ECE 300
Signals and Systems
Homework 4

Due Date: ***Thursday*** April 2, 2009 *at the beginning of class*

Exam 1, Monday April 6

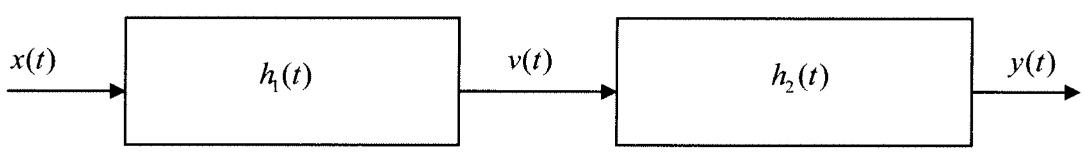
Problems

1. Determine the impulse responses for the following systems:

a) $y(t) = x(t) + \int_{-\infty}^{t} e^{-2(t-\lambda)} x(\lambda) d\lambda$ **b)** $y(t) = \int_{-\infty}^{t-1} e^{-(t-\lambda)} x(\lambda + 2) d\lambda$

c) $2\dot{y}(t) - y(t) = 3x(t+2)$ **d)** $y(t) = \frac{1}{I} \int_{t-I}^{t} x(\lambda) d\lambda$

2. Consider the following two subsystems, connected together to form a single LTI system.



Determine the impulse response $h(t)$ of the entire system if the impulse responses of the subsystems are given as:

a) $h_1(t) = \delta(t) \quad h_2(t) = 2e^{-t}u(t)$

b) $h_1(t) = e^{-t}u(t) \quad h_2(t) = 2\delta(t-1)$

c) $h_1(t) = e^{-(t+1)}u(t+1) \quad h_2(t) = e^{-(t-1)}u(t-1)$

d) $h_1(t) = u(t) - u(t-1) \quad h_2(t) = u(t)$

Use analytical convolution when evaluating the convolution integrals in this problem.

3. Consider a causal linear time invariant system with impulse response given by

$$h(t) = e^{-(t-1)}u(t-1)$$

The input to the system is given by

$$x(t) = u(t) - u(t-1) + u(t-3)$$

Using ***graphical convolution***, determine the output $y(t)$ **for** $2 \le t \le 5$. ***Note the limited range of t we are interested in !***

Specifically, you must

a) Flip and slide $h(t)$
b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
c) Determine the range of t for which each part of your solution is valid
d) Set up any necessary integrals to compute $y(t)$
e) Evaluate the integrals

You should get (in unsimplified form)

$$y(t) = \begin{cases} e^{-(t-1)}[e^1 - 1] & 2 \le t \le 4 \\ e^{-(t-1)}[e^1 - 1] + e^{-(t-1)}[e^{t-1} - e^3] & 4 \le t \le 5 \end{cases}$$

4. Consider a causal linear time invariant system with impulse response given by

$$h(t) = e^{-(t-1)}u(t-1)$$

a) Show that the step response of the system (the response to a unit step) is

$$y_s(t) = [1 - e^{-(t-1)}]u(t-1)$$

b) Using linearity and time-invariance, determine the response of the system to the input

$$x(t) = u(t-1) - 2u(t-2)$$

c) Use ***graphical convolution*** to determine the output of the system.

d) Show that your answers to **b** and **c** are the same.

e) Compute the derivative of the step response and show that you indeed obtain the impulse response.

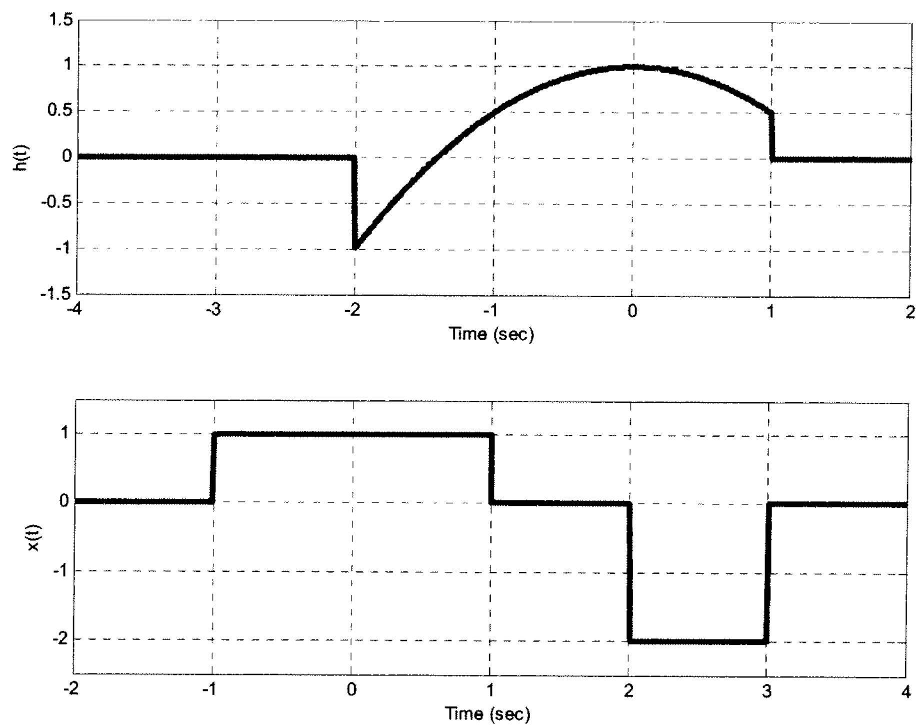
5. Consider a causal linear time invariant system with impulse response

$$h(t) = \left[1 - 0.5t^2\right][u(t+2) - u(t-1)]$$

The input to the system is

$$x(t) = u(t+1) - u(t-1) - 2u(t-2) + 2u(t-3)$$

These two functions are plotted below:



Using ***graphical convolution***, set up the integrals to determine the output $y(t)$

Specifically, you must

- Flip and slide $h(t)$
- Show graphs displaying $h(t-\lambda)$ relative to $x(\lambda)$ for each region of interest.
- Determine the ranges of t for which each part of your solution is valid.
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete and simplified as much as possible (no unit step functions)
- ***Do Not Evaluate the Integrals!!***

6. (Matlab/Prelab Problem, read the **Appendix** for help) From the class website download the files **homework4.m** and **convolution.m**. **homework4.m** is a script file that sets up the time arrays and functions, and the invokes the function **convolution.m** to compute the convolution of the two functions. **Homework4.m** then plots the two functions to be convolved, and then the resulting convolution of the two functions.

a) Complete the code for the function **convolution.m.**

b) Use the script **homework4.m** to compute and plot the convolution of the functions $x_1(t) = rect\left(\frac{t}{2}\right)$ and $x_2(t) = rect\left(\frac{t-1}{3}\right)$. If you have done this correctly, you results should like those shown in Figure 1. *Turn in your plot.*

c) Use the script **homework4.m** to compute and plot the convolution of the functions $x_1(t) = rect\left(\frac{t-2}{2}\right)$ and $x_2(t) = rect\left(\frac{t-2.5}{5}\right)$.If you have done this correctly, you results should like those shown in Figure 2. *Turn in your plot.*

d) For the remainder of this problem, assume we want t1 to go from -1 to 6 with an increment of 0.01 and t2 to go from -1 to 8 with an increment of 0.01. Then find the convolution of each of the following:

$$x_1(t) = \mathrm{rect}\left(\frac{t-2.5}{5}\right) \quad x_2(t) = e^{-t}u(t)$$

$$x_1(t) = \mathrm{rect}\left(\frac{t-1}{2}\right) \quad x_2(t) = e^{-t}u(t)$$

$$x_1(t) = \mathrm{rect}\left(\frac{t-0.1}{0.2}\right) \quad x_2(t) = e^{-t}u(t)$$

Turn in your plots and your code. *Note: In this part we can view this as looking at the response of a system with impulse response $h(t) = e^{-t}u(t)$ to inputs which are pulses of decreasing width. This is what we will be doing in Lab 3 when we try to model an impulse, $\delta(t)$, as a narrow pulse and look at how an RC circuit responds.*

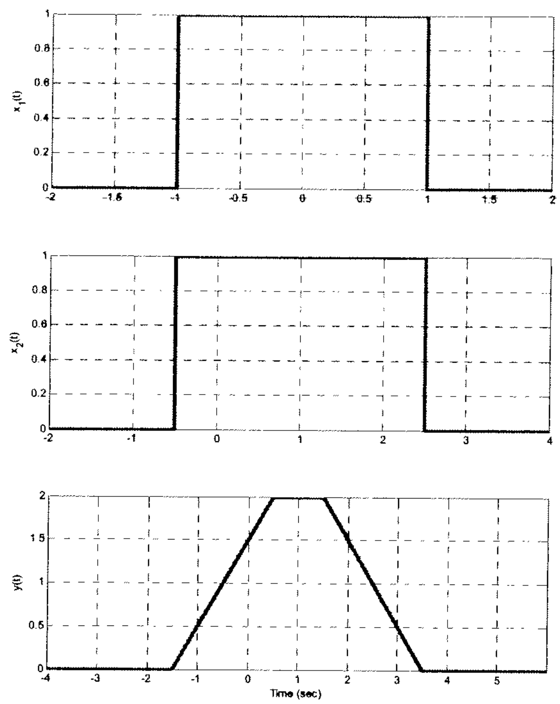


Figure 1: Results for problem 6-b

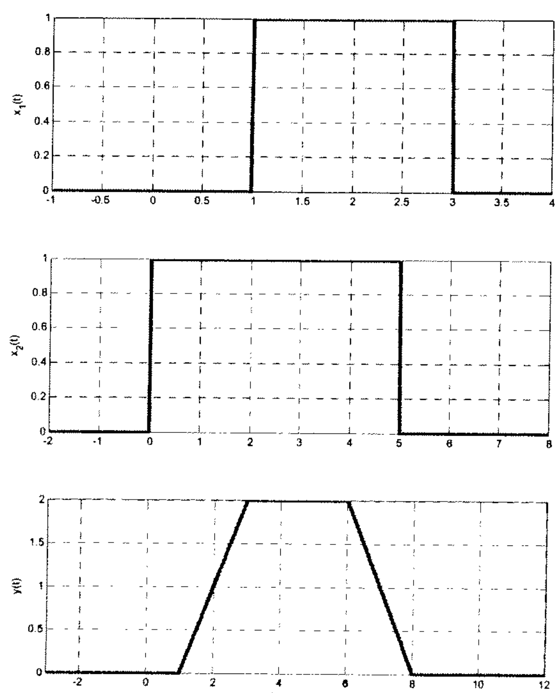


Figure 2: Results for Problem 6-c.

7. Pre-Lab Exercises (to be done by all students. Turn this in with your homework and bring a copy of this with you to lab!)

a) Calculate the impulse response of the RC lowpass filter shown in Figure 1, in terms of unspecified components R and C. Determine the time constant for the circuit.

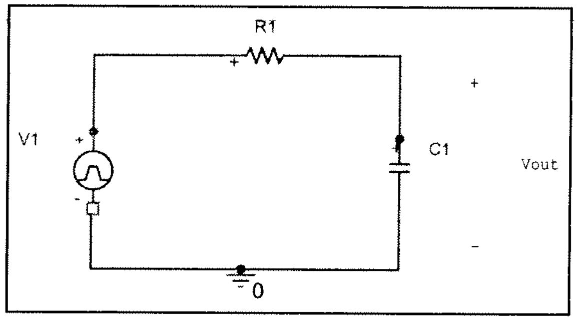


Figure 1. Simple RC lowpass filter circuit.

b) Show that the **step response** of the circuit (the response of the system when the input is a unit step) is $y_s(t) = (1 - e^{-t/\tau})u(t)$, and determine the 10-90% rise time. t_r, as shown below in Figure 2. The rise time is simply the amount of time necessary for the output to rise from 10% to 90% of its final value. Specifically, show that the rise time is given by $t_r = \tau \ln(9)$

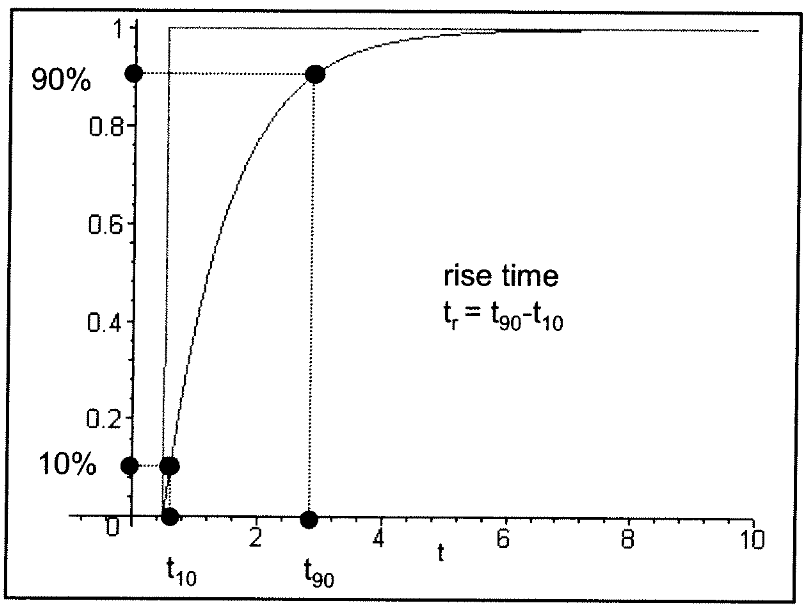


Figure 2. Step response of the RC lowpass filter circuit of Figure 1, showing the definition of the 10-90% risetime.

c) Specify values R and C which will produce a time constant of approximately 1 msec. Be sure to consider the fact that the capacitor will be asked to charge and discharge quickly in these measurements.

d) Using linearity and time-invariance, show that the response of the circuit to a pulse of length *T* and amplitude *A* (, i.e. a pulse of amplitude *A* starting at 0 and ending at *T*) is given by

$$y_{pulse}(t) = A(1-e^{-t/\tau})u(t) - A(1-e^{-(t-T)/\tau})u(t-T)$$

e) Assume the input is a pulse of amplitude A and width T, and use the results from part **d** to determine an expression for the amplitude of the output at the end of the pulse, $y_{pulse}(T)$. Next, assume that $\frac{T}{\tau} \ll 1$ (the duration for the pulse is much small than the time constant of the circuit) and use Taylor series approximations for the exponentials to show that $y_{pulse}(T) \approx \frac{AT}{\tau}$. This means the amplitude of the output at time T (the end of the pulse) is approximately the area of the pulse divided by the time constant.

Appendix

Although this is a continuous time course, and Matlab works in discrete-time, we can use Matlab to numerically do convolutions, under certain restrictions. The most important restriction is that the spaces between the time samples be the same for both functions. Another other restriction is that the functions really need to return to zero (or very close to zero) within the time frame we are examining them. Finally, we need fine enough resolution (the sampling interval must be sufficiently small) so that our sampled signals are a good approximation to the continuous signal. We will discuss sampling at the end of the course.

First, we need to have two functions to convolve. Let's assume we want to convolve the functions $x_1(t) = rect\left(\frac{t}{2}\right)$ and $x_2(t) = rect\left(\frac{t-1}{3}\right)$.

Let's denote the time vector that goes with x_1 as t1, and the time vector that goes with x_2 as t2. Then we create the functions with something like

```
t1 = [-3:0.01:3];
t2 = [-2:0.01:4];
x1 = @(t) 0*(t<-1)+1*((t>=-1)&(t<=1))+0*(t>1);
x2 = @(t) 0*(t<-0.5)+1*((t>=-0.5)&(t<=2.5))+0*(t>2.5);
```

Next we will need to determine the time interval between samples. We can determine this as

```
dt = t1(2)-t1(1);
```

It doesn't matter if we use t2 or t1, since the sample interval must be the same.

Now we can use Matlabs **conv** function to do the convolution. However, since we are trying to do continuous time convolution we need to do some scaling. To understand why, let's look again at convolution:

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

If we were to try and approximate this integral using discrete-time samples, with sampling interval Δt, we could write

$$y(t) \approx y(k\Delta t)$$
$$x(\lambda) \approx x(n\Delta t)$$
$$h(t-\lambda) \approx h([k-n]\Delta t)$$

We can then approximate the integral as

$$y(t)=\int_{-\infty}^{\infty}x(\lambda)h(t-\lambda)d\lambda \approx y(k\Delta t)=\sum_{n=-\infty}^{n=\infty}x(n\Delta t)h([k-n]\Delta t)\Delta t = \Delta t\sum_{n=-\infty}^{n=\infty}x(n\Delta t)h([k-n]\Delta t)$$

The Matlab function **conv** computes the sum $\sum_{n=-\infty}^{n=\infty}x(n\Delta t)h([k-n]\Delta t)$, so to approximate the continuous time integral we need to multiply (or scale) by Δt. Hence

```
y = dt*conv(x1,x2);
```

Finally, we need to determine a time vector that corresponds to y. To do this we need to determine where the starting point should be. Let's consider the convolution

$$y(t)=\int_{-\infty}^{\infty}x(\lambda)h(t-\lambda)d\lambda$$

Let's assume $x(t)$ is zero until time t_1, so we could write $x(t)=\tilde{x}(t)u(t-t_1)$ for some function $\tilde{x}(t)$. Similarly, let's assume $h(t)$ is zero until time t_2, so we could write $h(t)=\tilde{h}(t)u(t-t_2)$ for some function $\tilde{h}(t)$. The convolution integral is then

$$y(t)=\int_{-\infty}^{\infty}\tilde{x}(\lambda)u(\lambda-t_1)\tilde{h}(t-\lambda)u(t-\lambda-t_2)d\lambda=\int_{t_1}^{t-t_2}\tilde{x}(\lambda)\tilde{h}(t-\lambda)d\lambda$$

This integral will be zero unless $t-t_1\geq t_2$, or $t\geq t_1+t_2$. Hence we know that $y(t)$ is zero until $t=t_1+t_2$. This means for our convolution, the initial time of the output is the sum of the initial times:

```
initial_time = t1(1)+t2(1);
```

Every sample in y is separated by time interval dt. We need to determine how long y is, and create an indexed array of this length

```
n = [0:length(y)-1];
```

Finally we can construct the correct time vector (ty) that starts at the correct initial time and runs the correct length

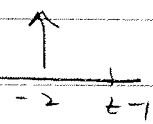
```
ty  = dt*n + initial_time;
```

#1

(a) $y(t) = x(t) + \int_{-\infty}^{t} e^{-2(t-\lambda)} x(\lambda)\,d\lambda$

$h(t) = \delta(t) + \int_{-\infty}^{t} e^{-2(t-\lambda)} \delta(\lambda)\,d\lambda = \boxed{\delta(t) + e^{-2t}u(t) = h(t)}$

(b) $y(t) = \int_{-\infty}^{t-1} e^{-(t-\lambda)} x(\lambda+2)\,d\lambda$



$t-1 > -2$

$t > -1$

$h(t) = \int_{-\infty}^{t-1} e^{-(t-\lambda)} \delta(\lambda+2) = \boxed{e^{-(t+2)} u(t+1) = h(t)}$

(c) $2\dot{y}(t) - y(t) = 3x(t+2)$

$\dot{h}(t) - \frac{1}{2}h(t) = \frac{3}{2}\delta(t+2)$

$\frac{d}{dt}\left[h(t)e^{-t/2}\right] = \frac{3}{2}e^{-t/2}\delta(t+2) = \frac{3}{2}e^{1}\delta(t+2)$

$h(t)e^{-t/2} = \frac{3}{2}e^{1}\int_{-\infty}^{t}\delta(\lambda+2)\,d\lambda = \frac{3}{2}e^{1}u(t+2)$

$h(t) = \frac{3}{2}e^{1}e^{t/2}u(t+2) = \boxed{\frac{3}{2}e^{(t+2)/2}u(t+2) = h(t)}$

(d) $y(t) = \frac{1}{I}\int_{t-I}^{t} x(\lambda)\,d\lambda$

$h(t) = \frac{1}{I}\int_{t-I}^{t}\delta(\lambda)\,d\lambda = \begin{cases} \frac{1}{I} & t > 0 \text{ and } t-I < 0 \\ 0 & \text{else} \end{cases}$

$\boxed{h(t) = \frac{1}{I}\left[u(t) - u(t-I)\right]}$

(#2)

(a) $h_1(t) = \delta(t) \quad h_2(t) = 2e^{-t}u(t)$

$$h(t) = h_1(t) * h_2(t) = \boxed{2e^{-t}u(t) = h(t)}$$

(b) $h_1(t) = e^{-t}u(t) \quad h_2(t) = 2\delta(t-1)$

$$h(t) = h_1(t) * h_2(t) = \boxed{2e^{-(t-1)}u(t-1) = h(t)}$$

(c) $h_1(t) = e^{-(t+1)}u(t+1) \quad h_2(t) = e^{-(t-1)}u(t-1)$

$$h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} e^{-(\lambda+1)} u(\lambda+1)\, e^{-(t-\lambda-1)} u(t-\lambda-1)\, d\lambda$$

$$= \int_{-1}^{t-1} e^{-\lambda-1-t+\lambda+1}\, d\lambda = e^{-t}\int_{-1}^{t-1} d\lambda = \boxed{te^{-t}u(t) = h(t)}$$

(d) $h_1(t) = u(t) - u(t-1) \quad h_2(t) = u(t)$

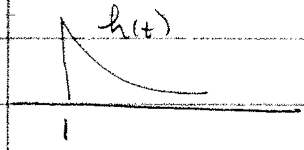
$$h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda)\, d\lambda$$

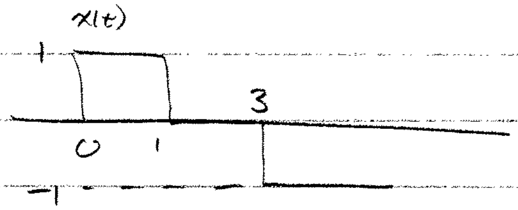
$$= \int_{-\infty}^{\infty} [u(\lambda) - u(\lambda-1)]\, u(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} u(\lambda)u(t-\lambda)\, d\lambda - \int_{-\infty}^{\infty} u(\lambda-1)u(t-\lambda)\, d\lambda$$

$$= \int_0^t d\lambda - \int_1^t d\lambda = \boxed{tu(t) - (t-1)u(t-1) = h(t)}$$

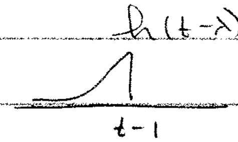
(#3) $h(t) = e^{-(t-1)} u(t-1) \qquad x(t) = u(t) - u(t-1) - u(t-3)$

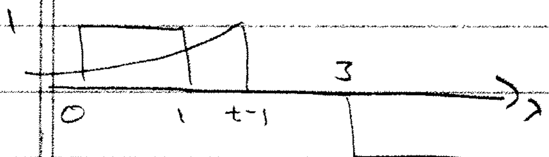
we want the output $y(t)$ for $2 \le t \le 5$





$h(1) = h(t-\lambda) \quad t-\lambda = 1 \quad t-1 = \lambda$



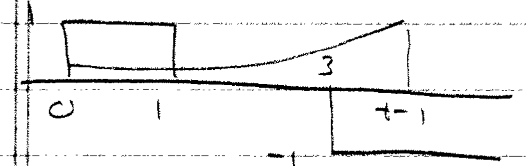


$2 \le t \le 4$

$$y(t) = \int_0^1 e^{-(t-\lambda-1)} d\lambda$$

$$= e^{-(t-1)} \int_0^1 e^{\lambda} d\lambda$$

$$\boxed{y(t) = e^{-(t-1)} \left[e^1 - 1\right] \qquad 2 \le t \le 4}$$



$4 \le t < \infty$

$$y(t) = \int_0^1 e^{-(t-\lambda-1)} d\lambda + \int_3^{t-1} e^{-(t-\lambda-1)} d\lambda$$

$$= e^{-(t-1)} \left[e^1 - 1\right] + e^{-(t-1)} \int_3^{t-1} e^{\lambda} d\lambda$$

$$\boxed{y(t) = e^{-(t-1)} \left[e^1 - 1\right] + e^{-(t-1)} \left[e^{t-1} - e^3\right] \qquad 4 \le t \le \infty}$$

(4) $h(t) = e^{-(t-1)}u(t-1)$

(a) step response $y_s(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\lambda)\, u(t-\lambda)\, d\lambda$

$$= \int_{-\infty}^{\infty} e^{-(\lambda-1)} u(\lambda-1)\, u(t-\lambda)\, d\lambda = \int_1^t e^{-(\lambda-1)} d\lambda = e^1 \int_1^t e^{-\lambda} d\lambda$$

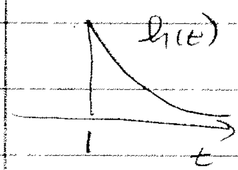
$$= e^1 \left[-e^{-\lambda} \Big|_1^t \right] u(t-1) = e^1 \left[e^{-1} - e^{-t} \right] u(t-1)$$

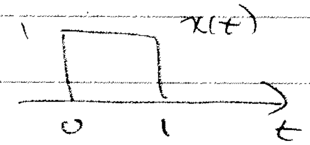
$$\boxed{= \left[1 - e^{-(t-1)} \right] u(t-1) = y_s(t)}$$

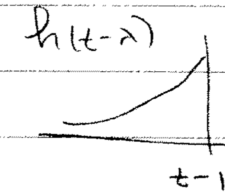
(b) for $x(t) = u(t) - u(t-1)$, $y(t) = y_s(t) - y_s(t-1)$

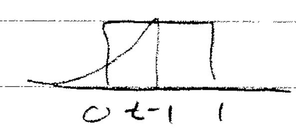
so $\boxed{y(t) = \left[1 - e^{-(t-1)}\right] u(t-1) - \left[1 - e^{-(t-2)}\right] u(t-2)}$

(c)





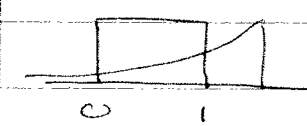




$1 \le t \le 2$

$$y(t) = \int_0^{t-1} e^{-(t-\lambda-1)} d\lambda = e^{-(t-1)} \int_0^{t-1} e^{\lambda} d\lambda$$

$$= e^{-(t-1)} \left[e^{t-1} - 1 \right] = 1 - e^{-(t-1)}$$



$t \ge 2$

$$y(t) = \int_0^1 e^{-(t-\lambda-1)} d\lambda = e^{-(t-1)} \int_0^1 e^{\lambda} d\lambda$$

$$= e^{-(t-1)} \left[e^1 - 1 \right] = e^{-(t-2)} - e^{-(t-1)}$$

$$\boxed{y(t) = \begin{cases} 1 - e^{-(t-1)} & 1 \le t \le 2 \\ e^{-(t-2)} - e^{-(t-1)} & t \ge 2 \end{cases}}$$

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(#4) (continued)

(d) for $1 \le t \le 2$ both answers are the same

for $t \ge 2$, $y(t) = [1-e^{-(t-1)}]u(t-1) - [1-e^{-(t-2)}]u(t-2)$

$$= 1 - e^{-(t-1)} - 1 + e^{-(t-2)}$$

$$= e^{-(t-2)} - e^{-(t-1)}$$, so both answers are the same

(e) $y_s(t) = [1 - e^{-(t-1)}]u(t-1)$

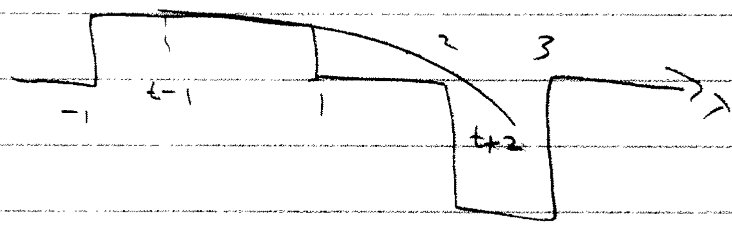
$$\frac{d}{dt} y_s(t) = \left\{\frac{d}{dt}[1-e^{-(t-1)}]\right\} u(t-1) + [1-e^{-(t-1)}]\left\{\frac{d}{dt} u(t-1)\right\}$$

evaluate at $t=1$

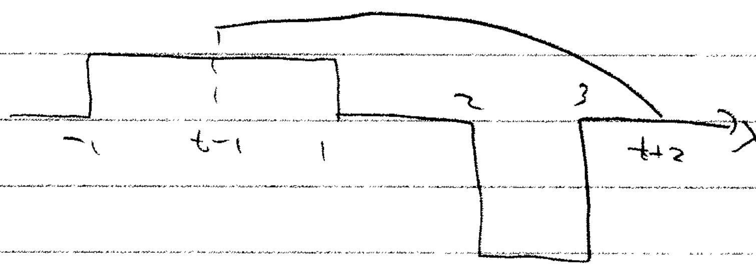
$$= +e^{-(t-1)}u(t-1) + [1-e^{-(t-1)}]\,\delta(t-1)$$

$$= e^{-(t-1)}u(t-1) + 0\,\delta(t-1) = \boxed{e^{-(t-1)}u(t-1) = h(t)}$$

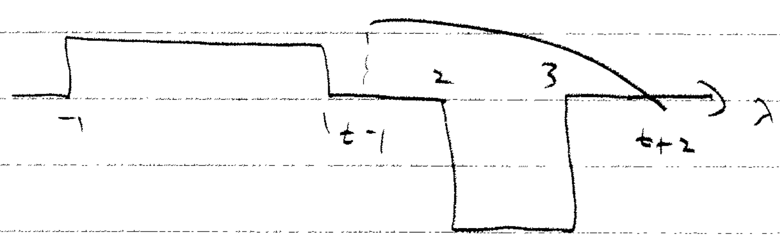
#5 (continued)



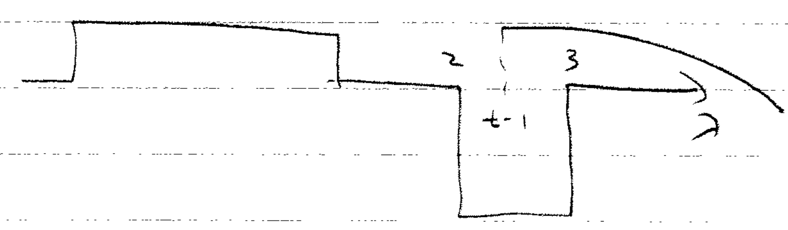
$0 \le t \le 1 \qquad y(t) = \int_{t-1}^{1} [1-0.5(t-\lambda)^2][1]\,d\lambda + \int_{2}^{t+2} [1-0.5(t-\lambda)^2][-2]\,d\lambda$



$1 \le t \le 2 \qquad y(t) = \int_{t-1}^{1} [1-0.5(t-\lambda)^2][1]\,d\lambda + \int_{2}^{3} [1-0.5(t-\lambda)^2][-2]\,d\lambda$



$2 \le t \le 3 \qquad y(t) = \int_{2}^{3} [1-0.5(t-\lambda)^2][-2]\,d\lambda$



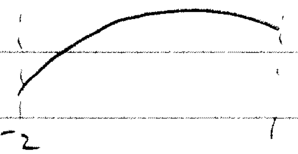
$3 \le t \le 4 \qquad y(t) = \int_{t-1}^{3} [1-0.5(t-\lambda)^2][-2]\,d\lambda$

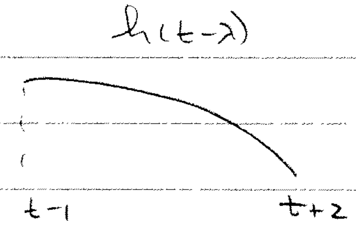
(#5) $h(t) = [1 - 0.5t^2][u(t+2) - u(t-1)]$

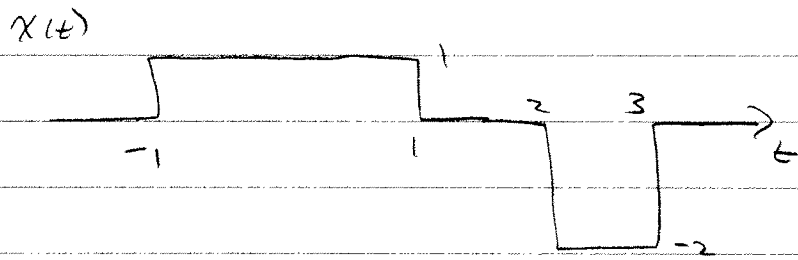
$x(t) = u(t+1) - u(t-1) - 2u(t-2) + 2u(t-3)$

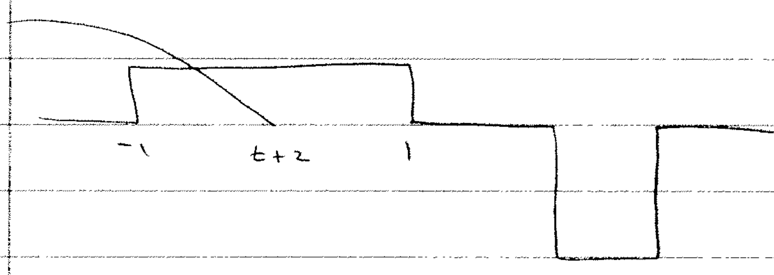
$h(1) = h(t-\lambda) \quad 1 = t-\lambda \quad \lambda = t-1$

$h(-2) = h(t-\lambda) \quad -2 = t-\lambda \quad \lambda = t+2$



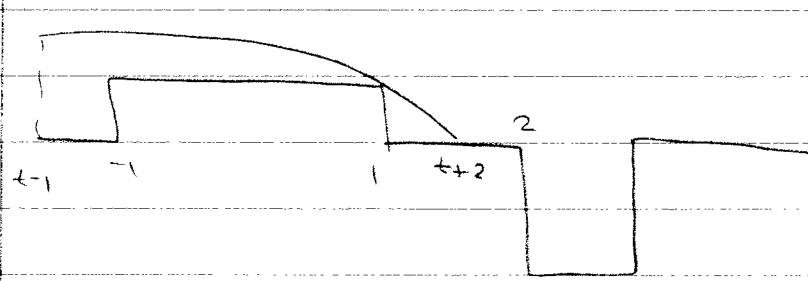






$-3 \le t \le -1$

$$y(t) = \int_{-1}^{t+2} [1 - 0.5(t-\lambda)^2][1]\, d\lambda$$



$-1 \le t \le 0 \qquad y(t) = \int_{-1}^{1} [1 - 0.5(t-\lambda)^2][1]\, d\lambda$

#5 (Continued)

Summary

$$y(t) = 0 \qquad t \le -3$$

$$y(t) = \int_{-1}^{t+2} [1-0.5(t-\lambda)^2]\,d\lambda \qquad -3 \le t \le -1$$

$$y(t) = \int_{-1}^{1} [1-0.5(t-\lambda)^2]\,d\lambda \qquad -1 \le t \le 0$$

$$y(t) = \int_{t-1}^{1} [1-0.5(t-\lambda)^2]\,d\lambda - 2\int_{2}^{t+2} [1-0.5(t-\lambda)^2]\,d\lambda \qquad 0 \le t \le 1$$

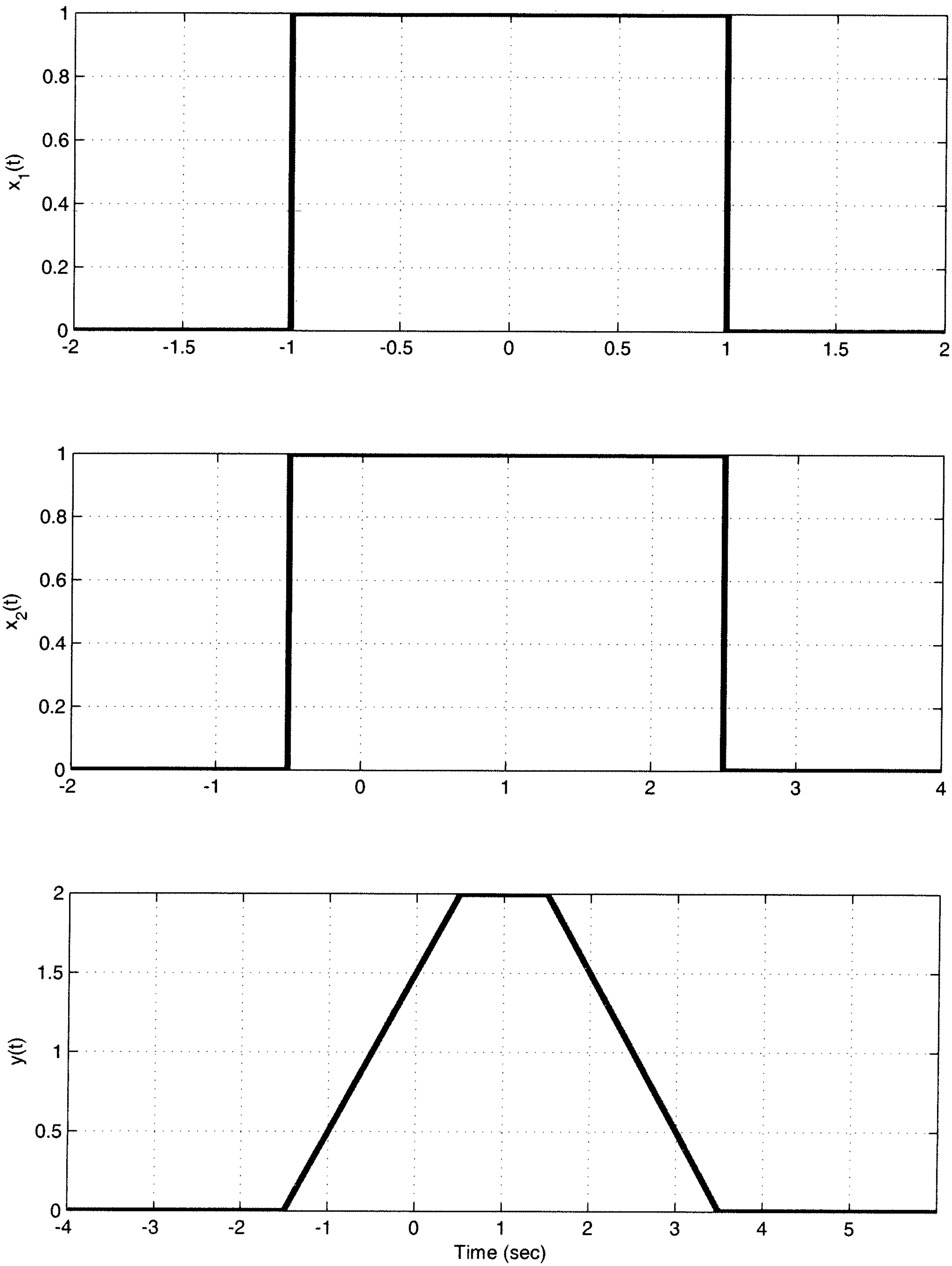
$$y(t) = \int_{t-1}^{1} [1-0.5(t-\lambda)^2]\,d\lambda - 2\int_{2}^{3} [1-0.5(t-\lambda)^2]\,d\lambda \qquad 1 \le t \le 2$$

$$y(t) = -2\int_{2}^{3} [1-0.5(t-\lambda)^2]\,d\lambda \qquad 2 \le t \le 3$$

$$y(t) = -2\int_{t-1}^{3} [1-0.5(t-\lambda)^2]\,d\lambda \qquad 3 \le t \le 4$$

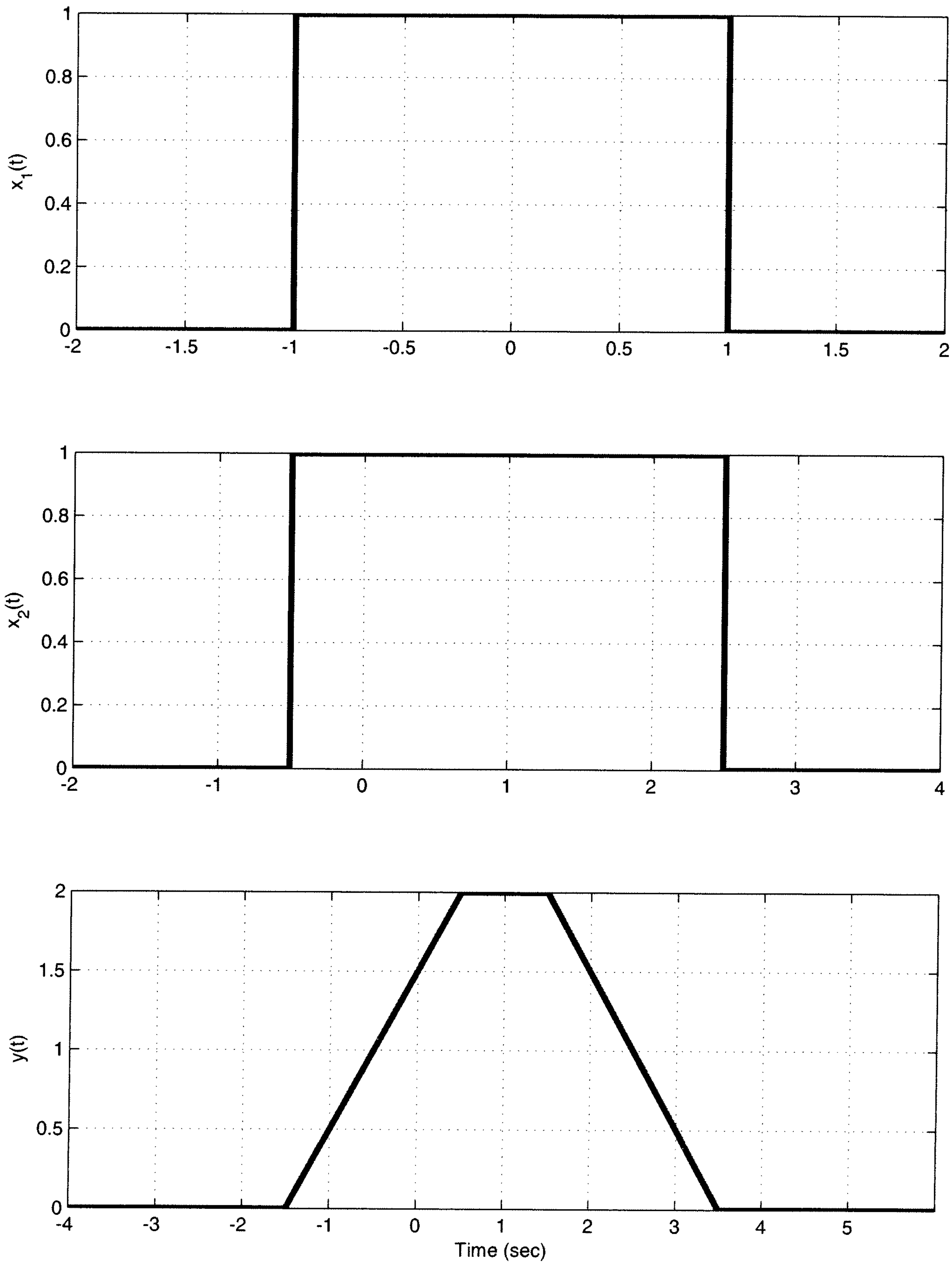
$$y(t) = 0 \qquad t \ge 4$$

```
%
% convolution problems for homework 4
%
  t1 = [-2:0.001:2];
  t2 = [-2:0.001:4];
%
  x1 = @(t) 0*(t<-1)+1*((t>=-1)&(t<=1))+0*(t>1);
  x2 = @(t) 0*(t<-0.5)+1*((t>=-0.5)&(t<=2.5))+0*(t>2.5)
%
  [y,ty] = convolve_bob(t1,t2,x1(t1),x2(t2));
%
  figure;
  orient tall
  subplot(3,1,1); plot(t1,x1(t1),'Linewidth',3); grid; ylabel('x_1(t)');
  subplot(3,1,2); plot(t2,x2(t2),'Linewidth',3); grid; ylabel('x_2(t)');
  subplot(3,1,3); plot(ty,y,'Linewidth',3);grid; axis('tight');xlabel('Time (sec)');↙
ylabel('y(t)');
```

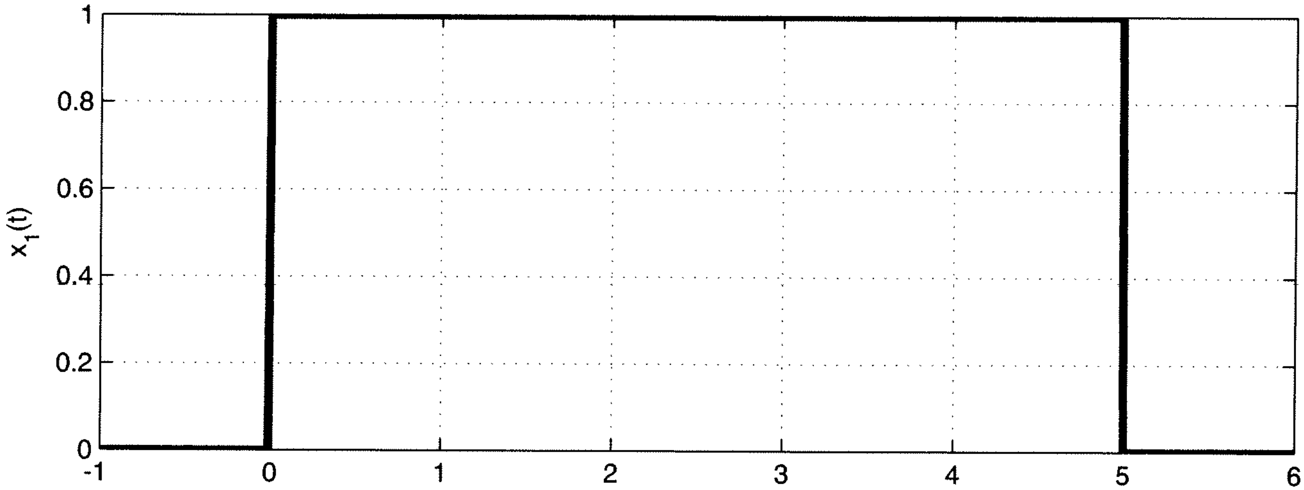


x_1(t)
x_2(t)
y(t)
Time (sec)

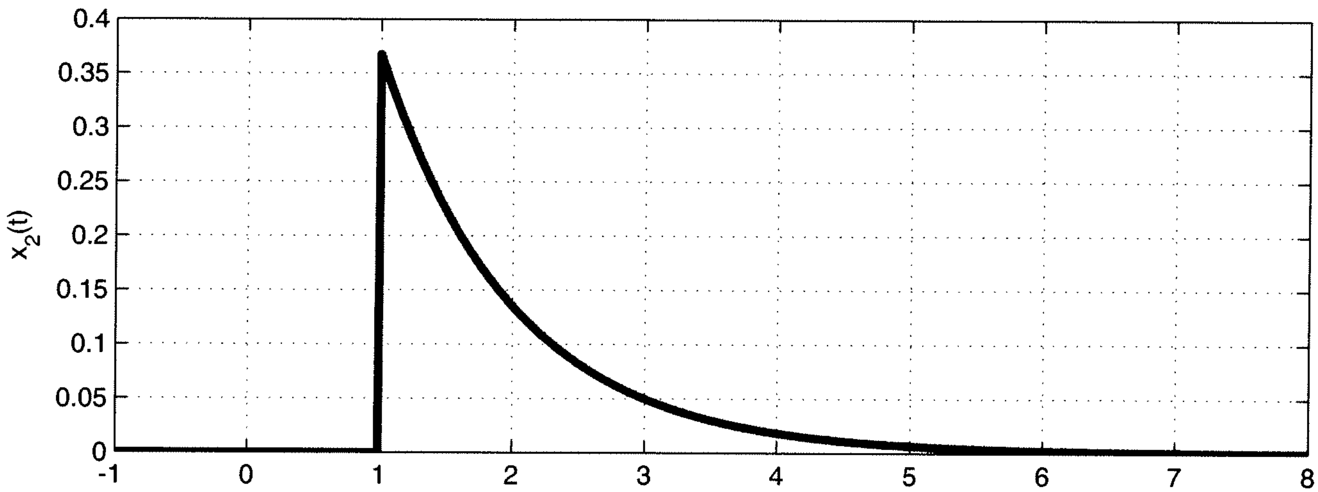
```
%
% convolution problems for homework 4
%
  t1 = [-2:0.001:2];
  t2 = [-2:0.001:4];
%
  x1 = @(t) 0*(t<-1)+1*((t>=-1)&(t<=1))+0*(t>1);
  x2 = @(t) 0*(t<-0.5)+1*((t>=-0.5)&(t<=2.5))+0*(t>2.5)
%
  [y,ty] = convolve_bob(t1,t2,x1(t1),x2(t2));
%
  figure;
  orient tall
  subplot(3,1,1); plot(t1,x1(t1),'Linewidth',3); grid; ylabel('x_1(t)');
  subplot(3,1,2); plot(t2,x2(t2),'Linewidth',3); grid; ylabel('x_2(t)');
  subplot(3,1,3); plot(ty,y,'Linewidth',3);grid; axis('tight');xlabel('Time (sec)');↙
ylabel('y(t)');
```



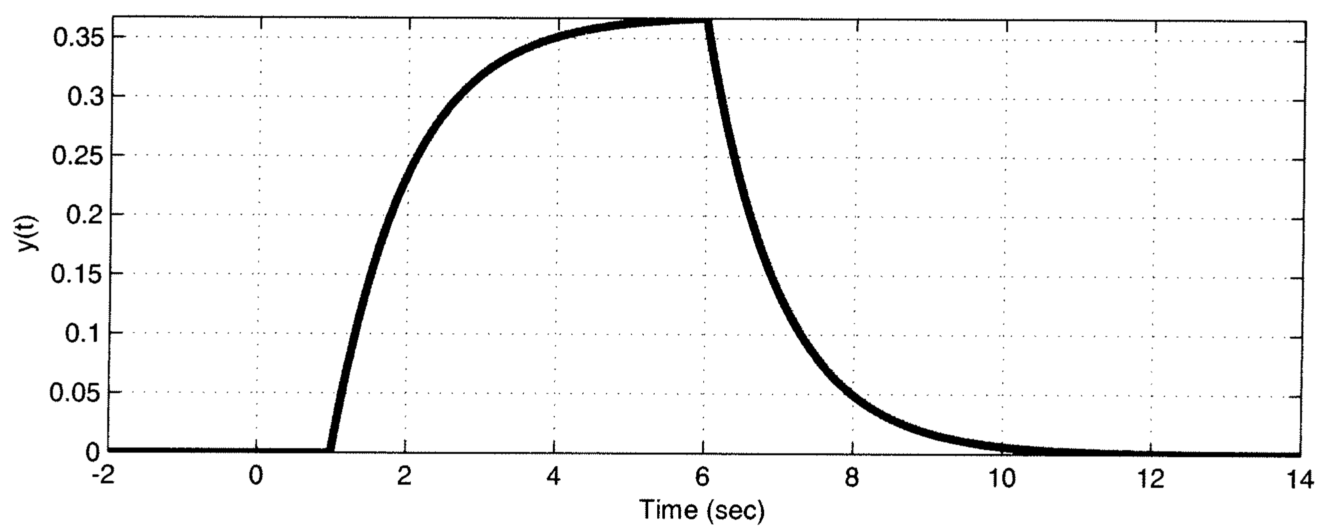
x_1(t)
1
0.8
0.6
0.4
0.2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x_2(t)
1
0.8
0.6
0.4
0.2
0
-2
-1
0
1
2
3
4
y(t)
2
1.5
1
0.5
0
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)



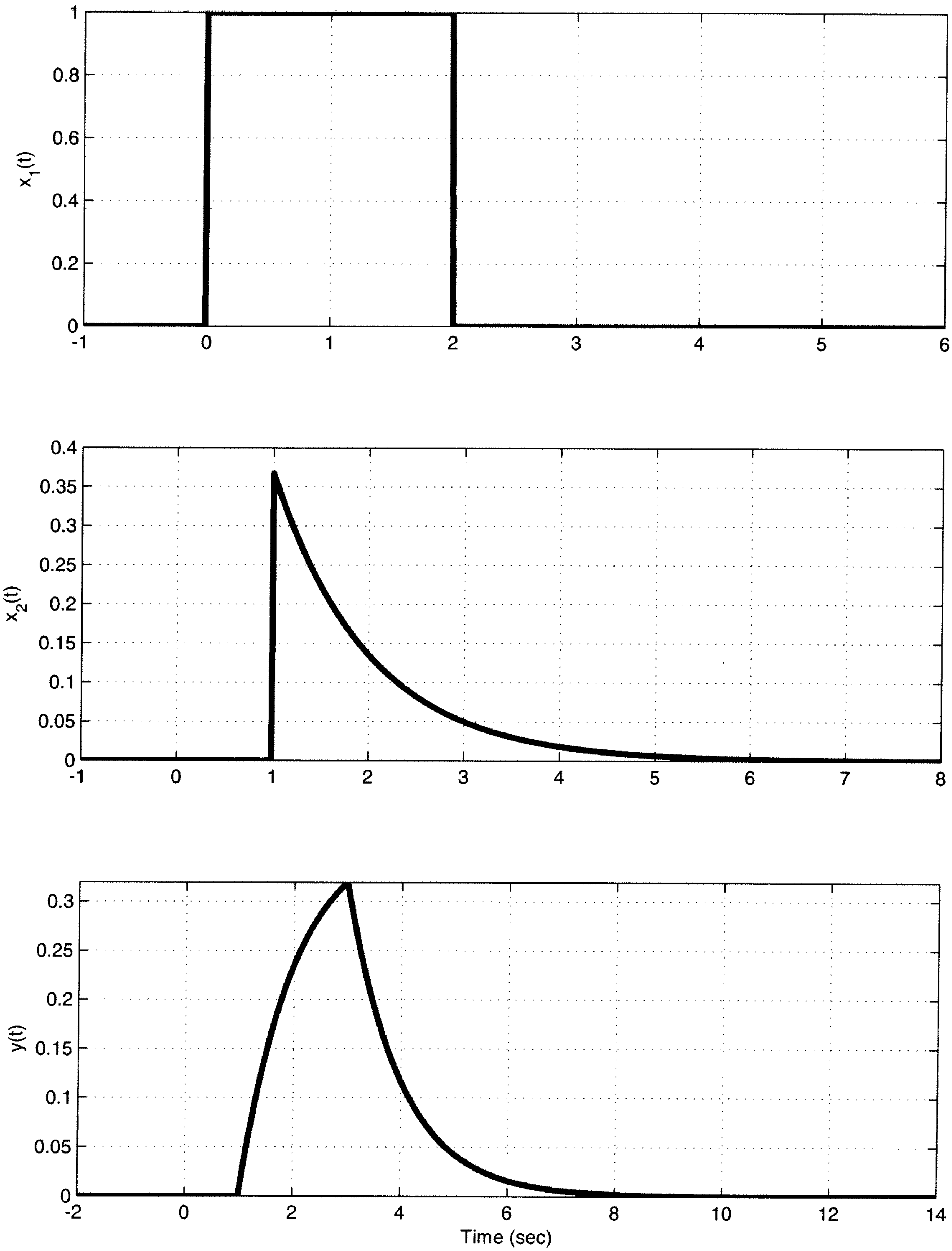
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
6
x_1(t)



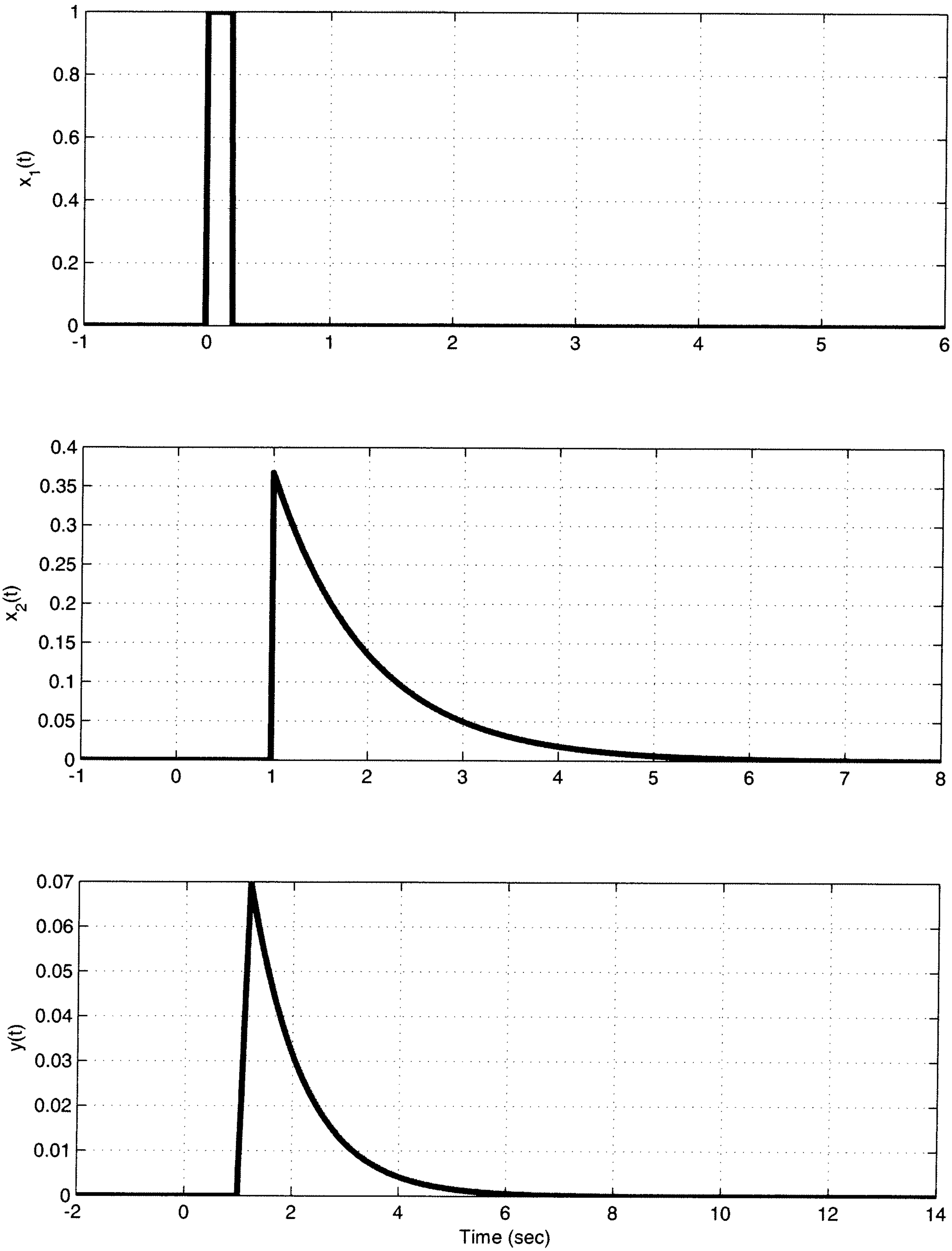
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-1
0
1
2
3
4
5
6
7
8
x_2(t)



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
0
2
4
6
8
10
12
14
Time (sec)
y(t)

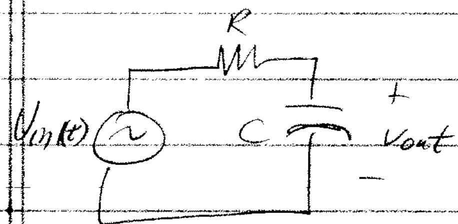


x_1(t)
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
6
x_2(t)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-1
0
1
2
3
4
5
6
7
8
y(t)
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
0
2
4
6
8
10
12
14
Time (sec)


x_1(t)
x_2(t)
y(t)
Time (sec)

#7

(a)



$$C\frac{dV_{out}}{dt} = \frac{V_{in}(t) - V_{out}(t)}{R}$$

$$RC\frac{dV_{out}}{dt} + V_{out}(t) = V_{in}(t) \qquad \dot{h}(t) + \tfrac{1}{RC} h(t) = \tfrac{1}{RC}\delta(t)$$

$$\frac{d}{dt}\left(h e^{t/RC}\right) = \frac{1}{RC} e^{t/RC}\delta(t) = \tfrac{1}{RC}\delta(t)$$

$$h(t) e^{t/RC} = \int_{-\infty}^{t} \frac{1}{RC}\delta(\lambda)\,d\lambda = \frac{1}{RC}u(t)$$

$$\boxed{h(t) = \frac{1}{RC} e^{-t/RC} u(t)} \qquad \boxed{\tau = RC}$$

(b)

$$y_s(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(t-\lambda)u(\lambda)\,d\lambda$$

$$= \int_{-\infty}^{\infty} \frac{1}{RC} e^{-(t-\lambda)/RC} u(t-\lambda)u(\lambda)\,d\lambda = \int_0^t \frac{1}{RC} e^{-(t-\lambda)/RC}\,d\lambda$$

$$= e^{-t/RC}\int_0^t \frac{1}{RC} e^{\lambda/RC}\,d\lambda = e^{-t/RC}\left[e^{\lambda/RC}\Big|_0^t\right]u(t)$$

$$= e^{-t/RC}\left[e^{t/RC} - 1\right]u(t) = \boxed{\left[1 - e^{-t/RC}\right]u(t) = y_s(t)}$$

$$y_s(t_9) = 0.9 = 1 - e^{-t_9/\tau} \qquad y(t_1) = 0.1 = 1 - e^{-t_1/\tau}$$

$$e^{-t_9/\tau} = 1 - 0.9 = 0.1 \qquad e^{-t_1/\tau} = 1 - 0.1 = 0.9$$

$$\frac{e^{-t_1/\tau}}{e^{-t_9/\tau}} = e^{(t_9 - t_1)/\tau} = \frac{0.9}{0.1} = 9 \qquad t_r = t_9 - t_1 = \boxed{\tau \ln 9 = t_r}$$

(c) $RC = 0.001$ (lots of choices)

(d) $u(t) \rightarrow y_s(t) = \left[1 - e^{-t/\tau}\right] u(t)$

$x(t) = \left[u(t) - u(t-T)\right] A$

$\rightarrow y(t) = \left[y_s(t) - y_s(t-T)\right] A$

$= A\left[1 - e^{-t/\tau}\right] u(t) - A\left[1 - e^{-(t-T)/\tau}\right] u(t-T) = y(t)$

(e) for $\frac{T}{\tau} \ll 1$ $\quad e^{-T/\tau} \approx 1 - T/\tau$ (Taylor Series)

$y_{pulse}(T) \approx A\left[1 - (1 - T/\tau)\right] = AT/\tau$