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ECE 300
Signals and Systems

Exam 2
30 April, 2009

NAME Solutions

This exam is closed-book in nature. *You may use a calculator for simple calculations during the exam, but not for integration.* Do not write on the back of any page, use the extra pages at the end of the exam.

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Exam 2 Total Score: _______ / 100

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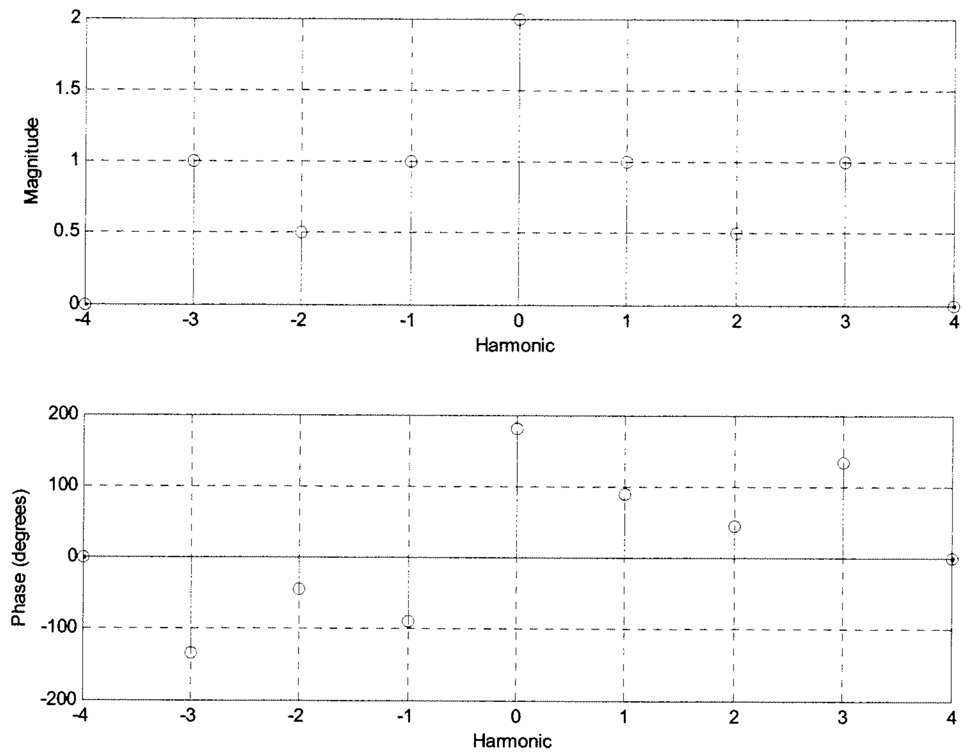
1) (20 points) The spectrum of periodic signal $x(t)$ is shown below. The period of this signal is $T_0 = 3$ seconds and all angles are multiples of 45 degrees.



a) Determine a closed-form expression for $x(t)$ in terms of cosines.

b) Sketch the single-sided power spectrum for this signal as power versus harmonic. Be sure to label all significant points (values) on your graph.

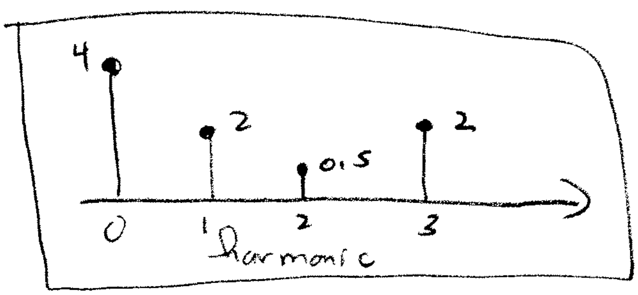
c) Compute the average power of this signal.

d) Compute the average value of this signal.

(a) $\boxed{x(t) = -2 + 2\cos\left(\frac{2\pi}{3}t + 90^\circ\right) + \cos\left(\frac{4\pi}{3}t + 45^\circ\right) + 2\cos\left(\frac{6\pi}{3}t + 135^\circ\right)}$

(c) $P_{ave} = 2^2 + 2(1^2) + 2(0.5^2) + 2(1^2) = \boxed{8.5 = P_{ave}}$

(d) $\boxed{C_0 = -2}$



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2) (25 points) The periodic function $x(t)$ is defined over one period ($T_0 = 4$ seconds) as

$$x(t) = \begin{cases} 2 & -2 \le t \le 0 \\ 0 & 0 \le t \le 2 \end{cases} \qquad \omega_o = \frac{2\pi}{T_o} = \frac{\pi}{2}$$

Determine the complex Fourier series coefficients, c_k.

Be sure to simplify your answer as much as possible and use a <u>*sinc*</u> *function if appropriate. Recall that* $1 = e^0$.

$$c_0 = \frac{1}{4}\int_{-2}^{0} 2\,dt = \frac{1}{4}(2)(2) = 1$$

$$c_K = \frac{1}{4}\int_{-2}^{0} 2e^{-jK\omega_o t}\,dt = \frac{1}{2}\left[\frac{e^{-jK\omega_o t}}{-jK\omega_o}\Bigg|_{-2}^{0}\right] = \frac{1}{2}\left[\frac{1 - e^{jK\omega_o 2}}{-jK\omega_o}\right]$$

$$= \frac{1}{K\omega_o}\left[\frac{e^{jK\omega_o 2} - 1}{2j}\right] = \frac{e^{jK\omega_o}}{K\omega_o}\left[\frac{e^{jK\omega_o} - e^{-jK\omega_o}}{2j}\right]$$

$$= \frac{e^{jK\omega_o}}{K\omega_o}\sin(K\omega_o) = \frac{e^{jK\pi/2}\,\sin\left(\frac{K\pi}{2}\right)}{\frac{K\pi}{2}} = \boxed{e^{jK\frac{\pi}{2}}\,\text{sinc}\left(\frac{K}{2}\right) = c_K}$$

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3) (15 points) Assume the periodic signal $x(t)$ with the Fourier series representation

$$x(t) = \sum_k c_k^x e^{jk\omega_o t}$$

is the input to an LTI system described by the differential equation

$$\dot{y}(t) + ay(t) = dx(t-b)$$

Since the system is LTI the output will be periodic with Fourier series representation

$$y(t) = \sum_k c_k^y e^{jk\omega_o t}$$

a) Determine an algebraic relationship between c_k^x and c_k^y

b) Determine the (continuous frequency) transfer function $H(j\omega)$ relating the input and output.

$$y(t) = \sum c_k^y e^{jk\omega_0 t} \qquad \dot{y}(t) = \sum c_k^y (jk\omega_0) e^{jk\omega_0 t}$$

$$x(t) = \sum c_k^x e^{jk\omega_0 t} \qquad x(t-b) = \sum c_k^x e^{jk\omega_0 (t-b)} = \sum c_k^x e^{-jk\omega_0 b} e^{jk\omega_0 t}$$

$$\dot{y}(t) + ay(t) = \sum \left[c_k^y (jk\omega_0) + a \right] e^{jk\omega_0 t} = dx(t-b) = \sum \left[c_k^x d e^{-jk\omega_0 b} \right] e^{jk\omega_0 t}$$

$$\boxed{c_k^y = c_k^x \frac{d e^{-jk\omega_0 b}}{jk\omega_0 + a}}$$

$$\boxed{H(j\omega) = \frac{d e^{-j\omega b}}{j\omega + a}}$$

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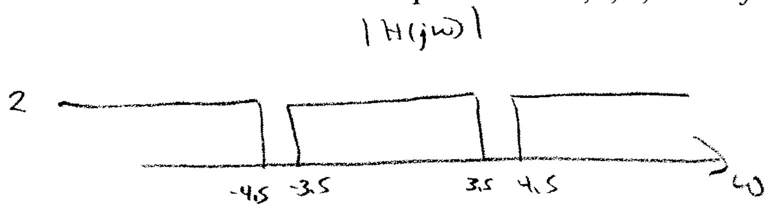
4) (20 points) The periodic signal $x(t)$ has the Fourier series representation

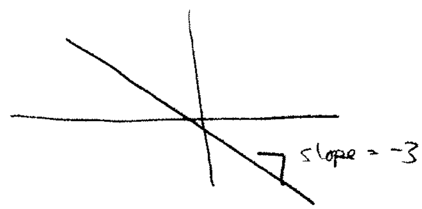
$$x(t) = \sum_{k=-\infty}^{k=\infty} \frac{1}{1+kj} e^{jk2t}$$

$x(t)$ is the input to an LTI system (a band reject or notch filter) with the transfer function

$$H(j\omega) = \begin{cases} 2e^{-j3\omega} & |0 \le |\omega| \le 3.5 \text{ and } 4.5 \le |\omega| < \infty \\ 0 & 3.5 < |\omega| < 4.5 \end{cases}$$

The steady state output of the system can be written as $y(t) = ax(t-b) + d\cos(et + f)$.
Determine numerical values for the parameters a, b, d, e and f





Assuming all frequencies pass, we would have $y(t) = 2x(t-3)$

so $\boxed{a=2,\ b=3}$

the filter removes the second harmonic ($k=2$), so $\omega = 2\cdot\omega_0 = \boxed{4 = e}$

Consider the 2^{nd} harmonic

$$2\left|\frac{1}{1+2j}\right| \cos\left(4t + \angle \frac{1}{1+2j}\right) = \frac{2}{\sqrt{5}}\cos(4t - 1.107) \rightarrow \boxed{2e^{-j3\omega}}$$

$$\rightarrow \frac{4}{\sqrt{5}}\cos(4t - 1.107 - 12)$$

so $\boxed{y(t) = 2x(t-3) - \frac{4}{\sqrt{5}}\cos(4t - 13.107 \text{ rad})}$

$\boxed{a = 2,\ b = 3,\ d = \frac{-4}{\sqrt{5}},\ e = 4,\ f = -13.107 \text{ rad}}$

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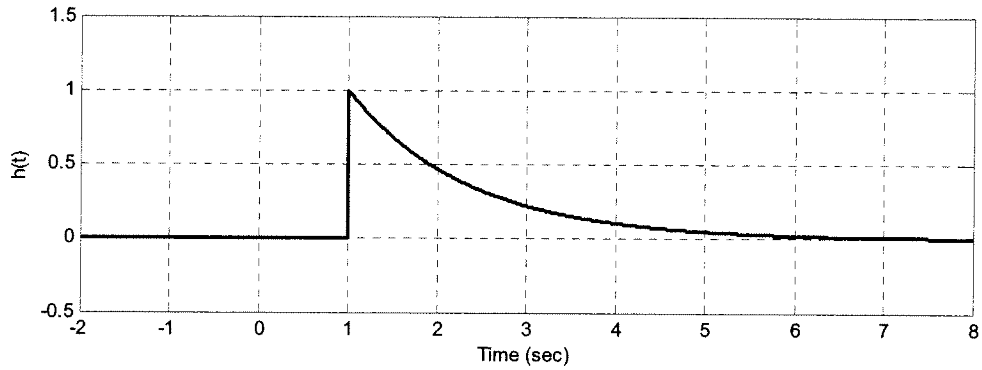
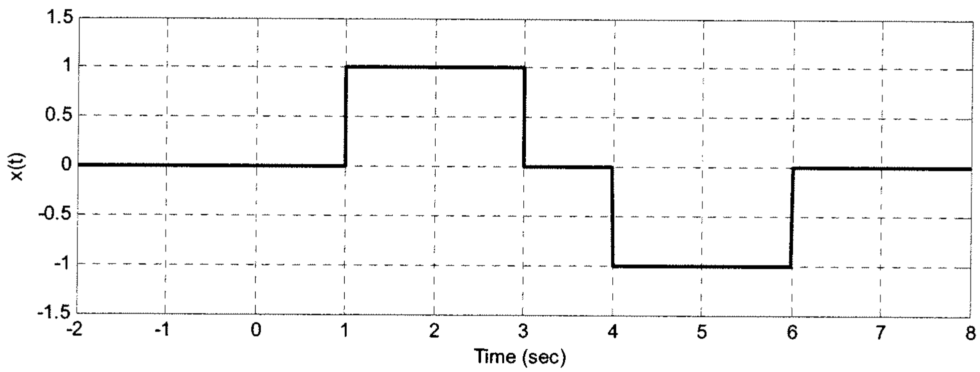
5) (20 points) Consider a causal linear time invariant system with impulse response given by

$$h(t) = e^{-0.75(t-1)}u(t-1)$$

The input to the system is given by

$$x(t) = u(t-1) - u(t-3) - u(t-4) + u(t-6)$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, <u>***NOT***</u> $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- <u>***DO NOT EVALUATE THE INTEGRALS!!***</u>

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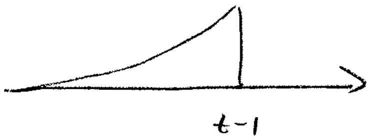
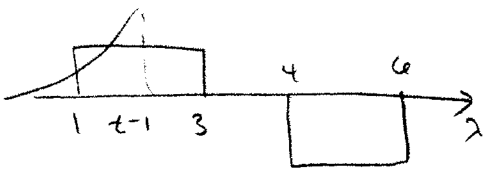
$$h(t-\lambda) = e^{-0.75(t-\lambda-1)}$$



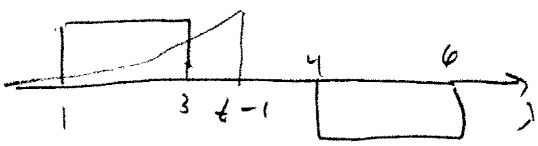
$2 \le t \le 4$



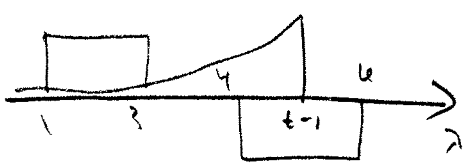
$$y(t) = \int_1^{t-1} e^{-0.75(t-\lambda-1)}\,d\lambda$$

$4 \le t \le 5$

$$y(t) = \int_1^{3} e^{-0.75(t-\lambda-1)}\,d\lambda$$



$5 \le t \le 7$



$$y(t) = \int_1^{3} e^{-0.75(t-\lambda-1)}\,d\lambda - \int_4^{t-1} e^{-0.75(t-\lambda-1)}\,d\lambda$$

$t \ge 7$

$$y(t) = \int_1^{3} e^{-0.75(t-\lambda-1)}\,d\lambda - \int_4^{6} e^{-0.75(t-\lambda-1)}\,d\lambda$$