**ECE 300**
**Signals and Systems**
Homework 9

**Due Date:** The beginning of lab, May 15, 2009

**Problems**

**1.** Determine the transfer function $H(\omega)$ that would produce the following input /output relationships. Simplify your answers as much as possible.

a) $y(t) = a\dot{x}(t-b)$

b) $y(t) = ax(t+b) + ax(t-b)$

c) $\dot{y}(t) = x(t) * e^{-t}u(t-b)$

**2.** Using the ***duality property***, find the corresponding Fourier transform for the following:

a) $g(t) = \text{sinc}^2(Bt)$

b) $g(t) = \text{sinc}(Wt)$

c) $g(t) = \delta(t)$

d) $g(t) = \cos(\omega_0 t)$

**Do not** just look up the pairs from the table (though you can use any other pairs except the one you are trying to find).

**3.** Consider a linear time invariant system with transfer function given by

$$H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \le 2 \\ 0 & else \end{cases}$$

with input $x(t) = \frac{8}{\pi}\text{sinc}^2\left(\frac{2(t-1)}{\pi}\right)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

d) Determine $y(t)$, the output of the system.

*Answer* $y(t) = \frac{20}{\pi}\text{sinc}\left[\frac{2}{\pi}(t-3)\right] + \frac{10}{\pi}\text{sinc}^2\left[\frac{1}{\pi}(t-3)\right]$

**4.** Consider a linear time invariant system with impulse response given by $h(t)=\frac{1}{2\pi}\text{sinc}\left(\frac{\text{t-2}}{2\pi}\right)$ with input $x(t)=\frac{4}{\pi}\text{sinc}\left(\frac{2\text{t}}{\pi}\right)\cos(t)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Determine the energy in $x(t)$

d) Determine $H(\omega)$.

e) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

f) Determine $y(t)$, the output of the system.

g) Determine the energy in $y(t)$.

**5.** Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t)=5\,\text{sinc}\left(\frac{t}{0.002}\right)+5\,\text{sinc}\left(\frac{t}{0.001}\right)$$ *Answer* 56%

**6.** In this problem we will go over some of the Fourier series and Fourier transform results we need to understand impulse sampling.

a) Assume we have periodic function $p(t)$ with Fourier series representation

$$p(t)=\sum_{k=-\infty}^{k=\infty}c_k e^{jk\omega_0 t}$$

and we construct the function $x_s(t)=x(t)p(t)$. Show that in the frequency domain we have

$$X_s(\omega)=\sum_{k=-\infty}^{k=\infty}c_k X(\omega-k\omega_0)$$

b) Show that the Fourier series for the (periodic) impulse train

$$p(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT_0)$$

Is given by

$$p(t) = \sum_{k=-\infty}^{k=\infty} \frac{1}{T_0} e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}$$

c) Combine the previous two parts to show

$$X_s(\omega) = \sum_{k=-\infty}^{k=\infty} \frac{1}{T_0} X(\omega - k\omega_0)$$

**7. (Matlab)** This problem is a continuation of Problem 6, but you only need to use the results of that problem. There are three important things you should know about impulse sampling

- If the original signal $x(t)$ is sampled with an impulse train with period $T_0$ (the time between samples is $T_0$) then the spectrum of the original signal, $X(f)$ (or $X(\omega)$), will be replicated every $f_0 = \frac{1}{T_0}$ Hz (or $\omega_0$ radians/sec) in the spectrum of the sampled signal, $X_s(f)$ (or $X_s(\omega)$)
- The replicated spectra $X(f)$ (or $X(\omega)$) will be scaled by $\frac{1}{T_0}$
- If we want to recover the original signal from the sampled signal, we need to be able to isolate one instance of the original signal's spectrum by lowpass filtering.

The Matlab routine **impulse_sampling.m** illustrates the effects of sampling the signal

$$x(t) = \text{sinc}^2\left(\frac{t}{0.0002}\right)$$

using a sampling rate of $f_0 = 20,000Hz$ and then filtering to try and reconstruct the original signal. You do not need to understand what this routine is doing, you will only be modifying it a bit and trying to understand what is happening. *Run the routine as it is for parts b, c, and d and turn in this plot.*

a) Show that for this function $X(\omega) = 0.0002\Lambda\left(\frac{\omega}{(2\pi)5000}\right)$

b) Verify that the spectrum of the replicated signal $X(\omega)$ has the correct bandwidth

c) Verify that the replicated signal $X_s(\omega)$ has maximum amplitude equal to $\frac{1}{T_0}\max\{|X(\omega)|\}$

d) Verify that the signal is replicated every $f_0 = 20,000Hz$

The bottom panel (and the second figure) shows the original signal and the signal we tried to reconstruct after sampling. If our sampling is effective we should be able to reconstruct the original signal.

e) Modify the lowpass filter (lines 59 and 60) so that we get a reasonably good approximation of the original signal. *Turn in your plots.*

f) Modify the sampling rate (line 31) so you are sampling at 15 kHz, 10 KHz, and then 7.5 kHz. *Turn in your plots.* Notice where the replicated signals are located. Are you able to reconstruct the original signal for these sampling rates?

**8. (Matlab/Prelab Problem)** A Butterworth filter has the property that it is maximally flat in the *passband*. An nth order Butterworth filter has the magnitude squared response

$$|H(\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_p}\right)^{2n}}$$

where $\omega_p$ is the *passband frequency.* At this frequency the power has been reduced by one half or 3 dB,

$$|H(\omega_p)|^2 = \frac{1}{1+\left(\frac{\omega_p}{\omega_p}\right)^{2n}} = \frac{1}{2} \quad \text{or} \; 10\log_{10}|H(\omega_p)|^2 = 10\log_{10}\left(\frac{1}{2}\right) = -3dB$$

To determine the required order of a filter we often look at the desired *stopband frequency*, $\omega_s$. Usually we want to indicate the minimum required power difference between the passband and the stopband, $\Delta$. $\Delta$ is the *rejection.*Hence we have

$$\Delta = 20\log_{10}|H(0)| - 20\log_{10}|H(\omega_s)|$$

or

$$\Delta = -10\log_{10}\frac{1}{1+\left(\frac{\omega_s}{\omega_p}\right)^{2n}}$$

The ratio $\frac{\omega_s}{\omega_p}$ is called the *transition ratio.*

a) Show that we can write

$$n = \frac{\ln\left(10^{\frac{\Delta}{10}} - 1\right)}{2\ln\left(\frac{\omega_s}{\omega_p}\right)}$$

Note that *n must be an integer*, so we always round up (to the next larger integer).

b) For $\omega_p = 10$ rad/sec, $\omega_s = 20$ rad/sec, and $\Delta = 18$ dB, determine the required Butterworth filter order for this filter. (Remember it must be an integer). Using the Table at the end of this problem, plot the Bode plot of your Butterworth filter and verify that all frequencies $\omega > \omega_s$ have magnitude (power) less than $\Delta_{max}$ .Matlab's **tf** command and **Bode** commands will be really useful here. Note that you can click on the curve on the Bode plot to read it more accurately. Turn in your plot.

c) Matlab's command **r = pole(H)**, where H is the transfer function of the Butterworth filter, returns the poles of the transfer function in the array **r**. Using Matlab's commands **abs** and **angle,** relate the magnitude of the poles to $\omega_p$, then plot the pole locations in the complex plane on a circle with radius $\omega_p$. (Note that **angle** returns angles in radians, and you probably want angle in degrees.) Note that the pole locations are all separated by an angle $\theta$. What is this angle?

d) For $\omega_p = 15$ rad/sec, $\omega_s = 35$ rad/sec, and $\Delta = 28$ dB, determine the required Butterworth filter order for this filter. (Remember it must be an integer). Using the Table at the end of this problem, plot the Bode plot of your Butterworth filter and verify that all frequencies $\omega > \omega_s$ have magnitude (power) less than $\Delta_{max}$ . Matlab's **tf** command and **Bode** commands will be really useful here. Turn in your plot.

e) Matlab's command **r = pole(H)**, where H is the transfer function of the Butterworth filter, returns the poles of the transfer function in the array **r**. Using Matlab's commands **abs** and **angle,** relate the magnitude of the poles to $\omega_p$, then plot the pole locations in the complex plane on a circle with radius $\omega_p$. (Note that **angle** returns angles in radians, and you probably want angle in degrees.) Note that the pole locations are all separated by an angle $\theta$. What is this angle?

| $n$ | denominator(s) |
|---|---|
| 1 | $\frac{s}{\omega_p}+1$ |
| 2 | $\left(\frac{s}{\omega_p}\right)^2+1.414\left(\frac{s}{\omega_p}\right)+1$ |
| 3 | $\left(\frac{s}{\omega_p}\right)^3+2\left(\frac{s}{\omega_p}\right)^2+2\left(\frac{s}{\omega_p}\right)+1$ |
| 4 | $\left(\frac{s}{\omega_p}\right)^4+2.6131\left(\frac{s}{\omega_p}\right)^3+3.4142\left(\frac{s}{\omega_p}\right)^2+2.6131\left(\frac{s}{\omega_p}\right)+1$ |
| 5 | $\left(\frac{s}{\omega_p}\right)^5+3.2361\left(\frac{s}{\omega_p}\right)^4+5.2361\left(\frac{s}{\omega_p}\right)^3+5.2361\left(\frac{s}{\omega_p}\right)^2+3.2361\left(\frac{s}{\omega_p}\right)+1$ |

**Table 1**: Denominators of Buterworth filter for filter orders 1-5. The numerator for the Butterworth filter is 1.

(#1)

(a) $y(t) = a\dot{x}(t-b)$

$Y(\omega) = a j\omega X(\omega) e^{-j\omega b}$ $\quad \boxed{H(\omega) = j\omega a e^{-j\omega b}}$

(b) $y(t) = ax(t+b) + ax(t-b)$

$Y(\omega) = a X(\omega) e^{j\omega b} + a X(\omega) e^{-j\omega b}$

$= a\left[e^{j\omega b} + e^{-j\omega b}\right] X(\omega)$

$= 2a\cos(\omega b)\, X(\omega)$ $\quad \boxed{H(\omega) = 2a\cos(\omega b)}$

(c) $y(t) = x(t) * e^{-t}u(t-b)$

$j\omega Y(\omega) = X(\omega)\,\mathcal{F}\{e^{-t}u(t-b)\}$

$$\int_{-\infty}^{\infty} e^{-t}u(t-b)\, e^{-j\omega t}\,dt = \int_{b}^{\infty} e^{-(1+j\omega)t}\,dt$$

$$= \frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\Bigg|_{b}^{\infty} = \frac{e^{-(1+j\omega)b}}{1+j\omega} = \frac{e^{-b}e^{-j\omega b}}{1+j\omega}$$

or $e^{-t}u(t-b) = e^{-(t-b)}e^{-b}u(t-b) = e^{-b}e^{-(t-b)}u(t-b)$

$$\mathcal{F}\{e^{-b}e^{-(t-b)}u(t-b)\} = \frac{e^{-b}e^{-j\omega b}}{1+j\omega}$$

so $\boxed{H(\omega) = \frac{1}{j\omega}\,\frac{e^{-b}e^{-j\omega b}}{1+j\omega}}$

#2

a) $g(t) = \text{sinc}^2(Bt)$

from the table,

$$g_1(t) = \Lambda\left(\frac{t}{W}\right) \longleftrightarrow G_1(\omega) = W\,\text{sinc}^2\left(\frac{W}{2\pi}\omega\right)$$

by duality

$$g_2(t) = G_1(t) = W\,\text{sinc}^2\left(\frac{W}{2\pi}t\right) \longleftrightarrow G_2(\omega) = 2\pi g_1(-\omega) = 2\pi\,\Lambda\left(\frac{\omega}{W}\right)$$

$B = \frac{W}{2\pi}$ so $W = 2\pi B$

$$g_2(t) = 2\pi B\,\text{sinc}^2(Bt) \longleftrightarrow G_2(\omega) = 2\pi\,\Lambda\left(\frac{\omega}{2\pi B}\right)$$

or $\boxed{\text{sinc}^2(Bt) \longleftrightarrow \frac{1}{B}\,\Lambda\left(\frac{\omega}{2\pi B}\right)}$

b) $g(t) = \text{sinc}(Wt)$

from the table

$$g_1(t) = \text{rect}\left(\frac{t}{T}\right) \longleftrightarrow G_1(\omega) = T\,\text{sinc}\left(\frac{T}{2\pi}\omega\right)$$

by duality

$$g_2(t) = G_1(t) = T\,\text{sinc}\left(\frac{T}{2\pi}t\right) \longleftrightarrow G_2(\omega) = 2\pi g_1(-\omega) = 2\pi\,\text{rect}\left(\frac{\omega}{T}\right)$$

$W = \frac{T}{2\pi}$ or $T = 2\pi W$

$$g_2(t) = 2\pi W\,\text{sinc}(Wt) \longleftrightarrow 2\pi\,\text{rect}\left(\frac{\omega}{2\pi W}\right)$$

$$\boxed{\text{sinc}(Wt) \longleftrightarrow \frac{1}{W}\,\text{rect}\left(\frac{\omega}{2\pi W}\right)}$$

(c) $g(t) = \delta(t)$

from the table

$$g_1(t) = 1 \longleftrightarrow G_1(\omega) = 2\pi\,\delta(\omega)$$

by duality

$$g_2(t) = G_1(t) = 2\pi\,\delta(t) \longleftrightarrow G_2(\omega) = 2\pi\, g_1(-\omega) = 2\pi$$

$$\boxed{\delta(t) \longleftrightarrow 1}$$

(d) $g(t) = \cos(\omega_0 t)$

for $G_1(\omega) = \cos(T\omega) = \frac{1}{2}e^{j\omega T} + \frac{1}{2}e^{-j\omega T} \longleftrightarrow g_1(t) = \frac{1}{2}\delta(t+T) + \frac{1}{2}\delta(t-T)$

$$\text{so } g_1(t) = \frac{1}{2}\left[\delta(t+T) + \delta(t-T)\right] \longleftrightarrow G_1(\omega) = \cos(T\omega)$$

by duality

$$g_2(t) = G_1(t) = \cos(Tt) \longleftrightarrow G_2(\omega) = 2\pi\, g_1(-\omega) = \pi\,\delta(-\omega+T) + \pi\,\delta(-\omega-T)$$

$T = \omega_0$

$$\boxed{\cos(\omega_0 t) \longleftrightarrow \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)}$$

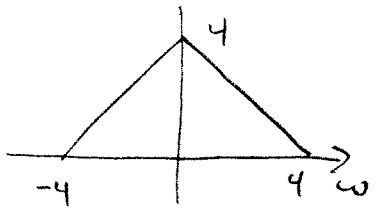
$\delta()$ is an <u>even</u> function

#3 $x(t) = \frac{8}{\pi}\text{sinc}^2\left(2\frac{(t-1)}{\pi}\right)$ $\quad H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \le 2 \\ 0 & \text{else} \end{cases}$

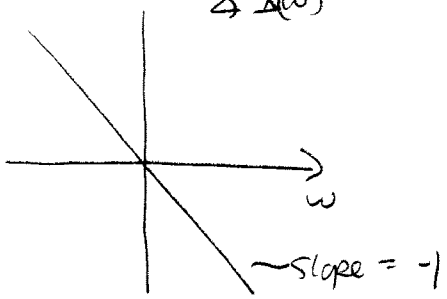
a) for $x(t) = \text{sinc}^2\left(\frac{2t}{\pi}\right) \longleftrightarrow X(\omega) = \frac{\pi}{2}\Lambda\left(\frac{\omega}{4}\right)$

so $\boxed{X(\omega) = 4\Lambda\left(\frac{\omega}{4}\right)e^{-j\omega}}$

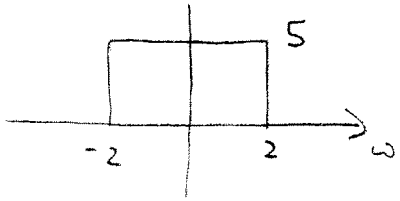
b) $|X(\omega)|$



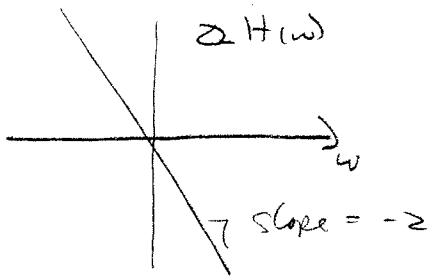

$\measuredangle X(\omega)$



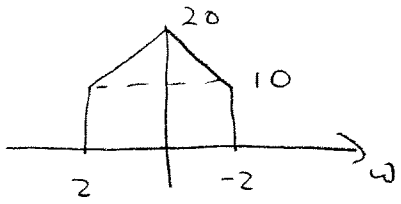

c) $|H(\omega)|$



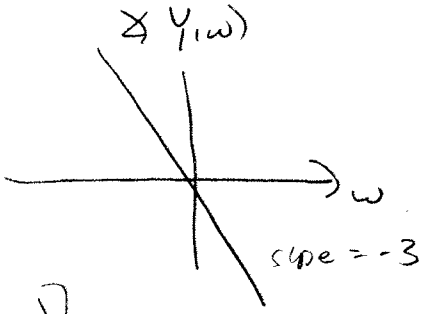

$\measuredangle H(\omega)$




d) $|Y|$




$\measuredangle Y(\omega)$




$$Y(\omega) = \left[10\,\text{rect}\left(\frac{\omega}{4}\right) + 10\Lambda\left(\frac{\omega}{2}\right)\right]e^{-j3\omega}$$

$$\text{rect}\left(\frac{\omega}{2\pi W}\right) \longleftrightarrow W\text{sinc}(Wt) \qquad W = \frac{2}{\pi}$$

$$\Lambda\left(\frac{\omega}{2\pi B}\right) \longleftrightarrow B\text{sinc}^2(Bt) \qquad B = \frac{1}{\pi}$$

$$\boxed{y(t) = \frac{20}{\pi}\text{sinc}\left(\frac{2}{\pi}(t-3)\right) + \frac{10}{\pi}\text{sinc}^2\left(\frac{1}{\pi}(t-3)\right)}$$

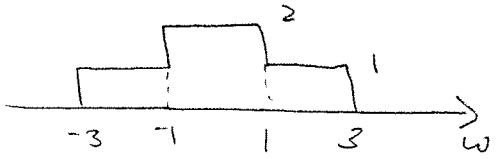
#4 $h(t) = \frac{1}{2\pi}\text{sinc}\left(\frac{t-2}{2\pi}\right)$ $x(t) = \frac{4}{\pi}\text{sinc}\left(\frac{2t}{\pi}\right)\cos(t)$

a) For $x_1(t) = \text{sinc}\left(\frac{2}{\pi}t\right)$ $\quad X_1(\omega) = \frac{\pi}{2}\,\text{rect}\left(\frac{\omega}{2\pi\cdot\left(\frac{2}{\pi}\right)}\right) = \frac{\pi}{2}\,\text{rect}\left(\frac{\omega}{4}\right)$

For $x_2(t) = \frac{4}{\pi}x_1(t)$ $\quad X_2(\omega) = 2\,\text{rect}\left(\frac{\omega}{4}\right)$

For $x_3(t) = x_2(t)\cos(t)$ $\quad \boxed{X_3(\omega) = \text{rect}\left(\frac{\omega+1}{4}\right) + \text{rect}\left(\frac{\omega-1}{4}\right)}$

b) $|X(\omega)|$




$\measuredangle X(\omega) = 0$

c) $E_x = \frac{1}{2\pi}\left[\int_{-3}^{-1} 1^2\,d\omega + \int_{-1}^{1} 2^2\,d\omega + \int_{1}^{3} 1^2\,d\omega\right] = \frac{1}{2\pi}\left[2 + 4\cdot 2 + 2\right] = \boxed{\frac{6}{\pi} = E_x}$

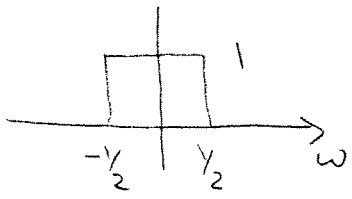
d) $h(t) = \frac{1}{2\pi}\text{sinc}\left(\frac{t-2}{2\pi}\right)$

For $h_1(t) = \text{sinc}\left(\frac{t}{2\pi}\right)$ $\quad H_1(\omega) = 2\pi\,\text{rect}\left(\frac{\omega}{2\pi\cdot\left(\frac{1}{2\pi}\right)}\right) = 2\pi\,\text{rect}(\omega)$

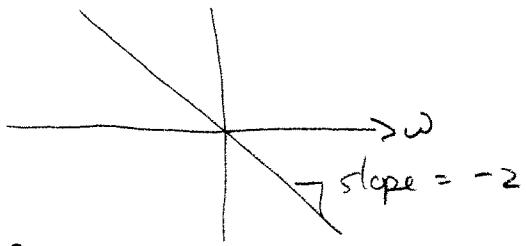
For $h_2(t) = \frac{1}{2\pi}h_1(t)$ $\quad H_2(\omega) = \text{rect}(\omega)$

For $h_3(t) = h_2(t-2)$ $\quad H_3(\omega) = e^{-j2\omega}H_2(\omega) = \boxed{\text{rect}(\omega)e^{-j2\omega} = H(\omega)}$

e) $|H(\omega)|$




$\measuredangle H(\omega)$




f) $Y_1(\omega) = 2\,\text{rect}(\omega)e^{-j2\omega}$

$\boxed{y_1(t) = \frac{1}{\pi}\text{sinc}\left(\frac{t-2}{2\pi}\right)}$

g) $E_y = \frac{1}{2\pi}\int_{-\frac{1}{2}}^{\frac{1}{2}} 2^2\,d\omega = \frac{4}{2\pi} = \boxed{\frac{2}{\pi} = E_y}$

#5

$$x(t) = 5\,\mathrm{sinc}\left(\frac{t}{0.002}\right) + 5\,\mathrm{sinc}\left(\frac{t}{0.001}\right)$$

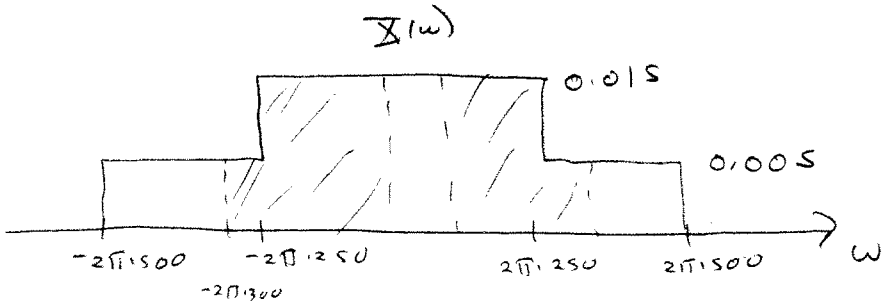
compute the % of energy between 100 and 300 Hz

for $\mathrm{sinc}(Wt) \leftrightarrow \frac{1}{W}\,\mathrm{rect}\left(\frac{\omega}{2\pi W}\right)$

here $W = \frac{1}{0.002} = 500$ or $W = \frac{1}{0.001} = 1000$

$$X(\omega) = \frac{5}{500}\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 500}\right) + \frac{5}{1000}\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 1000}\right)$$

$$= 0.01\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 500}\right) + 0.005\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 1000}\right)$$




$$E_{total} = \frac{1}{2\pi}\int_{-2\pi\cdot 500}^{2\pi\cdot 500} |X(\omega)|^2 d\omega = \frac{1}{2\pi}\left[2\cdot\int_{-2\pi\cdot 500}^{-2\pi\cdot 250}(0.005)^2 d\omega + 2\int_{-2\pi\cdot 250}^{0}(0.015)^2 d\omega\right]$$

$$= \frac{1}{2\pi}\left[2\cdot(2\pi\cdot 250)(0.005)^2 + 2\cdot(2\pi\cdot 250)(0.015)^2\right]$$

$$= 2\cdot 250\cdot(0.005)^2 + 2\cdot 250\cdot(0.015)^2 = \boxed{0.125 = E_{total}}$$

$$E_{Band} = \frac{1}{2\pi}\left[2\cdot\int_{-2\pi\cdot 300}^{-2\pi\cdot 250}(0.005)^2 d\omega + 2\int_{-2\pi\cdot 250}^{-2\pi\cdot 100}(0.015)^2 d\omega\right]$$

$$= \frac{1}{2\pi}\left[2\cdot(2\pi\cdot 50)(0.005)^2 + 2(2\pi\cdot 150)(0.015)^2\right]$$

$$= 2\cdot(50)(0.005)^2 + 2(150)(0.015)^2 = \boxed{0.070 = E_{Band}}$$

$$\text{ratio} = \frac{0.070}{0.125} = 0.560$$

56%

#6

(a) $p(t) = \sum c_k e^{jk\omega_0 t}$

$x_s(t) = x(t)\, p(t)$

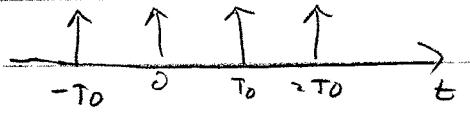
$X_s(\omega) = \frac{1}{2\pi} X(\omega) * P(\omega)$ $\qquad P(\omega) = \sum c_k \mathcal{F}\{e^{jk\omega_0 t}\}$

$= \sum c_k [2\pi \delta(\omega - k\omega_0)]$

$X_s(\omega) = \frac{1}{2\pi} X(\omega) * \sum_{k=-\infty}^{\infty} c_k [2\pi \delta(\omega - k\omega_0)]$

$$\boxed{X_s(\omega) = \sum_{k=-\infty}^{\infty} c_k X(\omega - k\omega_0)}$$

(b) $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$ periodic with period $T_0$




$c_k = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \delta(t) e^{-jk\omega_0 t} dt = 1/T_0$

$$\boxed{c_k = \frac{1}{T_0}} \qquad \boxed{p(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t}}$$

(c) $$\boxed{X_s(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0} X(\omega - k\omega_0)}$$ just substitute $c_k = \frac{1}{T_0}$ into (a)

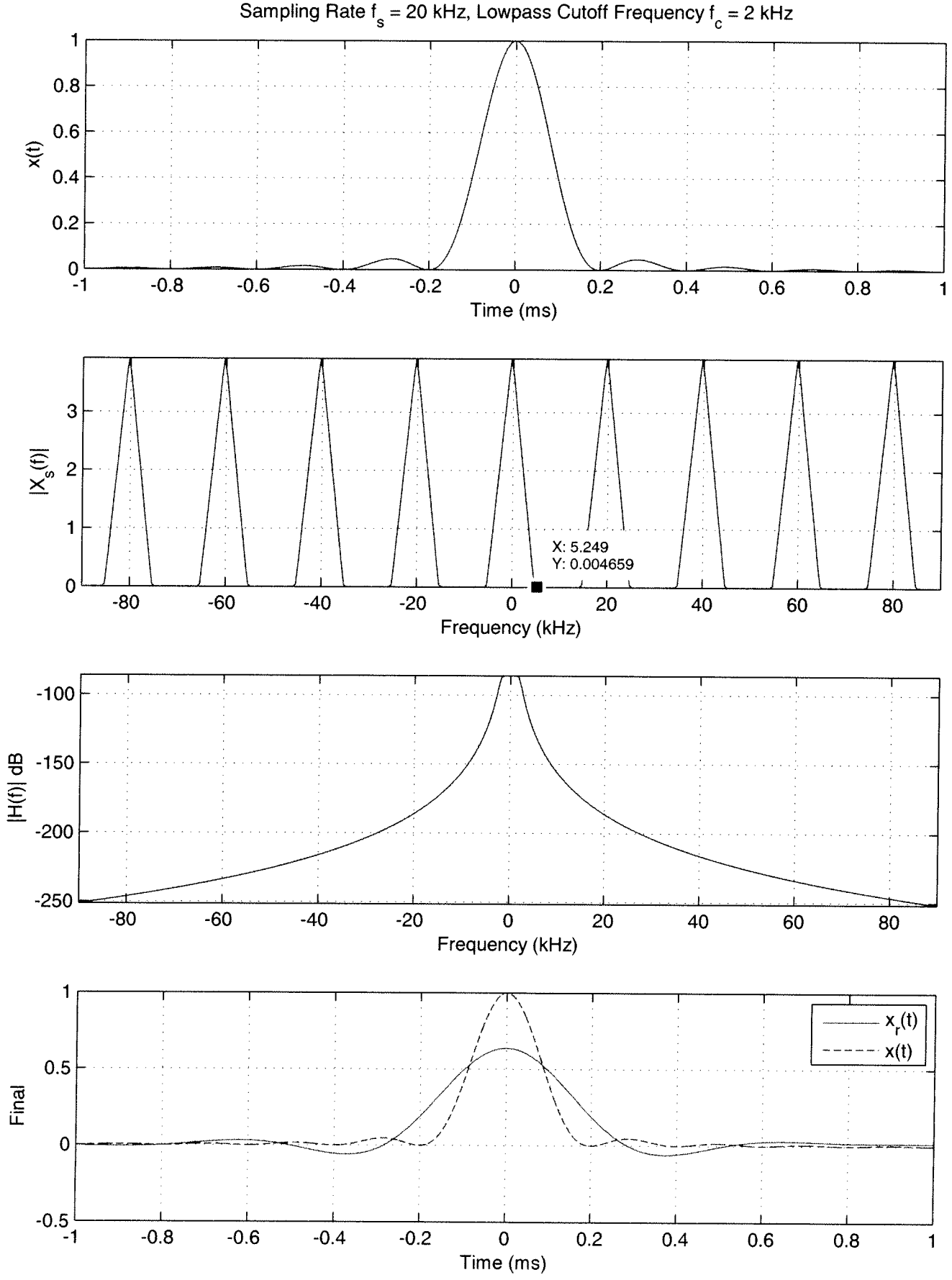
#7 $x(t) = \text{sinc}^2\left(\frac{t}{0.0002}\right) = \text{sinc}^2(Bt) \quad B = \frac{1}{0.0002}$

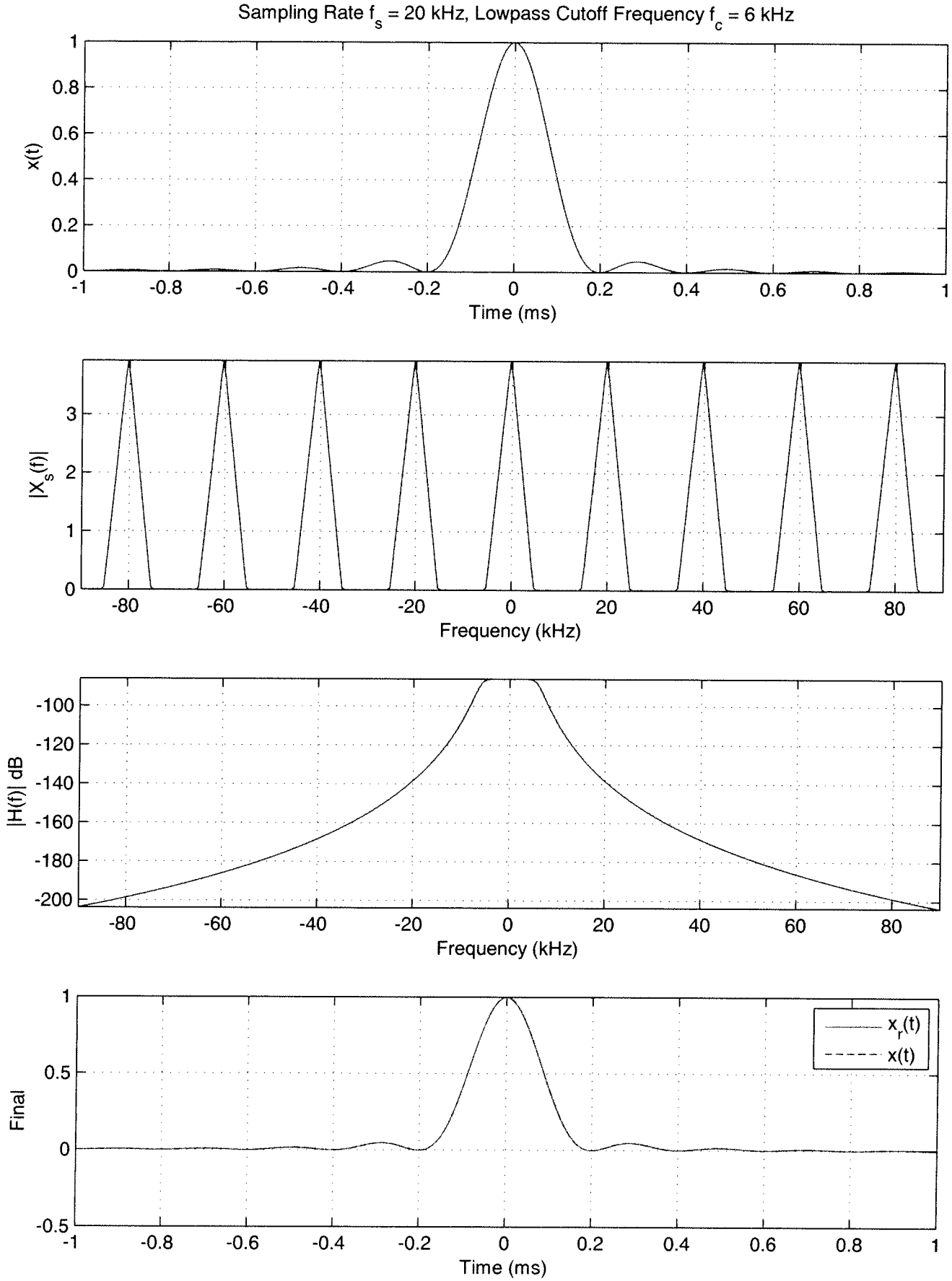
(a) $X(\omega) = \frac{1}{B}\Lambda\left(\frac{\omega}{2\pi B}\right) = 0.0002\,\Lambda\left(\frac{\omega}{2\pi(5000)}\right)$

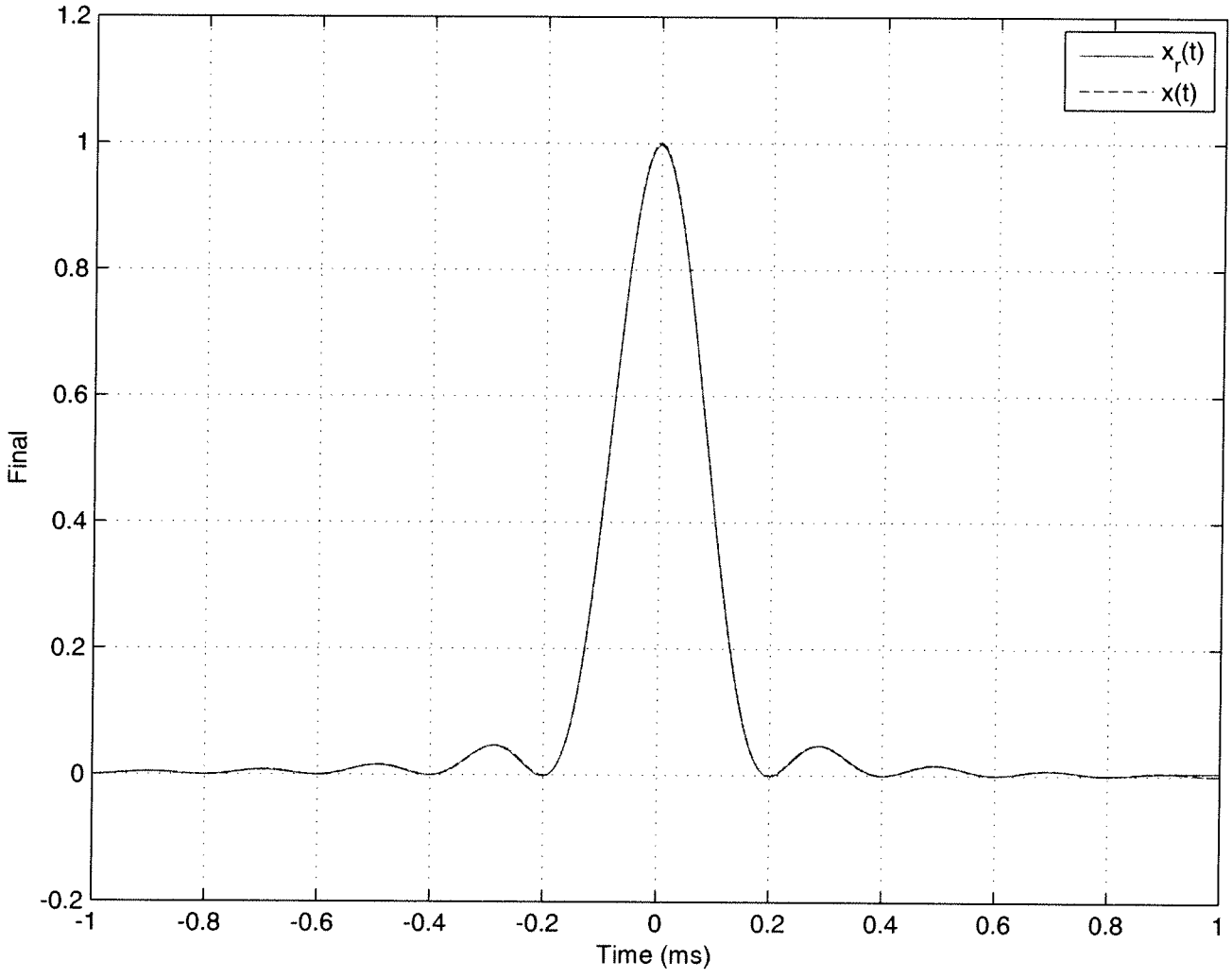
(b) $BW = 10{,}000\,Hz$ (5000 on each side)

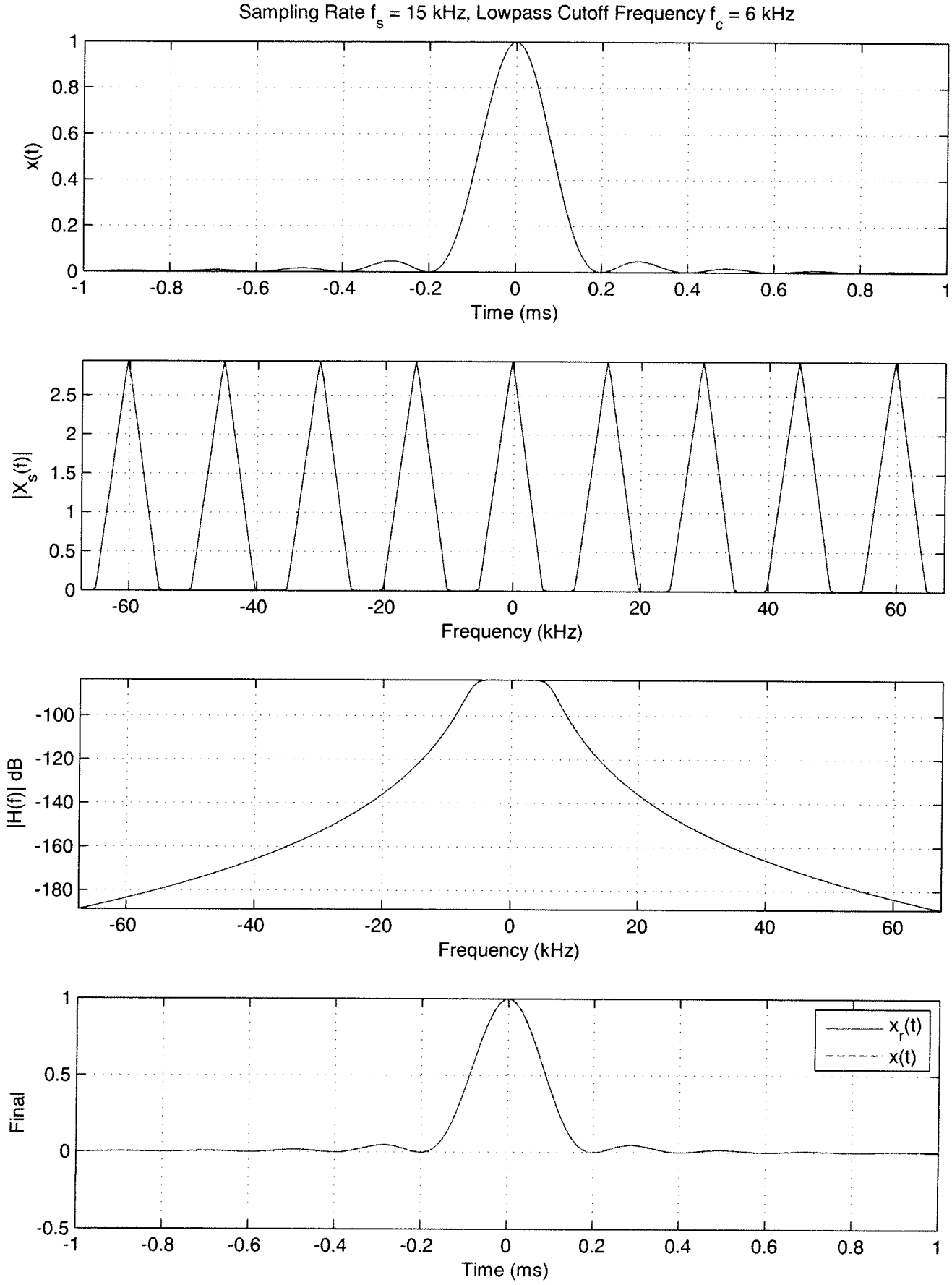
(c) $T_0 = 1/f_0 = \frac{1}{20{,}000} = 0.00005 \quad max = \frac{0.0002}{0.00005} = 4$

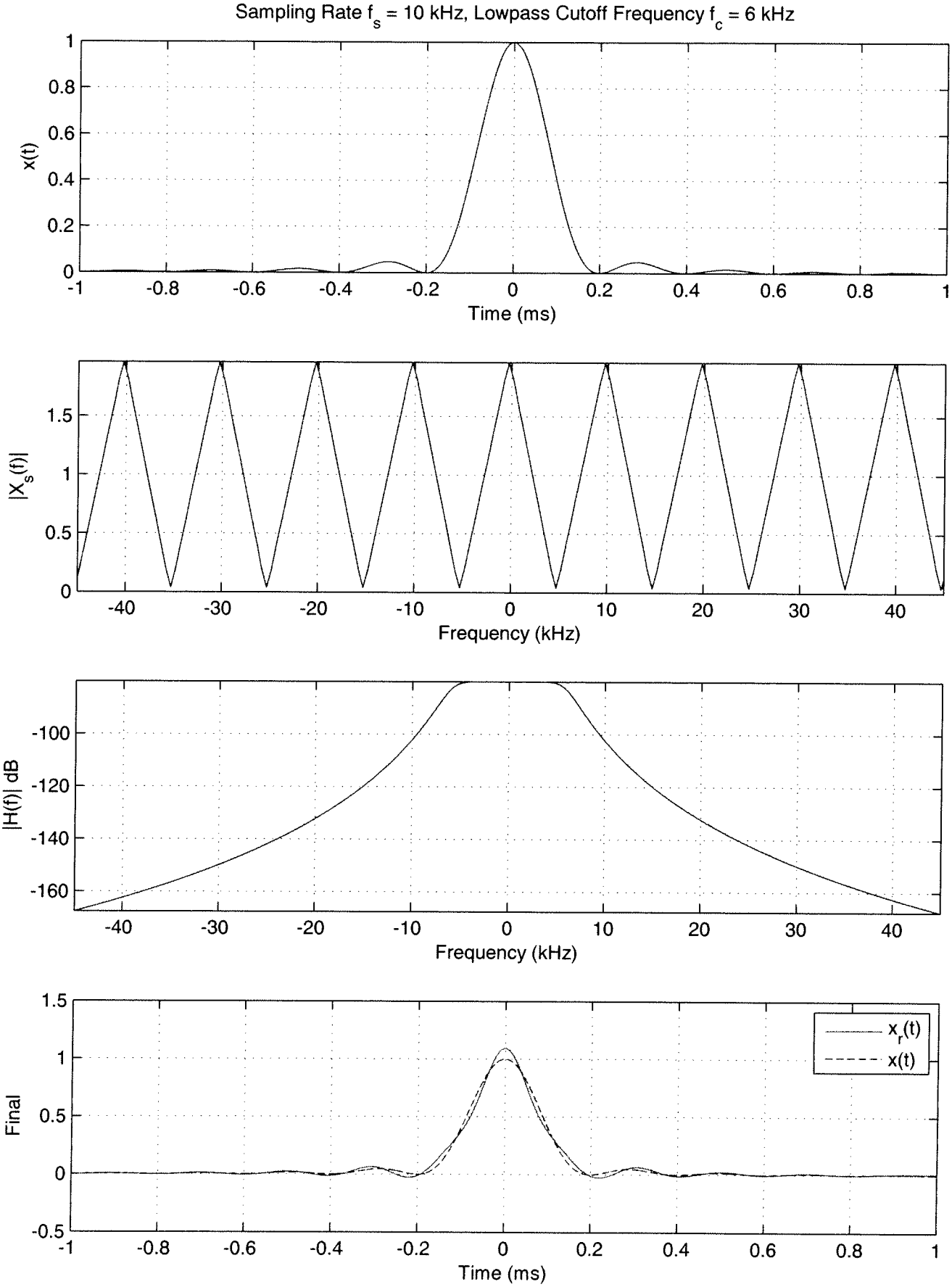
(d) see graph

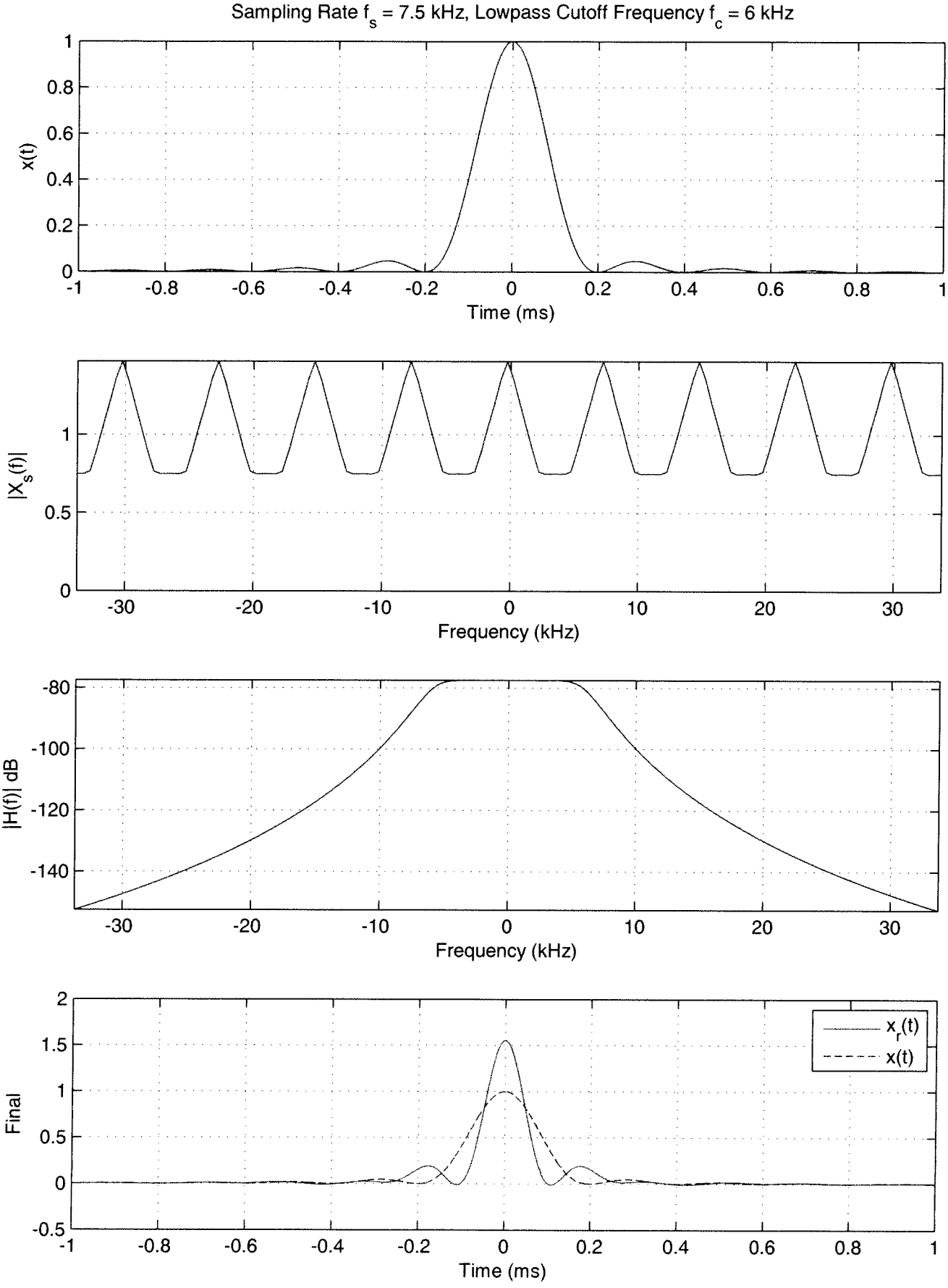

Sampling Rate $f_s$ = 20 kHz, Lowpass Cutoff Frequency $f_c$ = 2 kHz
x(t)
Time (ms)
$|X_s(f)|$
X: 5.249
Y: 0.004659
Frequency (kHz)
|H(f)| dB
Frequency (kHz)
Final
$x_r(t)$
x(t)
Time (ms)


Sampling Rate $f_s$ = 20 kHz, Lowpass Cutoff Frequency $f_c$ = 6 kHz
x(t)
Time (ms)
$|X_s(f)|$
Frequency (kHz)
|H(f)| dB
Frequency (kHz)
Final
$x_r(t)$
x(t)
Time (ms)


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
Final
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Time (ms)
x_r(t)
x(t)


Sampling Rate $f_s$ = 15 kHz, Lowpass Cutoff Frequency $f_c$ = 6 kHz
x(t)
Time (ms)
$|X_s(f)|$
Frequency (kHz)
|H(f)| dB
Frequency (kHz)
Final
$x_r(t)$
x(t)
Time (ms)


Sampling Rate $f_s$ = 10 kHz, Lowpass Cutoff Frequency $f_c$ = 6 kHz
x(t)
Time (ms)
$|X_s(f)|$
Frequency (kHz)
|H(f)| dB
Frequency (kHz)
Final
$x_r(t)$
x(t)
Time (ms)


Sampling Rate $f_s$ = 7.5 kHz, Lowpass Cutoff Frequency $f_c$ = 6 kHz
x(t)
Time (ms)
$|X_s(f)|$
Frequency (kHz)
|H(f)| dB
Frequency (kHz)
Final
$x_r(t)$
x(t)
Time (ms)

#8

(a) $\Delta = -10 \log_{10} \frac{1}{1+\left(\frac{\omega_s}{\omega_p}\right)^{2n}}$

$10^{-\Delta/10} = \frac{1}{1+\left(\frac{\omega_s}{\omega_p}\right)^{2n}}$ $\quad 1+\left(\frac{\omega_s}{\omega_p}\right)^{2n} = 10^{+\Delta/10}$

$\left(\frac{\omega_s}{\omega_p}\right)^{2n} = 10^{+\Delta/10} - 1$

$2n \ln\left(\frac{\omega_s}{\omega_p}\right) = \ln\left[10^{+\Delta/10} - 1\right]$

$$n = \frac{\ln\left[10^{+\Delta/10} - 1\right]}{2\ln\left(\frac{\omega_s}{\omega_p}\right)}$$

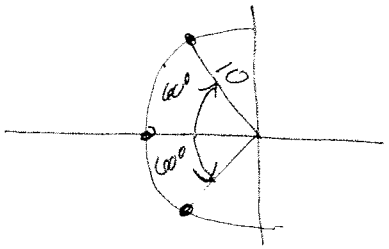
(b) $\omega_p = 10$ rad/sec $\omega_s = 20$ rad/sec $\Delta = +18$ dB

$n = \frac{\ln\left[10^{+1.8} - 1\right]}{2\ln\left(\frac{20}{10}\right)} = 2.978$ need $n = 3$ see plot

(c) For this filter the poles are at $-10, -5 \pm j8.660$

The magnitudes for each of these is 10 rad/sec $= \omega_p$

The phases are $-180, 120, -120$ poles separated by 60°




$\Theta = 60°$

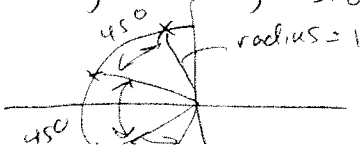
(d) $\omega_p = 15$ rad/sec $\omega_s = 35$ rad/sec $\Delta = +28$ dB

$n = \frac{\ln\left[10^{2.8} - 1\right]}{2\ln\left(\frac{35}{15}\right)} = 3.8$ need $n = 4$ see plot

(e) for this filter the poles are at $-5.7403 \pm j\,13.8582, -13.8550 \pm j\,5.7408$

The magnitudes are all 15 rad/sec $= \omega_p$

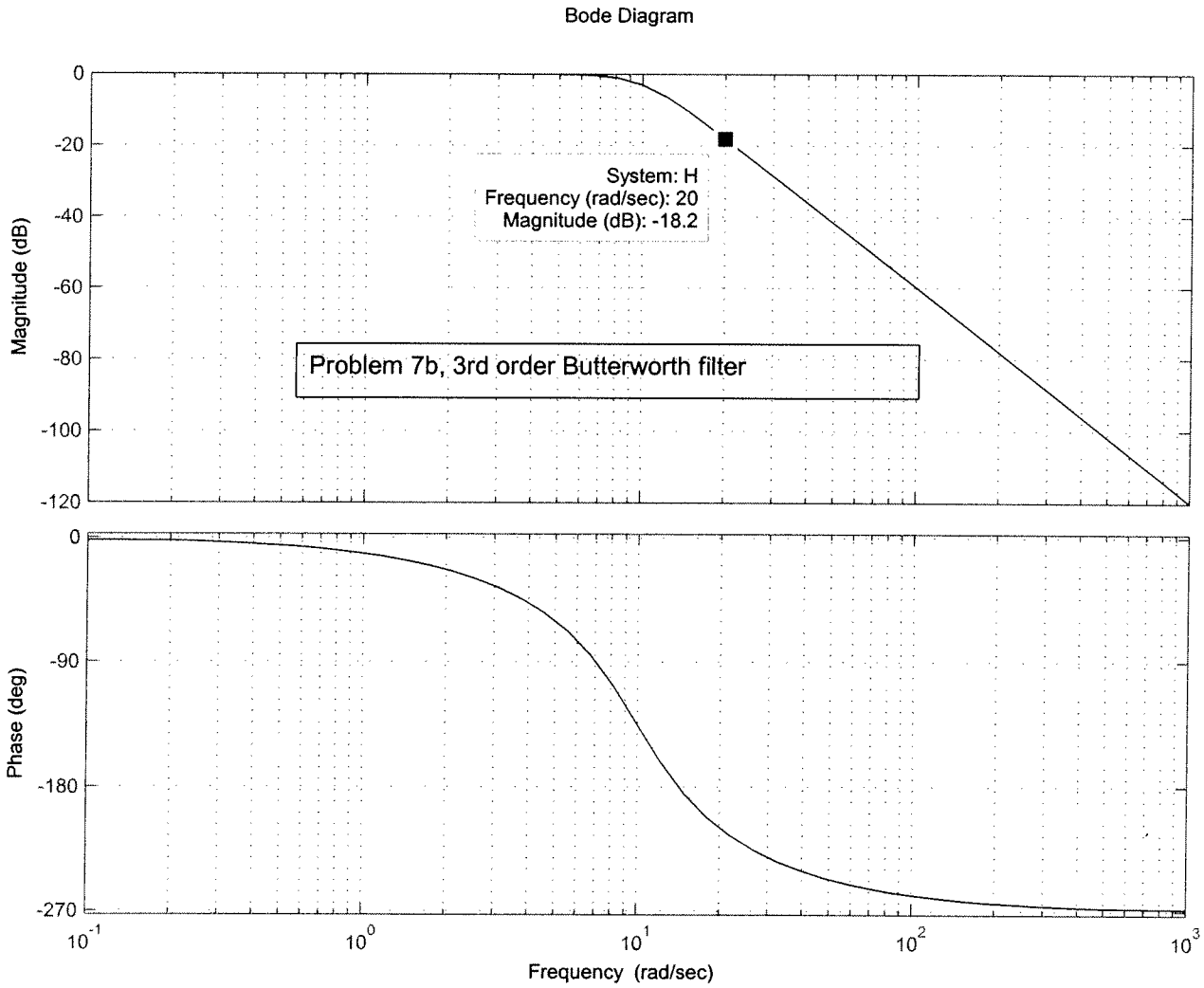
The phases are $\pm 112.5°, \pm 157.5°$

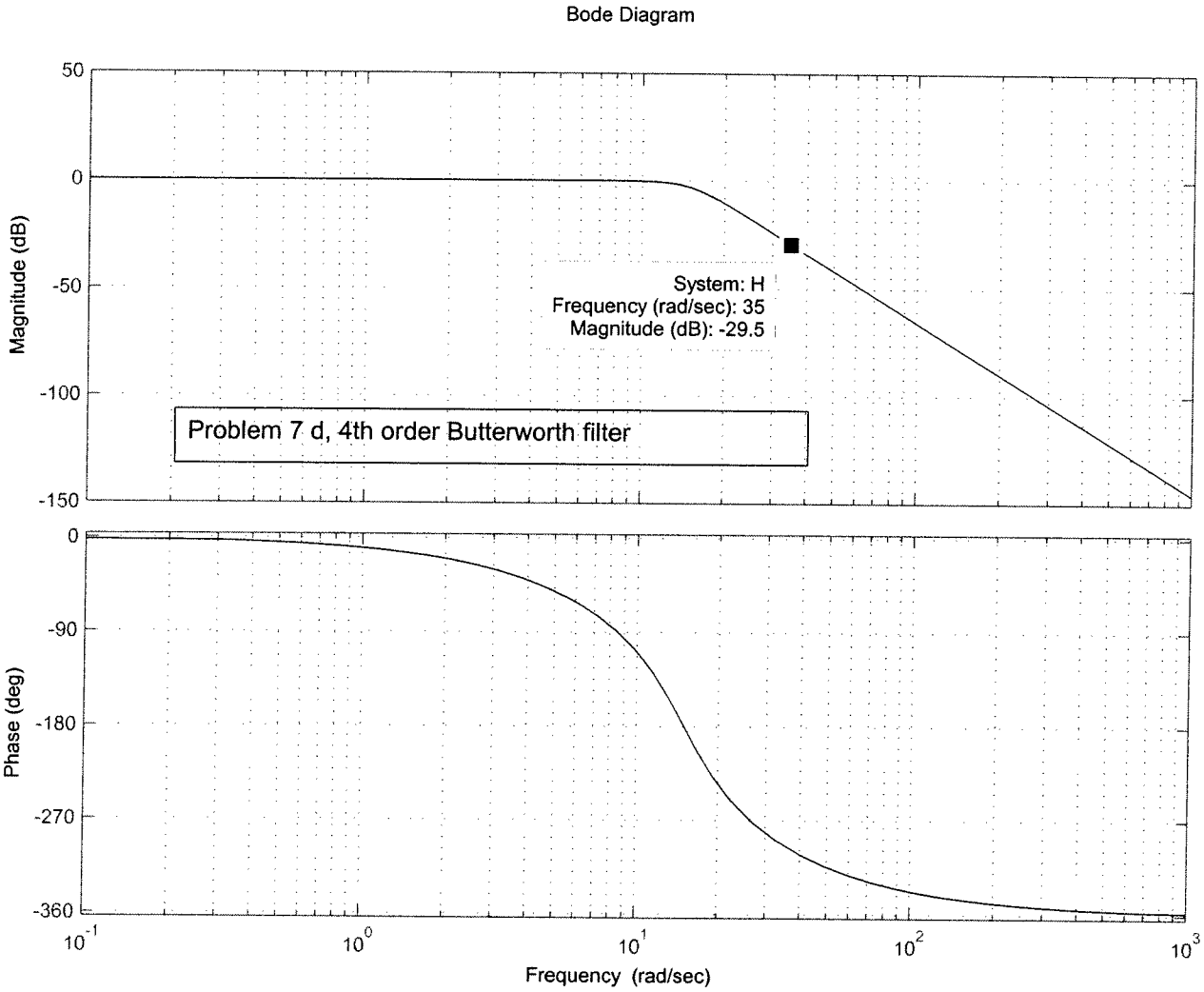



separated by 45°

$\Theta = 45°$

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
System: H
Frequency (rad/sec): 20
Magnitude (dB): -18.2
Problem 7b, 3rd order Butterworth filter
0
-20
-40
-60
-80
-100
-120
-90
-180
-270
10^-1
10^0
10^1
10^2
10^3


Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
System: H
Frequency (rad/sec): 35
Magnitude (dB): -29.5
Problem 7 d, 4th order Butterworth filter
Phase (deg)
0
-90
-180
-270
-360
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/sec)