SYSTEM LINEARITY
Lab 3
Mario F. Simoni, Robert Throne, and Bruce Ferguson

Prelab:

Obtain the necessary components as listed below from the parts room, C117, and wire up the circuit on your breadboard before coming to lab.

Objectives

The purpose of this lab is to gain a better understanding of what LINEARITY means in reference to a real system. You will begin the lab by building a transistor amplifier circuit and measuring the input and output signals as you vary the amplitude of your input signal. You will then compare the results of your experiment to the "expected" outputs by plotting your results in Matlab. You will also explore linearity through the use of sinusoidal analysis of systems by both listening to the input and output sounds, and by looking at the input and output waveforms.

1. Linearity of an amplifier

This lab will use the following components:

        1-100 uF electrolitic capacitor
        1-Q2N3904 NPN BJT transistor
        Resistors (1 ea.) – 1K, 1.2K, 9.1K, 10K, 51K, and 300K

Build the common-emitter amplifer, shown in Figure 1 below, on your breadboard before coming to lab. The pins for the transistor are shown next to the circuit diagram.
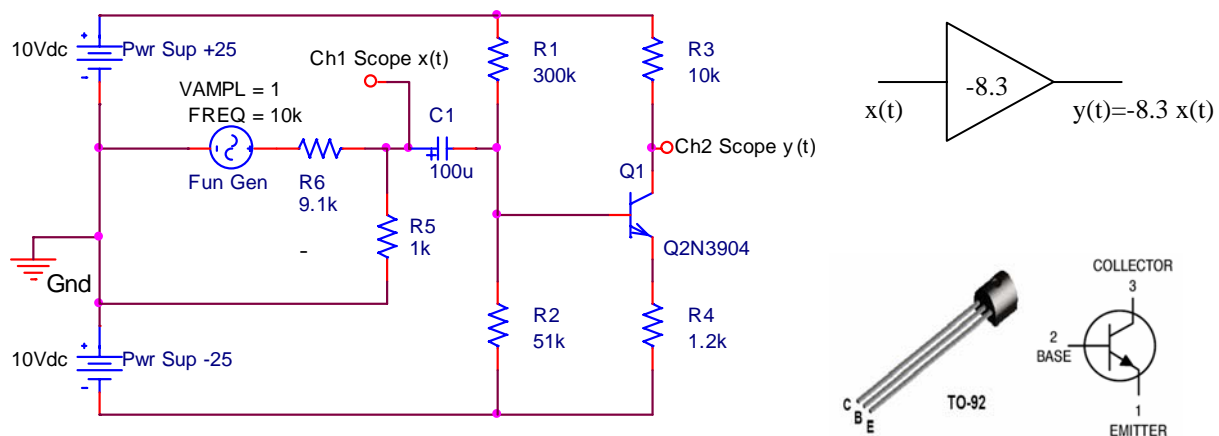

Figure 1: Circuit Diagram of the common-emitter amplifier. The pin layout for the transistor is shown to the right. The ideal block diagram representation for this amplifier is shown as the triangle in the upper right.

Ideally, this amplifier should have a gain of -8.3 regardless of the input signal's amplitude. In other words, in an ideal amplifier, if the input were to go to 100 V then the output should go to -

830 V. We know that this is not possible because our power supply only goes to 10 V. Saturation at the energy rails is a problem with any physical device because we cannot physically represent infinity. With transistor amplifiers the problem is even worse, because as the input gets larger, the nonlinearities of the transistors begin to affect the signal. When the input signal to the amplifier is "small", we can approximate the transistor with its small-signal model that is composed of linear elements. As long as the input signal stays small, the amplifier behaves like an ideal device. However, when the input to the amplifier is "large", the nonlinearities of the amplifier can be observed, and we have to use the large-signal model (exponential for BJT, square-law for FETs) of the transistor to understand the operation of the amplifier. "Large" and "small" are fuzzy terms and, as system designers, we would like to have a more quantitative definition for these values. With respect to the amplitude of the input signal, the amplifier's nonlinearities decrease the gain, called gain compression, and distort the shape of the output signal, called harmonic distortion. One way to quantitatively define "small" versus "large" signal is to measure the magnitude of the input voltage at which the gain begins to decrease, called the compression point. Another is to define an acceptable level of distortion and the amplitude threshold at which this distortion level occurs. You won't understand how to measure the distortion in the signal until the end of this quarter. This lab will focus on measuring the gain compression point while simply observing the distortion that occurs in the output due to the nonlinearities of the amplifier. The goal of the first part of this lab is to:
1. Measure the gain compression point for the amplifier
2. Record the output of the amplifier on the oscilloscope for both a "large" and "small" signal input to observe the distortion.

a) Build the amplifier as shown in the figure above. Hook up the power supplies and the function generator. Hook up the oscilloscope probes to the nodes labeled x(t) (the input to the amplifier) and y(t) (the output of the amplifier). Of special note when assembling and testing the amplifier:

1. Make sure the capacitor's positive node is connected as shown in the figure above.

2. The DC Bias points (node voltages relative to Gnd measured with the DMM set to DC Volts) of the amplifier should be **approximately**: V_{Base}=-7.4 V, $V_{Collector}$=-5.4 V, and $V_{Emitter}$=-8.1 V

3. Make sure you connect your power supplies with the right polarities.

4. The voltage divider connected to the function generator reduces the amplitude of the function generator by a factor of approximately 10. When doing your experiments, make sure you measure the input to the amplifier (on the positive node of the capacitor) not the output of the function generator.

5. Set the coupling of the oscilloscope to AC and the probe gains to 10:1 for each channel. Do not use autoscale because it will change your coupling back to DC.

6. Use the Acquire button on the oscilloscope to automatically measure the peak-peak voltages of both signals.

7. Turn on averaging (with 8 samples) in the oscilloscope to reduce the noise of the function generator.

Instructor Verification (see last page)

b) Set the function generator to be a sinusoid with frequency of 10 kHz. Start with the peak-peak amplitude of **x(t)**=50 mV. Measure the peak-peak voltage of both x(t) and y(t) as you sweep the amplitude of **x(t)** from 50mV to 490mV in 20mV increments (a total of 23 data points). NOTE: The the peak-peak amplitude of x(t) is different from the function generator because of the voltage divider. You will need to sweep the function generator's peak-peak amplitude from approximately 500mV to 4.9V in 200mV increments. Record your data in the table at the end of this handout and then enter the vectors into a Matlab m-file.

c) In the same m-file, plot the gain of the amplifier (vout/vin) for each input voltage amplitude. Your plot should look like the following figure. Include in your plot a line that indicates the ideal gain of your amplifier. The ideal gain can be found by averaging the measured gain for the first few "small-signal" data points.
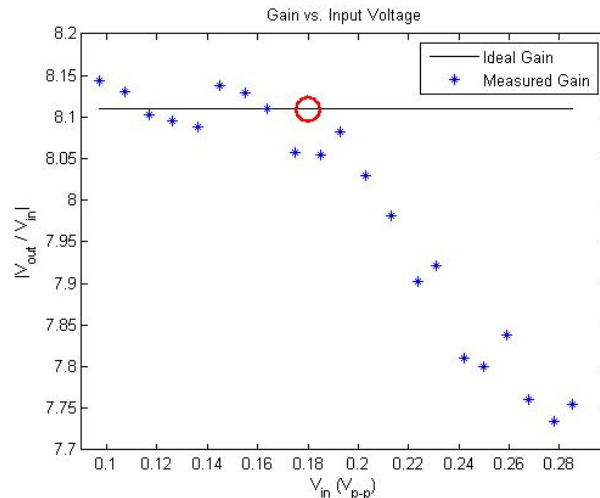

d) You can get a rough estimate of the gain compression point by looking at your plot. This point is defined as the input voltage at which the gain begins to decrease and is indicated by the red circle in the figure above. You should indicate your gain compression point similarly in your plot. Use the menu commands in the figure window to insert an ellipse.

e) In the same m-file, use the *polyfit* command in Matlab to fit a line to only the data **below** the gain compression point using x(t) as your x-coordinates and y(t) as your y-coordinates for the *polyfit* command.

f) Use the *figure()* command to assign figure(1) to the plot above and figure(2) for a new plot. In this new figure window, plot the measured output voltage versus the measured input voltages and the line that you generated from the previous step. You should get a figure that looks like the one below. This figure shows how your data compares to a purely linear amplifier. If done correctly, your data should deviate from the ideal line at the gain compression point. This figure is how the gain compression is typically displayed. Add axes labels and title to this figure and print it out to turn in.

Instructor Verification (see last page)

Measured Output vs. Input Voltage
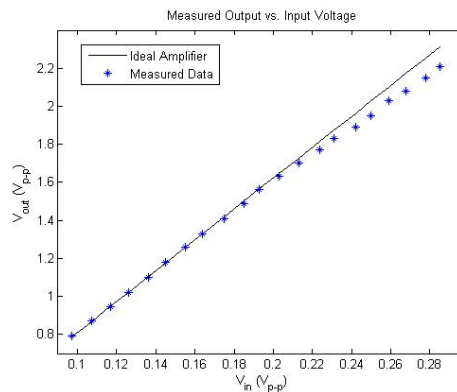
g) Now that you've found the gain compression point, you can quantitatively define "small" and "large" signal. Record a screen capture from the oscilloscope showing the output traces for the input and output voltages for both a "small-signal" value of x(t) and a "large-signal" value of x(t). Print out these screen captures to hand in with your lab. Indicate on the printouts where the nonlinearities of the amplifier cause distortion.

i) Also hand in your table with the recorded data files and the final m-file that includes your code for data analysis

Instructor Verification (see last page)

2. Experimentation with Sound, Small Signal Models

The MATLAB program **linear_systems_and_sounds.m** allows you to specify an input signal (lines 11-12) and system (line 24), and listen to how the sound changes as the signal passes through the system. We will use the sounds produced to try to identify the systems as linear or nonlinear. Recall from class that a system is linear if and only if an input signal at frequency f produces an output signal at frequency f. The phase angle and amplitude between the input and output may change, but the frequency cannot. Just think about how transfer functions affected periodic signals when you were using phasors to represent sinusoids- the transfer function could change the magnitude and phase but not the frequency of the input signal. *Note that we can only hear sounds when the speaker is vibrating.*

a) Using the code as is (i.e. with y=x), investigate how the frequency of the sinusoidal input signal affects the sound of the signal. Specifically, describe the sound you hear for input signal frequencies f_0 of 200, 1000, and 5000 Hz.. Note that you need to run the program, wait for the input sound (and graph) to appear, then hit enter and you will hear the output sound and see the output plotted in the lower panel.

b) Now change the system so the output is zero, i.e., $y = 0*x$ (this will make sure the y vector is the correct length). What do you hear for the system output, if anything?

c) Now change the system so the output is y=0*x+10; What do you hear? How fast (at what frequency) is the speaker vibrating?

d) For each of the mathematical models of systems in the following table, assume x is a periodic signal with frequency 200 Hz. For each of these models, determine if the mathematical model of the system is linear or not, and if you would classify the system models as linear or not based on both listening to and looking at the input and output. Discrete-time solutions for model (i) is included in the code, and is commented out. Uncomment the correct lines when you need to run these models.

e) For many system, such as transistors, we are really interested in **_small signal models_**. For these systems we only care about the behavior of the system about a set or bias point for an input with a small amplitude. For $x(t)$ small, we can use a Taylor series approximation of $y(x)$ around $x(t) = 0$ of the form

$$y(x) \approx y_0 + \nu x(t)$$

where y_0 and ν are determined by the Taylor series expansion of $y(x)$ about the point $x = 0$. If we subtract the bias point (y_0) from both sides of the equation and rename variables we have

$$y(x) - y_0 \equiv \Delta y(x) = \nu x(t) \quad .$$

This is now a description of a linear system between input $x(t)$ and output $\Delta y(x)$. This relationship is only approximately linear if the input $x(t)$ is sufficiently small that the Taylor series approximation is valid.

For the systems that are mathematically nonlinear, but appeared/sounded linear, determine the small signal linear approximation, that is, determine y_0 and ν.

Note that our input amplitude is 1, and hence something like $z = x/10$ would be considered small. The following formula's for Taylor series about the point $z = 0$ may be helpful.

$$y(z) = \left[a + bz\right]^c \approx a^c + ca^{c-1}bz \qquad y(z) = \sin(az) \approx az$$

As a check, for those systems where the small signal model is valid for this particular input signal , if you look at the plots from *linear_systems_and _sounds.m*, the output signal should be centered at y_o and should oscillate between $y_o + \nu$ and $y_o - \nu$.

System	Mathematical Model	Mathematical Model Linear? (Y/N)	Sounds/Looks Linear?(Y/N)	Small Signal Model (if valid)
a	$y(t) = \dfrac{1}{2 + x(t)}$			
b	$y(t) = \dfrac{1}{2 + \dfrac{x(t)}{10}}$			
c	$y(t) = \sin(2x(t))$			
d	$y(t) = \sin\left(\dfrac{x(t)}{10}\right)$			
e	$y(t) = \lvert x(t) \rvert$			
f	$y(t) = e^{x(t)}$			
g	$y(t) = \log(1.1 + x(t))$			
h	$y(t) = \left(10.1 + \dfrac{x(t)}{10}\right)^{0.2}$			
i	$\dot{y}(t) + ay(t) = bx^2(t)$			

Instructor Verification (see last page)

Report
For this lab, turn in all of your Matlab code and plots in addition to this verification sheet.

Lab 03 Introduction to MATLAB
Instructor Verification Sheet

Name _____ Date of Lab: _____

Part 1(a) Demonstrate that your circuit works appropriately.

Verified: _____ Date/Time: _____

Part 1(f) Present your plots.

Verified: _____ Date/Time: _____

Part 1(i) Present your plots.

Verified: _____ Date/Time: _____

Part 2(e) Present your table.

Verified: _____ Date/Time: _____

x(t) (mVp-p)	y(t) (mVp-p)
50	
70	
90	
110	
130	
150	
170	
190	
210	
230	
250	
270	
290	
310	
330	
350	
370	
390	
410	
430	
450	
470	
490	