ECE 300
Signals and Systems

Exam 1
6 April, 2009

NAME ____

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam.

Problems 1-4________ / 16
Problem 5 ________ / 29
Problem 6 ________ / 25
Problem 7 ________ / 25

Exam 1 Total Score: _______ / 95

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Problems 1-4 are worth 4 points each.

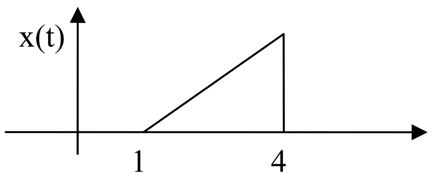
1. Which of the following statements is the best simplification of: $\int_{-2}^{t} x(\lambda - t_0)\delta(\lambda)d\lambda$

a) 0 b) $x(t-t_0)\delta(t)$ c) $x(-t_0)u(t)$ d) $x(-t_0)\delta(t)$ e) none of these

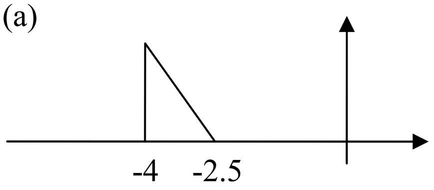
2 . The the signal $x(t) = \cos(t) - j\sin(t)$ is

a) and energy signal b) a power signal c) neither energy nor power

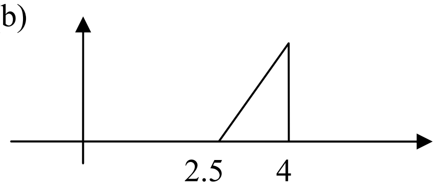
3. Given x(t) below, which of the plots labeled (a) − (d) represents $x(2(-t-2))$.



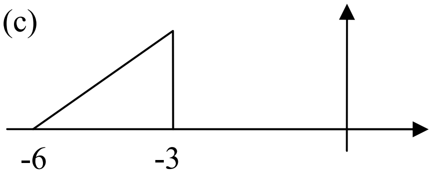
(a)



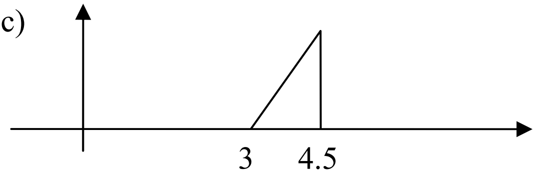
(b)



(c)



(c)



4. The signal $x(t) = \cos(4\pi t + \pi/2) + e^{j6\pi t} + 1$ is

a) not periodic
b) periodic with fundamental period 6π seconds
c) periodic with fundamental period 1 second
d) periodic with fundamental period 3/2 seconds
e) none of the above

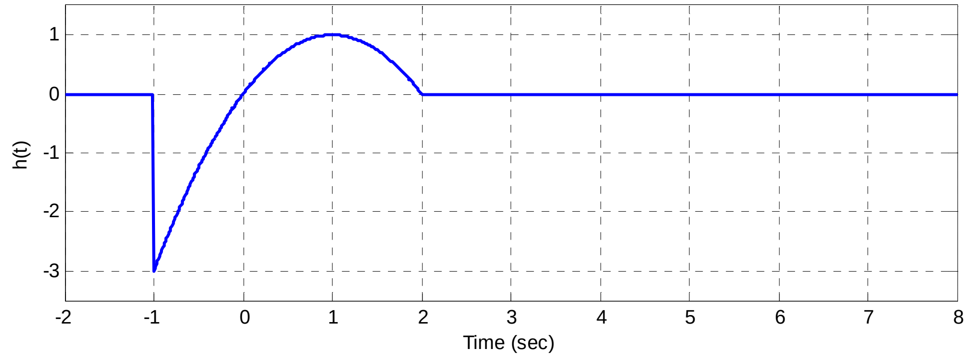
5. Graphical Convolution (29 points)

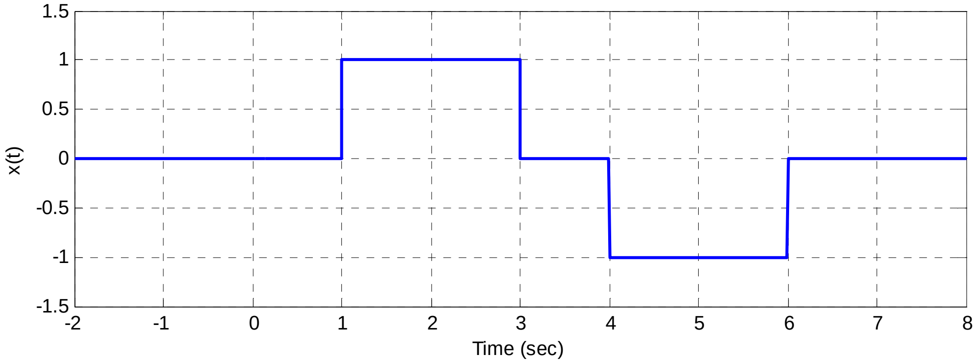
Consider a noncausal linear time invariant system with impulse response given by

$$h(t) = [1-(t-1)^2][u(t+1)-u(t-2)]$$

The input to the system is given by

$$x(t) = u(t-1)-u(t-3)-u(t-4)+u(t-6)$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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6. Impulse Response (25 points)
For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions.*

a) $y(t) = \int_{-\infty}^{t-2} e^{-(t-\lambda)} x(\lambda+1)\,d\lambda$

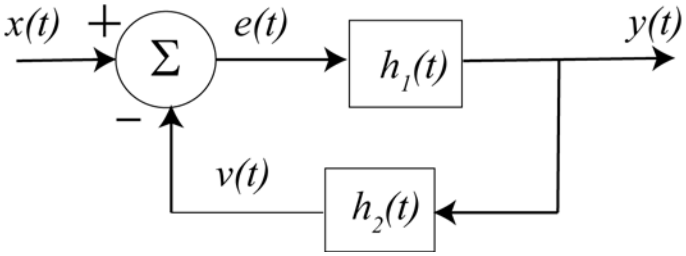
b) $2\dot{y}(t) + y(t) = x(t-1)$

c) For the interconnected feedback system below, determine a relationship between the input $x(t)$ and the output $y(t)$ in terms of $h_1(t)$ and $h_2(t)$. Your final answer will be of the form

$$y(t)*[\delta(t)+A(t)] = x(t)*[B(t)]$$

You need to determine $A(t)$ and $B(t)$.

Hints: $e(t) = x(t) - v(t)$ and $v(t) = y(t)*h_2(t)$



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7. System Properties (25 points)

a) Fill in the following table with a Y (Yes) or N (No). Only your responses in the table will be graded, not any work. Assume $x(t)$ is the system input and $y(t)$ is the system output. Also assume we are looking at all times (positive and negative times).

System	Linear ?	Time-Invariant?	Memoryless?	Causal?
$\dot{y}(t)+t^2y(t)=x(t+1)$				
$y(t)=x\left(-\frac{t}{2}\right)$				
$y(t)=x(t)+2$				
$y(t)=\vert x(t)\vert$				

b) For the system described below, determine the value of "c" that will make the system time-invariant. Use a formal technique such as we used in class (and on the homework) and justify your answer. *You will be graded more on your method of arriving at an answer than the answer itself!*

$$y(t)=e^{t}\int_{c}^{t}e^{-\lambda}x(\lambda)d\lambda$$

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Some Potentially Useful Relationships

$$E_\infty = \lim_{T\to\infty}\int_{-T}^{T}\left|x(t)\right|^2 dt = \int_{-\infty}^{\infty}\left|x(t)\right|^2 dt$$

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left|x(t)\right|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\mathrm{rect}\left(\frac{t-t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$