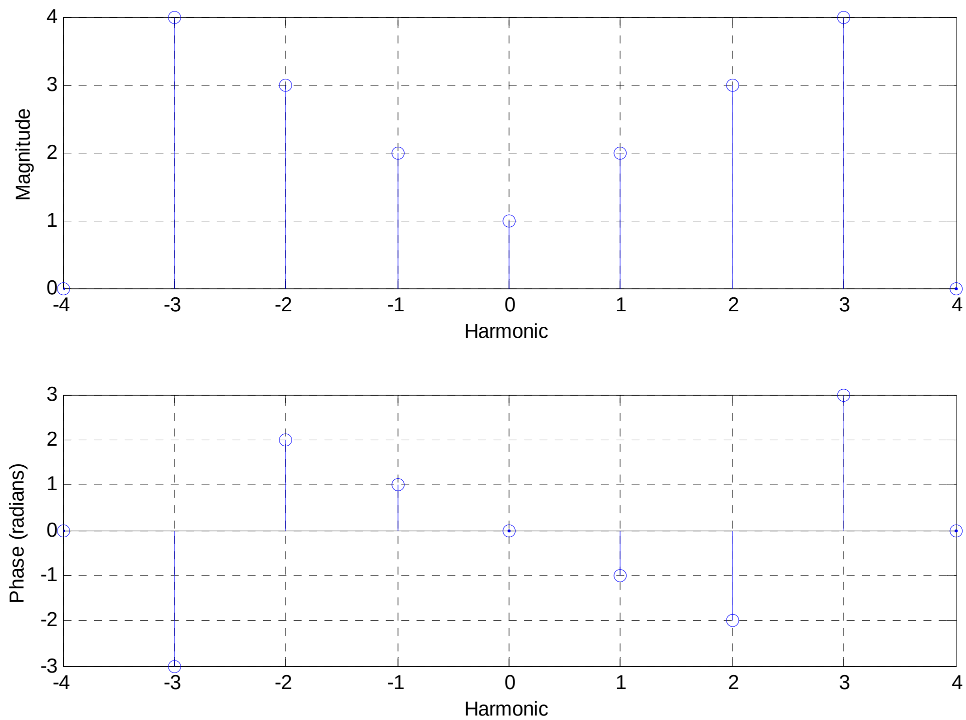
ECE 300
Signals and Systems
Homework 7

<u>Due Date:</u> Tuesday April 28, 2009 at the beginning of class

<u>Exam 2, Thursday April 30, 2009</u>

<u>Problems:</u>

1. Assume $x(t)$ has the spectrum shown below (the phase is shown in radians) and a fundamental frequency $\omega_o = 2 \text{ rad/sec}$:



Assume $x(t)$ is the input to a system with the transfer function

$$H(\omega) = \begin{cases} e^{-j\omega} & 1 \le |\omega| < 3 \\ 2e^{-j2\omega} & 3 < |\omega| < 5 \\ 0 & else \end{cases}$$

Determine an expression for the steady state output $y(t)$. Be as specific as possible, simplifying all values and using actual numbers wherever possible.

2. A periodic signal $x(t)$ is the input to an LTI system with output $y(t)$. The signal $x(t)$ has period 2 seconds, and is given over one period as

$$x(t) = e^{-t} \quad 0 < t < 2$$

$x(t)$ has the Fourier series representation

$$x(t) = \sum_k \frac{0.4323}{1 + jk\pi} e^{jk\pi t}$$

The system is an ideal highpass filter that eliminates all signals with frequency content less than 0.75 Hz.

a) Find the average power in $x(t)$.

b) Determine an expression for the output, $y(t)$. Your expression for $y(t)$ must be real.

(*Answer:* $y(t) = e^{-t} - 0.4323 - 0.2622\cos(\pi t - 1.2626)$)

c) Determine the average power in $y(t)$.

d) What fraction of the average power in $x(t)$ is contained in the DC and fundamental frequency components?

3. Assume $x(t) = t^2 \quad -\pi \le t \le \pi$ with Fourier Series representation

$$x(t) = \sum_k c_k^x e^{jkt}$$

where

$$c_k^x = \begin{cases} \dfrac{\pi^2}{3} & k = 0 \\ \dfrac{2(-1)^k}{k^2} & k \neq 0 \end{cases}$$

a) Assume $x(t)$ is the input to a system that eliminates all signals with frequencies outside the range 0.5 to 0.7 Hz. What is the output of the system $y(t)$ and what fraction of the average power in $x(t)$ is in $y(t)$? (Note: your answers must be real, no e^{ja} terms.)

b) Assume $x(t)$ is the input to a system that eliminates all signals with frequencies in the range 0.5 to 0.7 Hz. What is the output of the system $y(t)$ and what fraction of the average power in $x(t)$ is in $y(t)$?

4. Assume two periodic signals have the Fourier series representations

$$x(t) = \sum X_k e^{jk\omega_o t} \quad y(t) = \sum Y_k e^{jk\omega_o t}$$

For the following system (input/output) relationships:

a) $y(t) = bx(t-a)$
b) $y(t) = b\dot{x}(t-a)$
c) $y(t) = bx(t)\cos(\omega_o t)$ (Answer: $Y_n = \frac{b}{2}\left(X_{n-1} + X_{n+1}\right)$)
d) $\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = K\omega_n^2 x(t)$

i) write Y_k in terms of the X_k
ii) If possible, determine the system transfer function $H(j\omega)$
iii) A system must be both linear and time-invariant to have a transfer function. If you cannot determine the transfer function, indicate which system property is not satisfied (**L** or **TI).**

5. A periodic signal $x(t)$ with period T_0 has the constant component $c_0 = 2$. The signal $x(t)$ is applied to an LTI system with transfer function

$$H(j\omega) = \begin{cases} 10e^{-j5\omega} & |\omega| > \dfrac{\pi}{T_0} \\ 0 & otherwise \end{cases}$$

The output of the system $y(t)$ can be written

$$y(t) = ax(t-b) + c$$

Determine the constants $a, b,$ and c.