Name ______________________________ CM__________

ECE 300
Signals and Systems

Exam 1
6 April, 2009

NAME Key

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam.

Problems 1-4	________	/ 16
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Problem 7	________	/ 25

Exam 1 Total Score: ________ / 95

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Problems 1-4 are worth 4 points each.

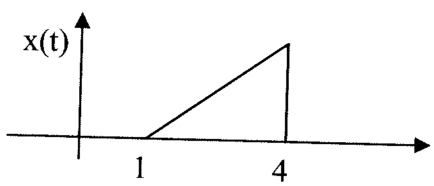
1. Which of the following statements is the best simplification of: $\int_{-2}^{t} x(\lambda - t_0)\delta(\lambda)d\lambda$

a) 0 b) $x(t-t_0)\delta(t)$ c) $x(-t_0)u(t)$ d) $x(-t_0)\delta(t)$ e) none of these

2. The the signal $x(t) = \cos(t) - j\sin(t)$ is $x(t) = e^{-jt}$

a) and energy signal b) a power signal c) neither energy nor power

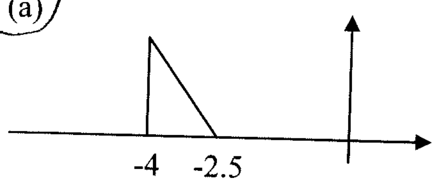
3. Given x(t) below, which of the plots labeled (a) – (d) represents $x(2(-t-2))$.



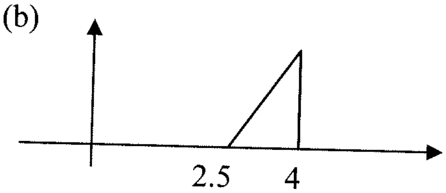
$1 = 2(-t-2)$ $\frac{1}{2} = -t-2$ $t = -2.5$

$4 = 2(-t-2)$ $2 = -t-2$ $t = -4$

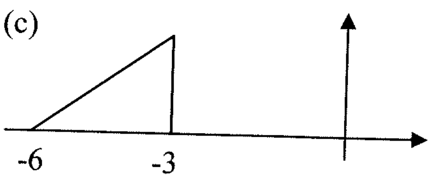
(a)



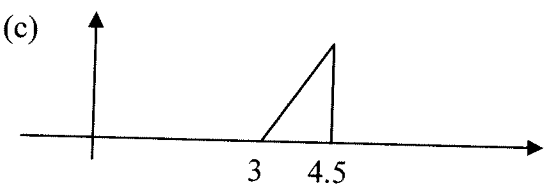
(b)



(c)



(c)



4. The signal $x(t) = \cos(4\pi t + \pi/2) + e^{j6\pi t} + 1$ is

a) not periodic

b) periodic with fundamental period 6π seconds

c) periodic with fundamental period 1 second

d) periodic with fundamental period 3/2 seconds

e) none of the above

$4\pi T = g(2\pi)$

$6\pi T = r(2\pi)$

$T = \frac{g}{2} = \frac{r}{3}$ $g = 2, r = 3$ $T = 1$

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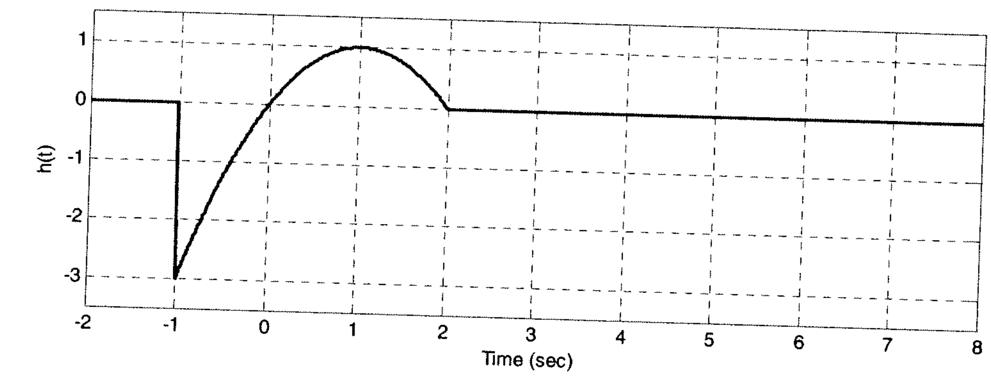
5. Graphical Convolution (29 points)

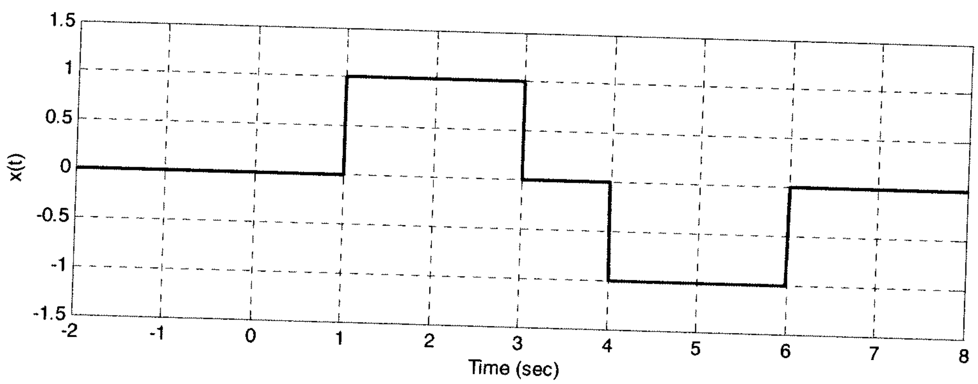
Consider a noncausal linear time invariant system with impulse response given by

$$h(t) = [1-(t-1)^2][u(t+1)-u(t-2)]$$

The input to the system is given by

$$x(t) = u(t-1) - u(t-3) - u(t-4) + u(t-6)$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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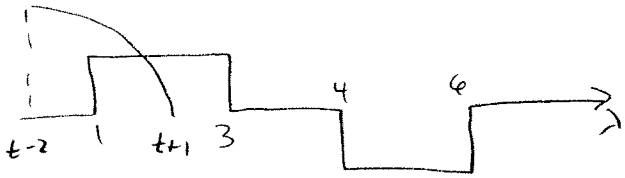
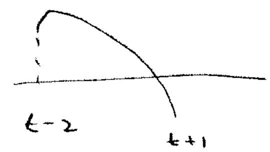
$y(t) = 0 \quad t \leq 0$

$h(2) = h(t-\lambda) \quad 2 = t-\lambda \quad \lambda = t-2$

$h(-1) = h(t-\lambda) \quad -1 = t-\lambda \quad \lambda = t+1$

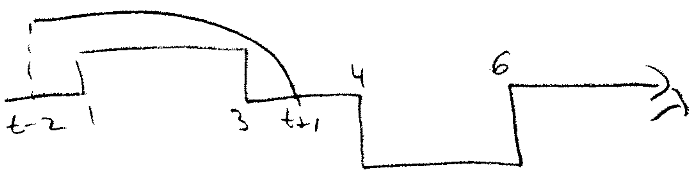
$h(t-\lambda) = 1-(t-\lambda-1)^2$





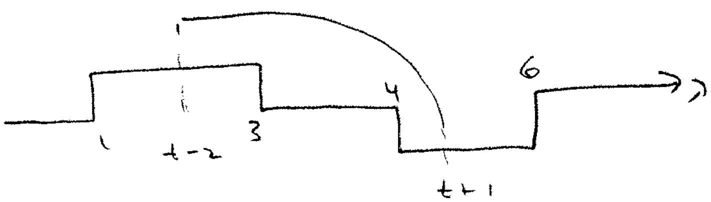
$0 \leq t \leq 2$

$$y(t) = \int_1^{t+1} [1-(t-\lambda-1)^2][1]\,d\lambda$$



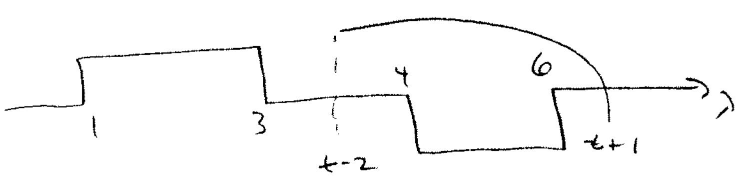
$2 \leq t \leq 3$

$$y(t) = \int_1^3 [1-(t-\lambda-1)^2][1]\,d\lambda$$



$3 \leq t \leq 5$

$$y(t) = \int_{t-2}^3 [1-(t-\lambda-1)^2][1]\,d\lambda + \int_4^{t+1} [1-(t-\lambda-1)^2][-1]\,d\lambda$$



$5 \leq t \leq 6$

$$y(t) = \int_4^6 [1-(t-\lambda-1)^2][-1]\,d\lambda$$

$y(t) = 0 \quad t \geq 8$

$6 \leq t \leq 8$

$$y(t) = \int_{t-2}^6 [1-(t-\lambda-1)^2][-1]\,d\lambda$$

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6. Impulse Response (25 points)

For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions.*

a) $y(t) = \int_{-\infty}^{t-2} e^{-(t-\lambda)} x(\lambda+1)\,d\lambda$

$\boxed{h(t) = e^{-(t+1)} u(t-1)}$

$-\infty \quad -1 \quad t-2$

$t-2 > -1$
$t > 1$

b) $2\dot{y}(t) + y(t) = x(t-1)$

$\dot{h} + \frac{1}{2} h = \frac{1}{2}\delta(t-1)$

$\frac{d}{dt}\left(h e^{t/2}\right) = \frac{1}{2} e^{t/2}\delta(t-1) = \frac{1}{2} e^{1/2}\delta(t-1)$

$h(t) e^{t/2} = \int_{-\infty}^{t} \frac{1}{2} e^{1/2}\delta(\lambda-1)\,d\lambda = \frac{1}{2} e^{1/2} u(t-1)$

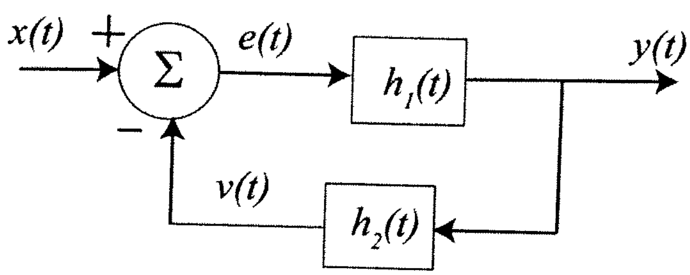
$\boxed{h(t) = \frac{1}{2} e^{-\frac{1}{2}(t-1)} u(t-1)}$

c) For the interconnected feedback system below, determine a relationship between the input $x(t)$ and the output $y(t)$ in terms of $h_1(t)$ and $h_2(t)$. Your final answer will be of the form

$$y(t) * [\delta(t) + A(t)] = x(t) * [B(t)]$$

You need to determine $A(t)$ and $B(t)$.

Hints: $e(t) = x(t) - v(t)$ and $v(t) = y(t) * h_2(t)$



From the hint (and graph)

$y(t) = e(t) * h_1(t) = [x(t) - v(t)] * h_1(t)$

$= [x(t) - y(t) * h_2(t)] * h_1(t)$

$y(t) + y(t) * h_1(t) * h_2(t) = x(t) * h_1(t)$

$y(t) * [\delta(t) + h_1(t) * h_2(t)] = x(t) * [h_1(t)]$

$\boxed{A(t) = h_1(t) * h_2(t) \qquad B(t) = h_1(t)}$

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7. System Properties (25 points)

a) Fill in the following table with a Y (Yes) or N (No). Only your responses in the table will be graded, not any work. Assume $x(t)$ is the system input and $y(t)$ is the system output. Also assume we are looking at all times (positive and negative times).

System	Linear ?	Time-Invariant?	Memoryless?	Causal?
$\dot{y}(t)+t^2y(t)=x(t+1)$	Y	N	N	N
$y(t)=x\left(-\frac{t}{2}\right)$	Y	N	N	N
$y(t)=x(t)+2$	N	Y	Y	Y
$y(t)=\lvert x(t)\rvert$	N	Y	Y	Y

b) For the system described below, determine the value of "c" that will make the system time-invariant. Use a formal technique such as we used in class (and on the homework) and justify your answer. *You will be graded more on your method of arriving at an answer than the answer itself!*

$$y(t)=e^{t}\int_{c}^{t}e^{-\lambda}x(\lambda)d\lambda$$

$$z_1 = H\{x(t-t_0)\} = e^{t}\int_{c}^{t}e^{-\lambda}x(\lambda-t_0)d\lambda$$

$$z_2 = H\{x(t)\}\Big|_{t=t-t_0} = e^{t-t_0}\int_{c}^{t-t_0}e^{-\lambda}x(\lambda)d\lambda$$

in z_1, let $\sigma=\lambda-t_0$ $\lambda=\sigma+t_0$

$$z_1 = e^{t}\int_{c-t_0}^{t-t_0}e^{-\sigma}e^{-t_0}x(\sigma)d\sigma = e^{t-t_0}\int_{c-t_0}^{t-t_0}e^{-\sigma}x(\sigma)d\sigma$$

comparing to z_1, we need $c=c-t_0$ or $\boxed{c=-\infty}$