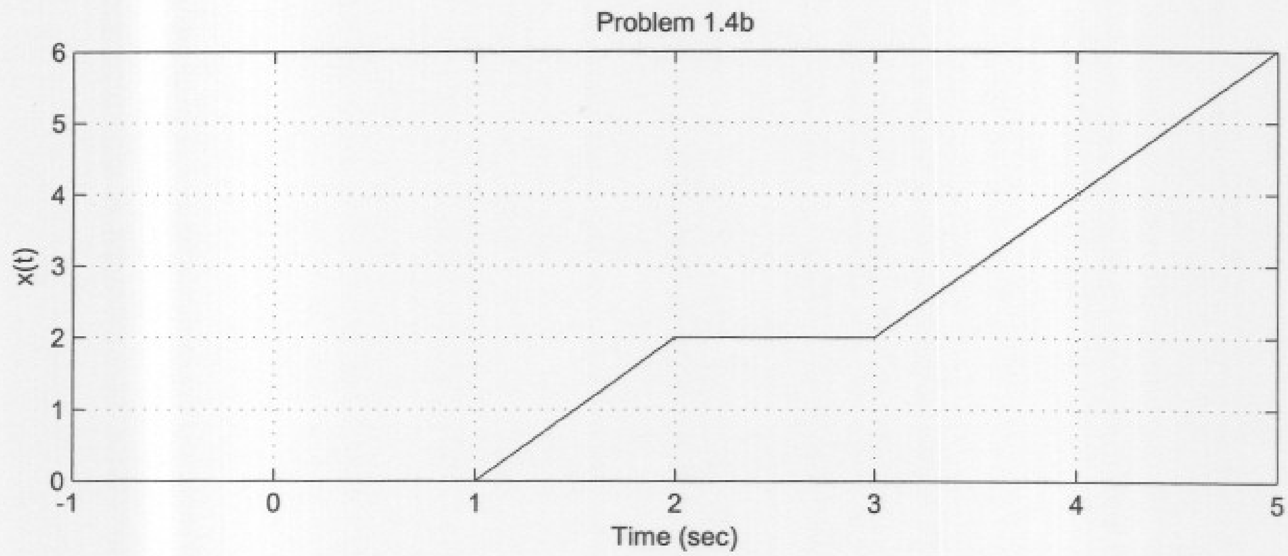

Problem 1.1e
x(t)
4
3
2
1
0
-3
-2
-1
0
1
2
3
Time (sec)



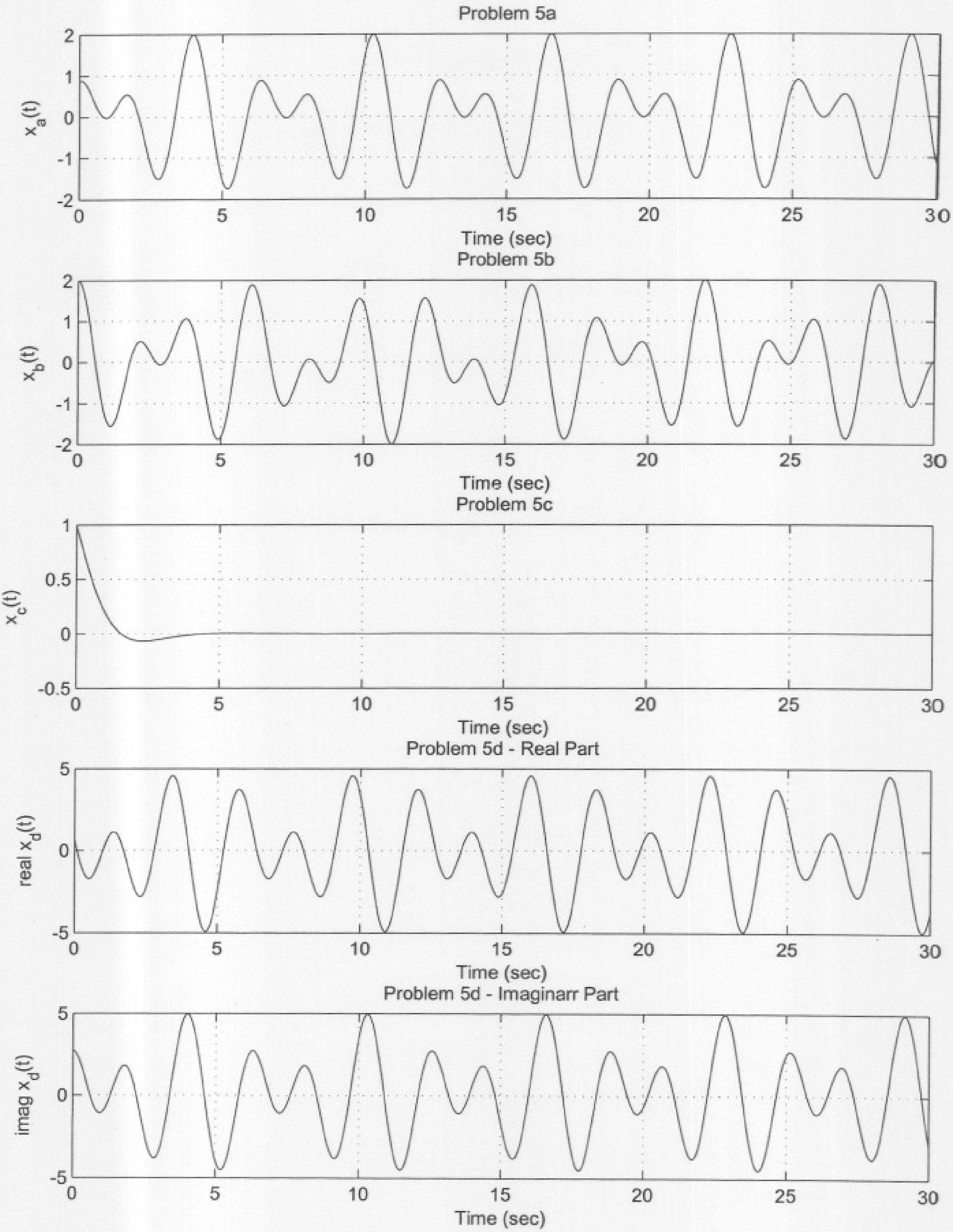
Problem 1.4a
1
0.5
0
-0.5
-1
x(t)
-2
-1
0
1
2
3
4
5
Time (sec)



Problem 1.4b
1
0.5
0
-0.5
-1
x(t)
-1
-0.5
0
0.5
1
1.5
2
2.5
3
Time (sec)



Problem 1.4b
6
5
4
3
2
1
0
x(t)
-1
0
1
2
3
4
5
Time (sec)


Problem 5a
x_a(t)
Time (sec)
Problem 5b
x_b(t)
Time (sec)
Problem 5c
x_c(t)
Time (sec)
Problem 5d - Real Part
real x_d(t)
Time (sec)
Problem 5d - Imaginarr Part
imag x_d(t)
Time (sec)
0
5
10
15
20
25
30