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ECE 300
Signals and Systems

Exam 3
19 MAY, 2009

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam.

Problem 1	________ / 45
Problem 2	________ / 25
Problem 3	________ / 30

Exam 3 Total Score: _______ / 100

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1. (45 points) Random Fourier transform problems.

a) **(10 points)** If $x(t) = \dfrac{2}{2 + j(t-2)}$, determine $X(\omega)$

b) **(10 points)** If $X(\omega) = rect\left(\dfrac{2\omega - 2}{3}\right)$, determine $x(t)$

c) **(10 points)** If $x(t)$ and $y(t)$ are related through the relationship
$\dot{y}(t) = x(t-b) \star e^{-t}u(t-c)$ determine the transfer function for the system.

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d) **(10 points)** If we have the Fourier transform pair $x(t) \leftrightarrow X(\omega)$, use the definition of the Fourier transform or inverse Fourier transform to show $tx(t) \leftrightarrow j\frac{dX(\omega)}{d\omega}$ if the Fourier transform of $tx(t)$ exists. In this problem you are to prove this relationship from the Fourier transform (or inverse Fourier transform) definitions.

e) **(5 points)** If $X(\omega) = \frac{1}{T}\text{sinc}\left(\frac{2\omega T}{\pi}\right)$ determine the location of the first nulls.

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2. (25 points) The periodic function $x(t)$ is defined over one period ($T_0 = 4$ seconds) as

$$x(t) = \begin{cases} 2 & -2 \le t \le 0 \\ 0 & 0 \le t \le 2 \end{cases}$$

Determine the complex Fourier series coefficients, c_k.

Be sure to simplify your answer as much as possible and use a <u>*sinc*</u> *function if appropriate. Recall that* $1 = e^0$.

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3. (30 points) Assume $x(t) = 4\,\mathrm{sinc}\left[\frac{1}{\pi}(t-2)\right]\cos(4(t-2))$ is the input to an LTI system with transfer function $H(\omega) = \begin{cases} \frac{1}{\pi}e^{-j\omega 3} & |\omega| < 4 \\ 0 & else \end{cases}$

a) Determine the Fourier transform $X(\omega)$ of $x(t)$

b) Accurately sketch the magnitude and phase of $X(\omega)$

c) Determine the system output $y(t)$

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Some Potentially Useful Relationships

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\mathrm{rect}\left(\frac{t-t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$