

ECE 300
Signals and Systems
Homework 6

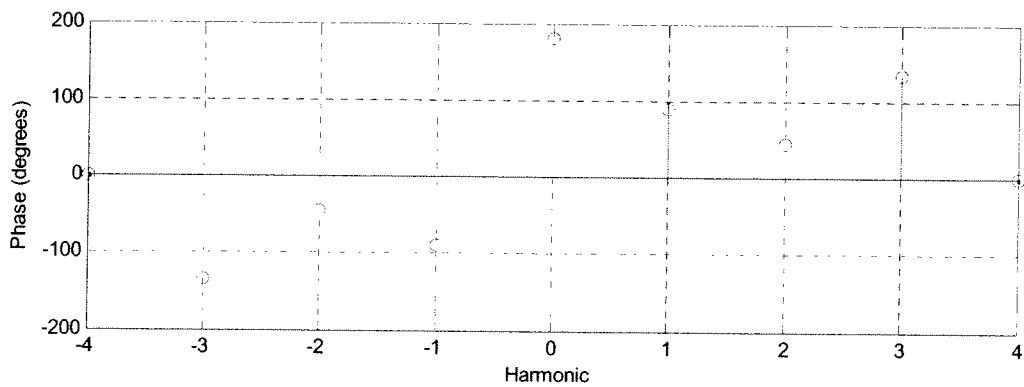
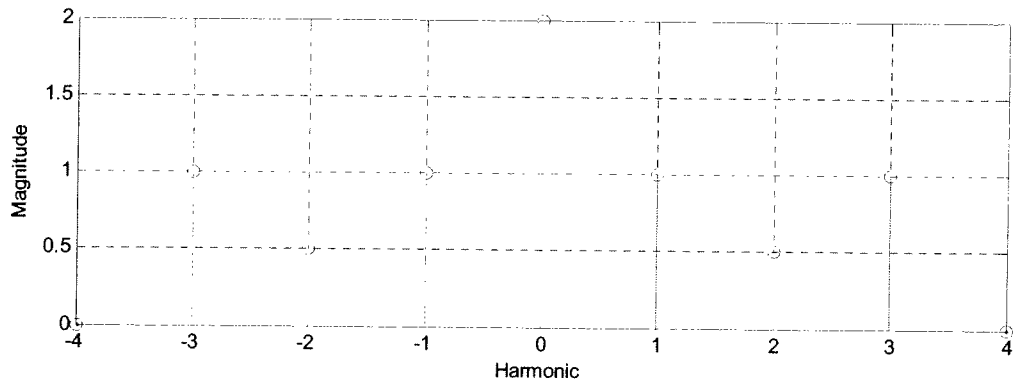
Due Date: April 23, 2009 at the beginning of class

Homework 7 due Tuesday April 28, 2009

Exam 2, Thursday April 30, 2009

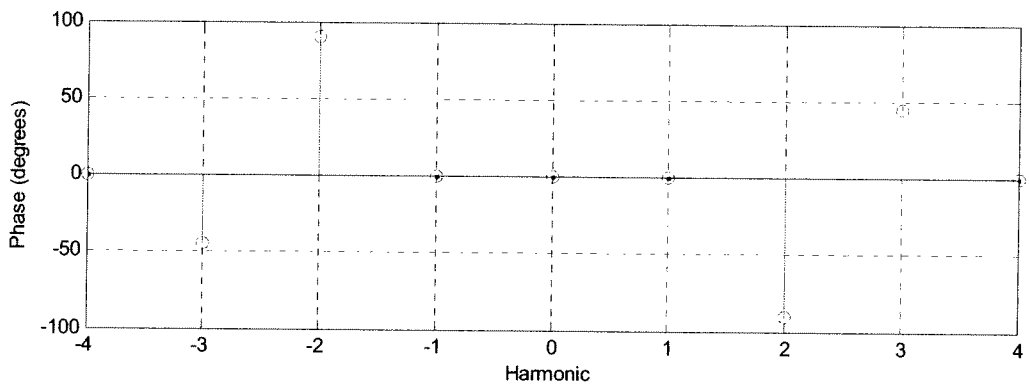
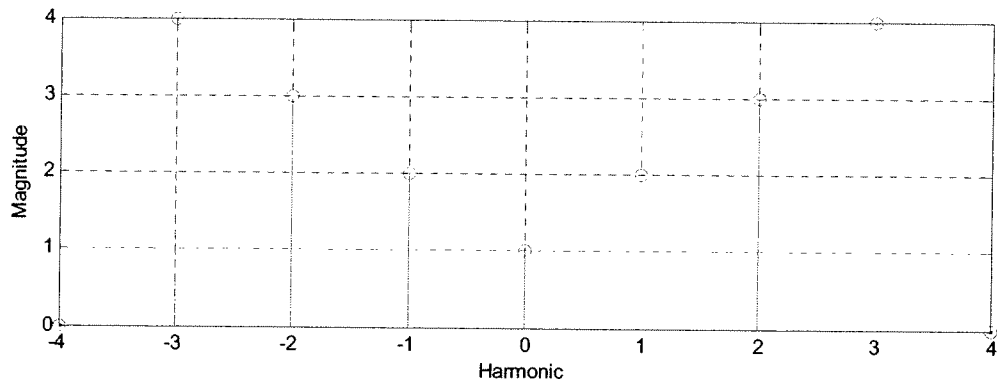
Problems:

1. Assume $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45 degrees)



- a) What is $x(t)$? Your expression must be real.
- b) What is the average value of $x(t)$?
- c) What is the average power in $x(t)$?

2. Assume $x(t)$ has the spectrum shown below (the phase is shown in radians) and a fundamental frequency $\omega_0 = 2 \text{ rad/sec}$:



- What is $x(t)$? Your expression must be real.
- What is the average value of $x(t)$?
- What is the average power in $x(t)$?
- What is the average power in the second harmonic of $x(t)$?

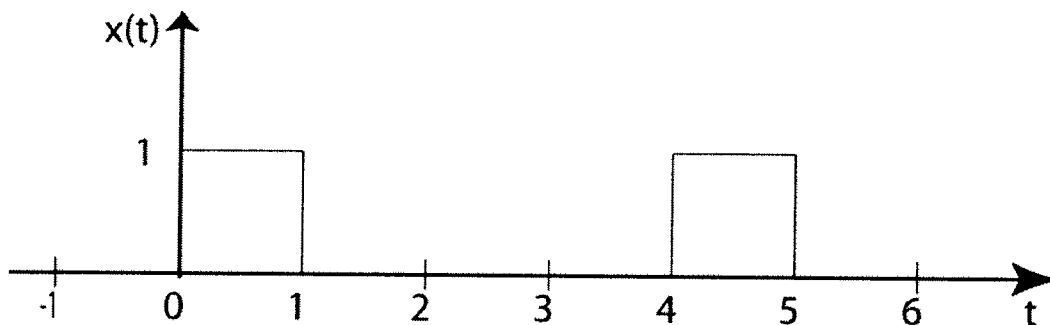
3. Simplify each of the following into the form $c_k = \alpha(k)e^{-j\beta(k)}\text{sinc}(\lambda k)$

$$\text{a) } c_k = \frac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$$

$$\text{b) } c_k = \frac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$$

$$\text{c) } c_k = \frac{e^{j5k} - e^{j2k}}{k}$$

Scrambled Answers $c_k = 3\pi e^{-j\frac{7\pi k}{2}} \text{sinc}\left(\frac{3k}{2}\right)$, $c_k = 3e^{j\left(\frac{7k+\pi}{2}\right)} \text{sinc}\left(\frac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi} \text{sinc}\left(k\frac{9}{2}\right)$



4. For the periodic signal shown above, with period $T = 4$

a) Determine the fundamental frequency ω_0 .

b) Determine the average value.

c) Determine the average power in the DC component of the signal.

d) Determine an expression for the expansion coefficients, c_k . You must write your expression in terms of the **sinc** function, and possibly a leading phase term.

5. The Fourier transform of an impulse response, $h(t)$, gives us the system transfer function, $H(j\omega)$. This is more often written as $H(\omega)$. The Fourier transform of the impulse response is defined as

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t} dt$$

Show that if the impulse response is real-valued, then the transfer function will have the properties

$$|H(-\omega)| = |H(\omega)| \quad (\text{the magnitude is even})$$

$$\angle H(-\omega) = -\angle H(\omega) \quad (\text{the phase is odd})$$

6. **(Matlab/Prelab)** When we are using a Fourier series approximation to a function, we need to be able to determine how many terms to use in the expansion. One method for choosing the number of terms is the just “eyeball” the original signal and the approximation, which is pretty much what we have been doing.. Another approach is to determine the average power in the approximation and compare this to the average power in the signal to be approximated. Using Parseval’s Theorem, we can compute the average power in a periodic signal $x(t)$ using the formulas

$$P_{ave} = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{k=\infty} |c_k|^2 = c_0^2 + \sum_{k=1}^{k=\infty} 2|c_k|^2$$

a) Modify your existing code to determine the average power in a signal using the integral formula.

b) Modify your existing code to estimate the average power in a Fourier series representation using the DC and N harmonics (the formula on the right)

c) For the signals

Half-wave rectifier ($V_m = 100\sqrt{2}$ volts, $f_0 = 60$ Hz)

$$x_1(t) = \begin{cases} V_m \sin(2\pi f_0 t) & 0 \leq t < \frac{T_0}{2} \\ 0 & \frac{T_0}{2} \leq t < T_0 \end{cases}$$

Full-wave rectifier ($V_m = 100\sqrt{2}$ volts, $f_0 = 60$ Hz)

$$x(t) = V_m |\sin(2\pi f_0 t)| \quad 0 \leq t \leq T_0$$

Exponential pulse ($T_0 = 1/60$)

$$x(t) = e^{-t/T_0} u(t) \quad 0 \leq t \leq T_0$$

show that the average power in these signals is 1000.0, 500.0, and 0.4323

d) For the above signals, determine the minimum number of harmonics N necessary so the Fourier series representation has 99.9% of the total energy in the original signal. *Print out your plots and your code and turn them in. Your code should not contain anything that is really not necessary for this problem.*

You may want to use a statement like the following for your plots:

```
title(['N = ', num2str(N), ', Percent of Average Power = ',  
num2str(P/P_ave,4) ]);
```

7. (Matlab/Prelab) Microcontrollers often need to output an analog signal in order to control sensors or motors. Although it is possible to use analog to digital converters, they take up a lot of space on a chip. An alternative is to use a form of pulse width modulation (PWM) and a lowpass filter. For our purposes, the pulse width modulated signal is a periodic square wave which is either at 5 volts or 0 volts. We only change *period* of the pwm signal and the *duty cycle* of the pwm signal since this can easily be done with timers.

a) Construct a square wave signal $x(t)$ with an amplitude of 5 volts (it goes from a minimum of 0 volts to a maximum of 5 volt), a period of 1 microsecond (the signal should start at zero), and a duty cycle of 80%. Leave the duty cycle a variable in your program.

b) Plot the original signal and the Fourier series representation of the signal (using 10 terms) over one period to verify everything is working before going on.

c) Plot the original signal and the Fourier series representation over 5 periods. Note that you will probably need to use the *mod* function in your original $x(t)$ but not in the Fourier series. Your figure should look like that in Figure 1 on the next page. *Print out your plots and code and turn them in.*

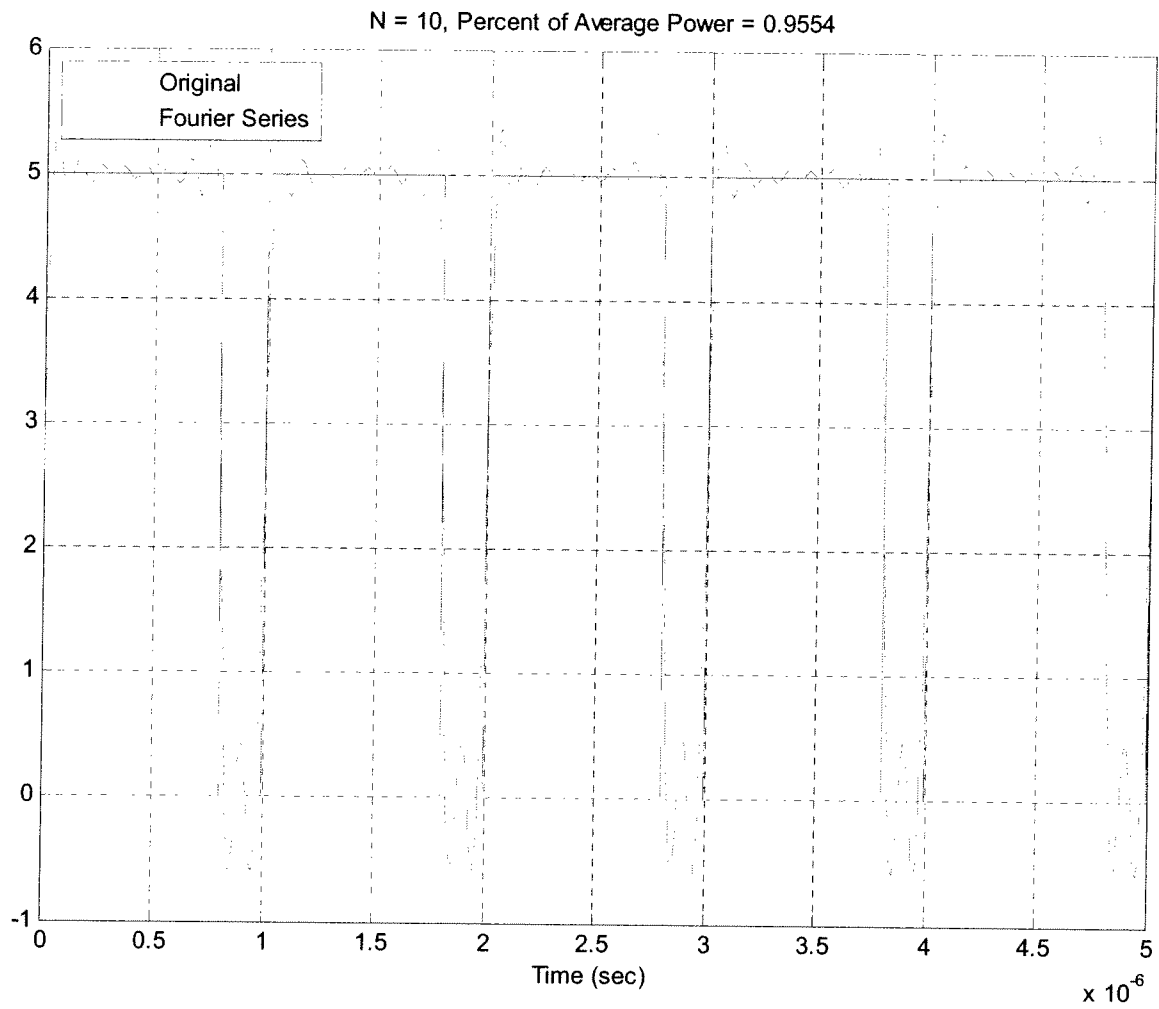


Figure 1: Pulse Width Modulated (PWM) signal with 80% duty cycle.

#1

$$x(t) = c_0 + \sum_{k=1}^{\infty} 2|c_k| \cos(k\omega_0 t + \angle c_k) \quad \omega_0 = \frac{2\pi}{2} = \pi$$

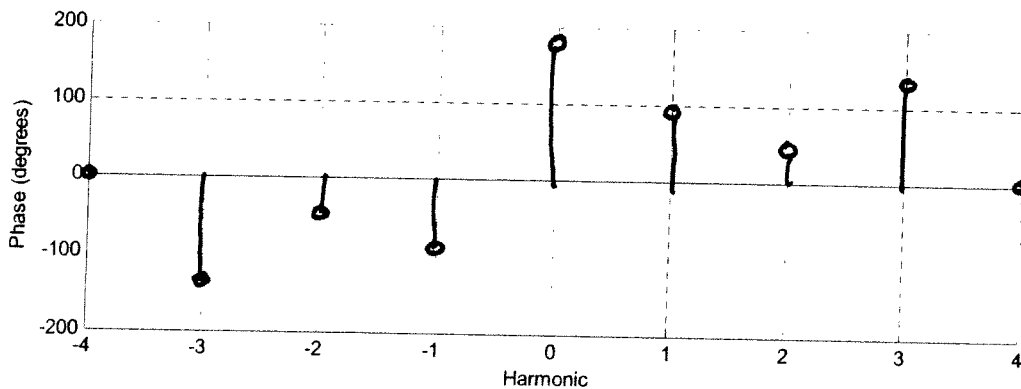
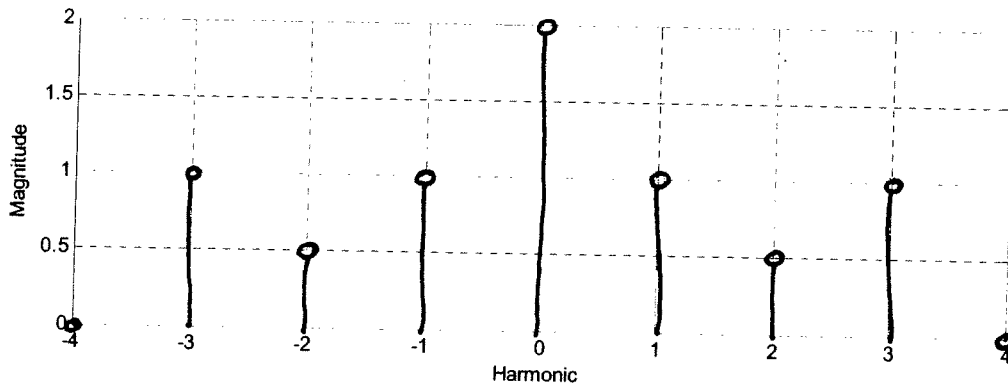
$$c_0 = -2 \quad c_2 = 0.5 \angle 45^\circ \quad c_4 = 0$$

$$c_1 = 1 \angle 90^\circ \quad c_3 = 1 \angle 135^\circ$$

(a) $x(t) = -2 + 2 \cos(\pi t + 90^\circ) + \cos(2\pi t + 45^\circ) + 2 \cos(3\pi t + 135^\circ)$

(b) $\bar{x} = -2$ (c) $P_{ave} = c_0^2 + \sum_{k=1}^{\infty} 2|c_k|^2 = (-2)^2 + 2(1^2) + 2(0.5^2) + 2(1^2) = 8.5 = P_{ave}$

1. Assume $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45 degrees)



#2

$$x(t) = C_0 + \sum_{k=1}^{\infty} 2|C_k| \cos(k\omega_0 t + \phi_k) \quad \omega_0 = 2$$

$$C_0 = 1 \quad C_2 = 3 \angle -90^\circ \quad C_4 = 0$$

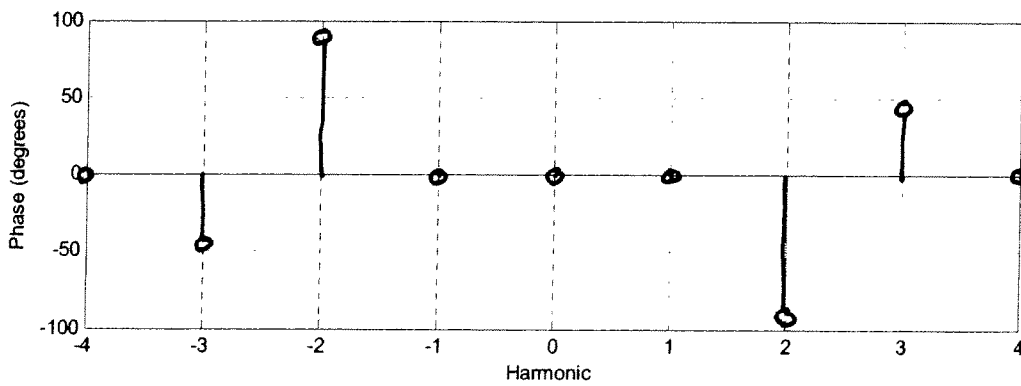
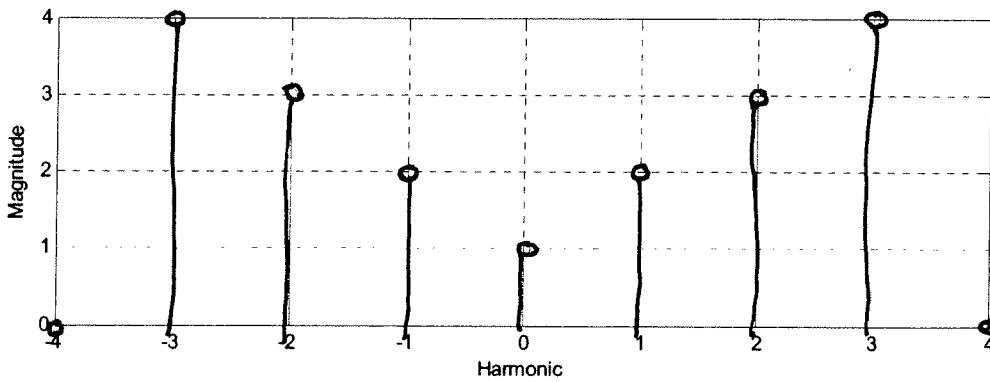
$$C_1 = 2 \angle 0^\circ \quad C_3 = 4 \angle 45^\circ$$

(a) $x(t) = 1 + 4 \cos(2t) + 6 \cos(4t - 90^\circ) + 8 \cos(6t + 45^\circ)$

(b) $\bar{x} = 1$

(c) $P_{ave} = 1^2 + 2(2^2) + 2(3^2) + 2(4^2) = 59 = P_{ave}$

(d) $P_{ave} = 2|C_2|^2 = 2|3|^2 = 18 = P_{ave}^2$



$$\textcircled{\#3} \quad (a) \quad C_k = \frac{e^{j\frac{1}{2}k\pi} - e^{-j\frac{1}{2}k\pi}}{jk\pi} = e^{j\frac{1}{2}\pi k} \frac{[e^{j\frac{1}{2}\pi k} - e^{-j\frac{1}{2}\pi k}]}{jk\pi}$$

$$= \frac{e^{j\frac{1}{2}\pi k}}{k\pi} \cdot 2 \left[\frac{e^{j\frac{1}{2}\pi k} - e^{-j\frac{1}{2}\pi k}}{2j} \right]$$

$$= \frac{2e^{j\frac{1}{2}\pi k}}{k\pi} \sin\left(\frac{1}{2}\pi k\right)$$

$$= \frac{2e^{j\frac{1}{2}\pi k}}{\pi k \left(\frac{1}{2}\right) \left(\frac{2}{1}\right)} = \boxed{e^{j\frac{1}{2}\pi k} \operatorname{sinc}\left(\frac{k}{2}\right) = C_k}$$

$$(b) \quad C_k = \frac{e^{-j2k\pi} - e^{-j5k\pi}}{jk} = e^{-j\frac{7}{2}k\pi} \frac{[e^{+j\frac{3}{2}k\pi} - e^{-j\frac{3}{2}k\pi}]}{jk}$$

$$= 2 \frac{e^{-j\frac{7}{2}k\pi}}{k} \sin\left(\frac{3}{2}k\pi\right) = 2e^{-j\frac{7}{2}k\pi} \frac{\sin\left(\frac{3}{2}k\pi\right)}{k \cdot \left(\frac{3}{2}\right) \left(\frac{2}{3\pi}\right)}$$

$$= 2e^{-j\frac{7}{2}k\pi} \operatorname{sinc}\left(\frac{3k}{2}\right) \cdot \frac{3\pi}{2}$$

$$\boxed{C_k = 3\pi e^{-j\frac{7}{2}k\pi} \operatorname{sinc}\left(\frac{3k}{2}\right)}$$

$$(c) \quad C_k = \frac{e^{j5k} - e^{j2k}}{k} = \frac{e^{j\frac{7}{2}k} (e^{j\frac{3}{2}k} - e^{-j\frac{3}{2}k})}{k}$$

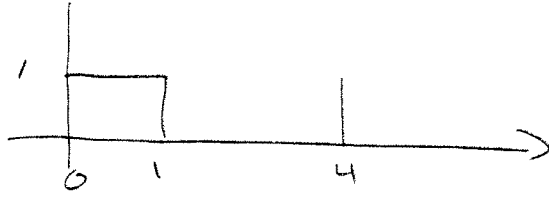
$$= \frac{2j}{k} e^{j\frac{7}{2}k} \left(\frac{e^{j\frac{3}{2}k} - e^{-j\frac{3}{2}k}}{2j} \right) = \frac{2j}{k} e^{j\frac{7}{2}k} \sin\left(\frac{3}{2}k\right)$$

$$= \frac{2e^{j\left(\frac{7}{2}k + \frac{\pi}{2}\right)}}{k} \sin\left(\frac{3}{2}k\right) = \frac{2}{k} e^{j\left(\frac{7}{2}k + \frac{\pi}{2}\right)} \sin\left(\pi \cdot \frac{3k}{2\pi}\right)$$

$$= 2e^{j\left(\frac{7}{2}k + \frac{\pi}{2}\right)} \frac{\sin\left(\pi \cdot \frac{3k}{2\pi}\right)}{\frac{\pi \cdot \frac{3k}{2\pi} \cdot \frac{2\pi}{3\pi}}$$

$$\boxed{C_k = 3e^{j\left(\frac{7}{2}k + \frac{\pi}{2}\right)} \operatorname{sinc}\left(\frac{3k}{2\pi}\right)}$$

#4



$$\textcircled{a} T_0 = 4 \text{ so } \omega_0 = \frac{2\pi}{4} = \boxed{\frac{\pi}{2} = \omega_0}$$

$$\textcircled{b} c_0 = \frac{1}{T_0} \int_{T_0} x(t) dt = \frac{1}{4} \int_0^1 1 dt = \boxed{\frac{1}{4} = c_0}$$

$$\textcircled{c} P_0 = c_0^2 = \boxed{\frac{1}{16} = P_0}$$

$$\textcircled{d} c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T_0} \int_0^1 1 e^{-jk\omega_0 t} dt = \frac{1}{T_0} \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_0^1$$

$$= \frac{e^{-jk\omega_0} - 1}{-jk\omega_0 T_0} = \frac{1 - e^{-jk\omega_0}}{2\pi k j} = \frac{e^{-jk\frac{\omega_0}{2}}}{\pi k (2j)} \left[e^{jk\frac{\omega_0}{2}} - e^{-jk\frac{\omega_0}{2}} \right]$$

$$= \frac{e^{-jk\frac{\omega_0}{2}}}{\pi k} \sin(k\frac{\omega_0}{2}) = e^{-jk\frac{\pi}{4}} \frac{\sin(k\frac{\pi}{4})}{\pi k (\frac{1}{4})}$$

$$\boxed{c_k = \frac{1}{4} e^{-jk\frac{\pi}{4}} \text{sinc}\left(\frac{k}{4}\right)}$$

(#3)

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \quad h(t) \text{ real-valued}$$

$$H^*(\omega) = \int_{-\infty}^{\infty} h(t) e^{j\omega t} dt = H(-\omega)$$

$$|H^*(\omega)| = |H(-\omega)| = |H(\omega)| \quad \text{since } |H(\omega)| = |H^*(\omega)|$$

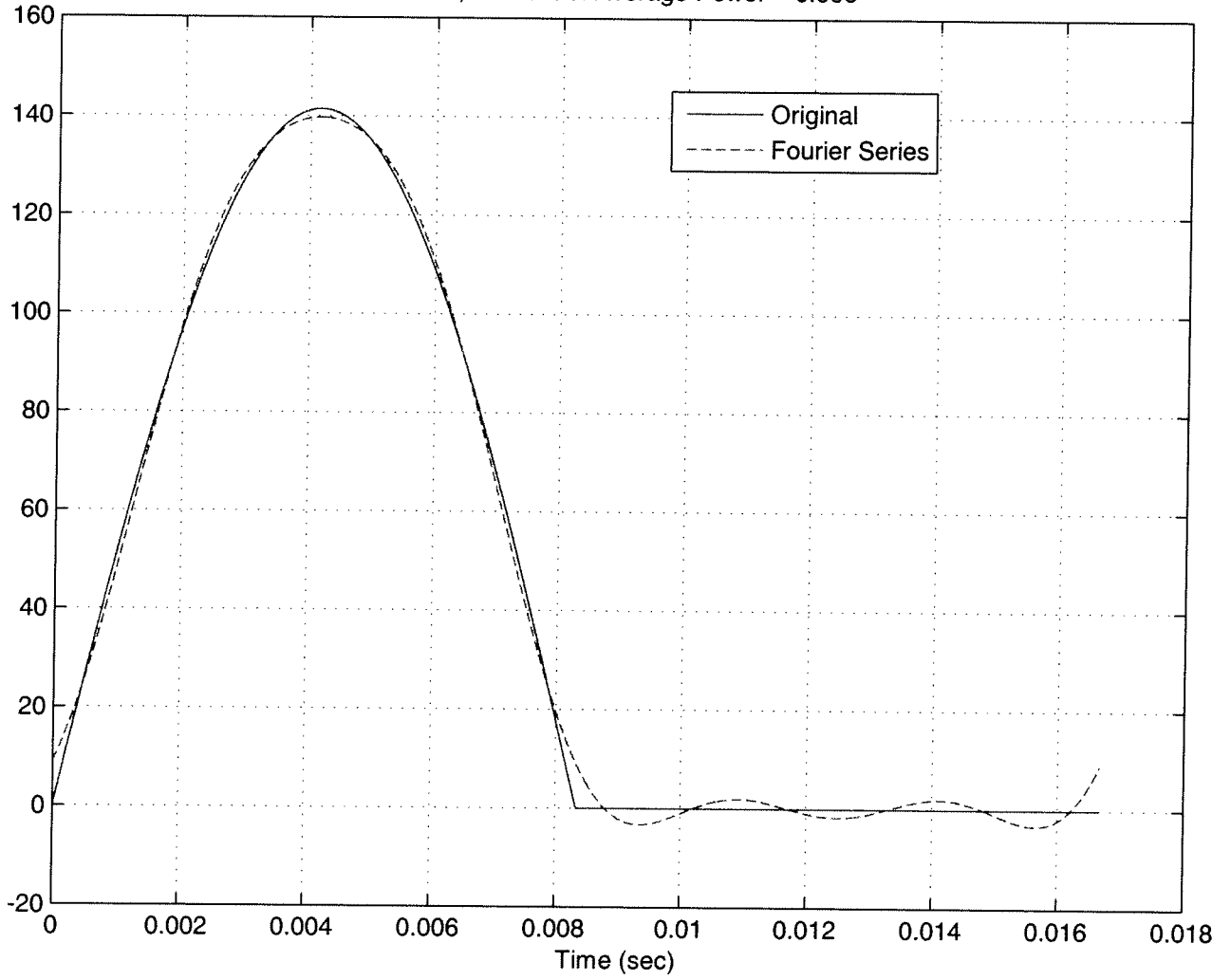
$$H(\omega) = |H(\omega)| e^{j\phi_H(\omega)}$$

$$H^*(\omega) = |H(\omega)| e^{-j\phi_H(\omega)}$$

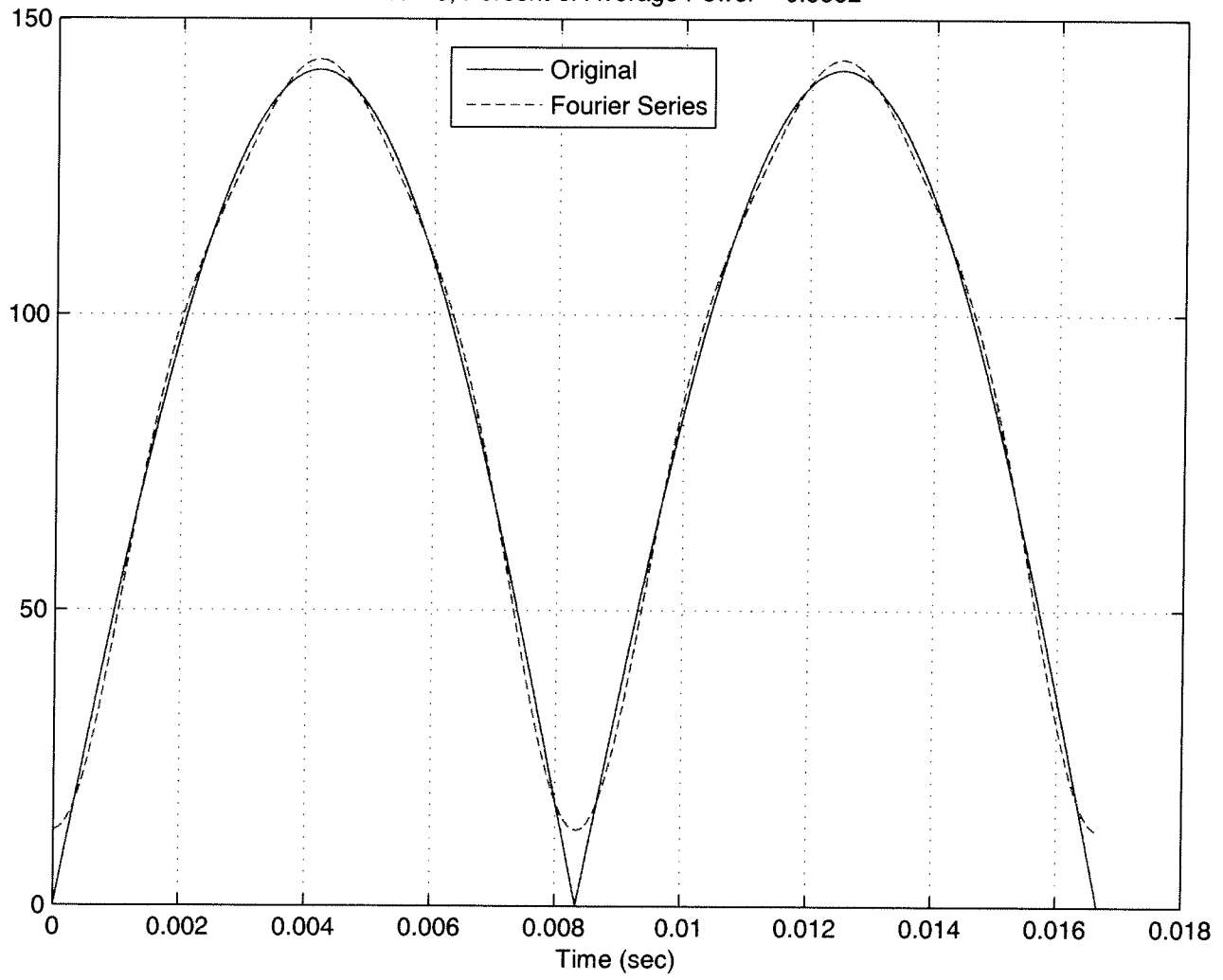
$$H(-\omega) = |H(-\omega)| e^{j\phi_H(-\omega)} = |H(\omega)| e^{j\phi_H(-\omega)}$$

since $H^*(\omega) = H(-\omega)$ $\boxed{-\phi_H(\omega) = \phi_H(-\omega)}$

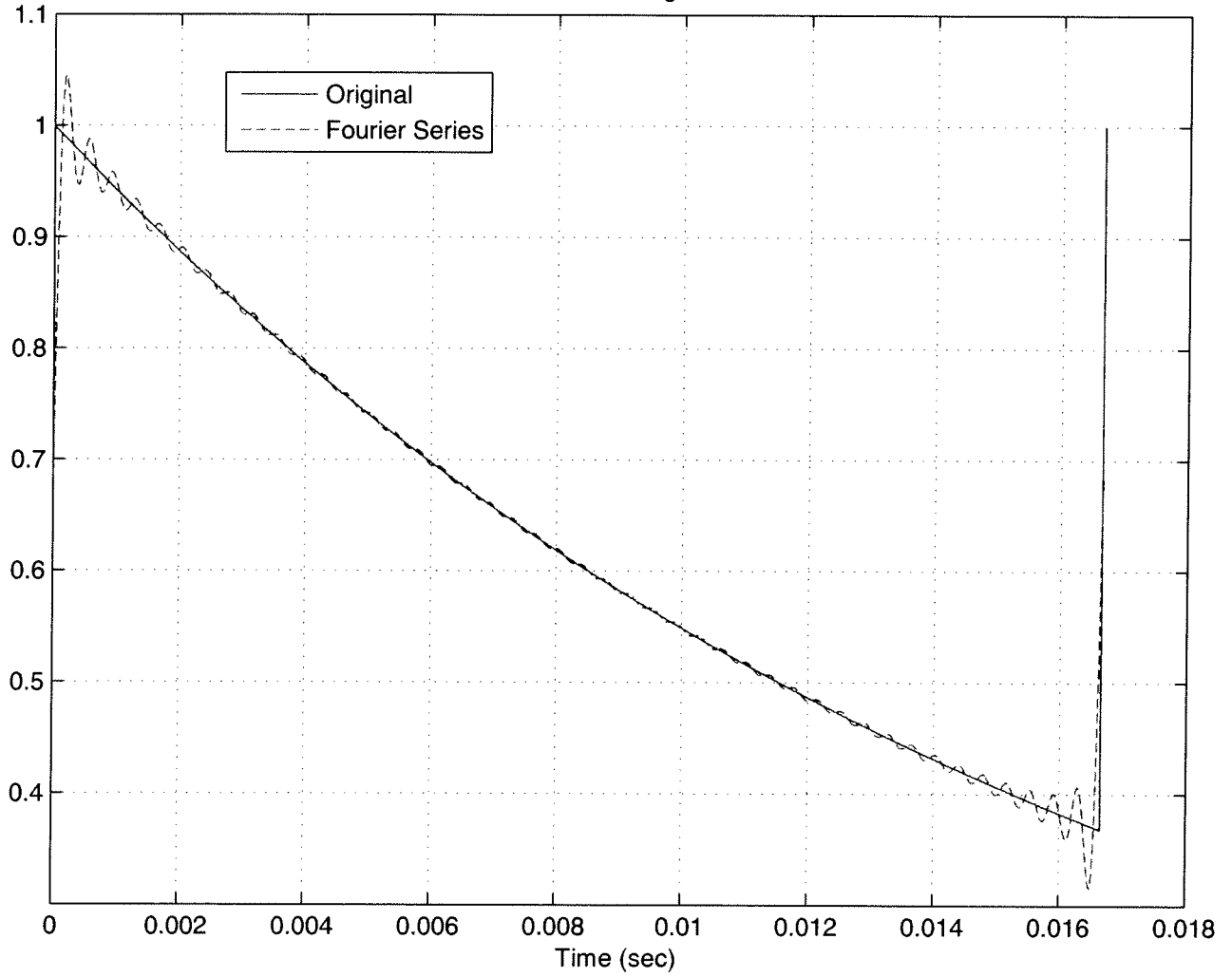
N = 4, Percent of Average Power = 0.999



N = 6, Percent of Average Power = 0.9992



N = 44, Percent of Average Power = 0.999



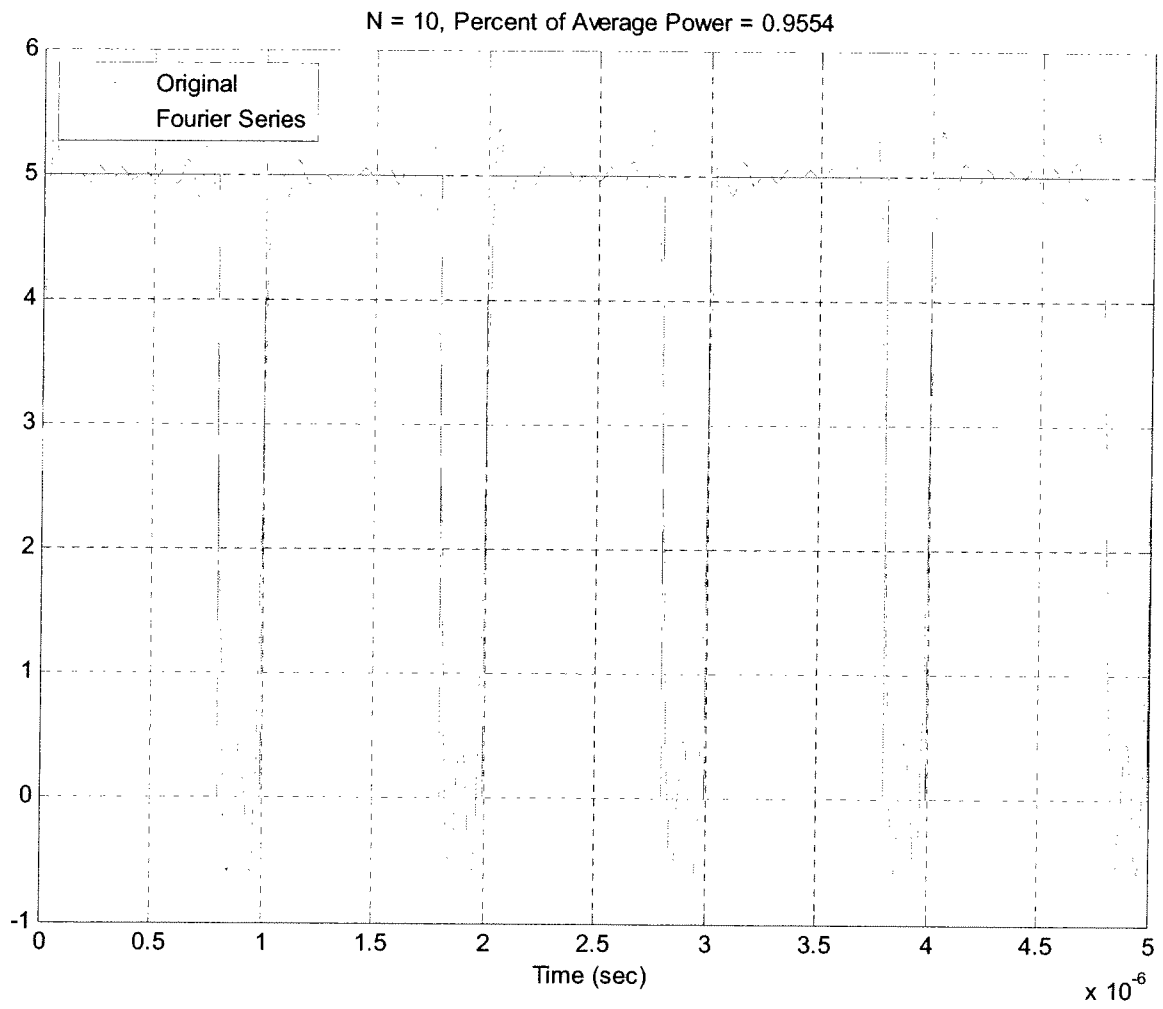


Figure 1: Pulse Width Modulated (PWM) signal with 80% duty cycle.