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ECE 300
Signals and Systems

Exam 2
24 April, 2008

NAME ____

This exam is closed-book in nature. You may use a calculator for simple calculations, but not for things like integrals. You must show all of your work. Credit will not be given for work not shown.

Problem 1	________	/ 15
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Problem 4	________	/ 25
Problem 5	________	/ 25

Exam 2 Total Score: _______ / 100

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1. (15 points) Assume $x(t)$ and $y(t)$ are periodic signal with Fourier series representations, and

$$x(t) = \sum_k X_k e^{jk\omega_0 t} \qquad y(t) = \sum_k Y_k e^{jk\omega_0 t}$$

Assume also that $x(t)$ and $y(t)$ are related by the differential equation

$$y(t-a) + 2y(t) = \dot{x}(t)$$

a) Write the Y_k in terms of the X_k

b) If $x(t)$ is the input to an LTI system with transfer function $H(j\omega)$ with output $y(t)$, what is the transfer function $H(j\omega)$?

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2. (15 points) Assume we are computing the Fourier series coefficients, and after evaluating the integrals we end up with

$$X_k = \frac{e^{jk} - e^{j3k}}{jk}$$

Write X_k in terms of the **sinc** function.

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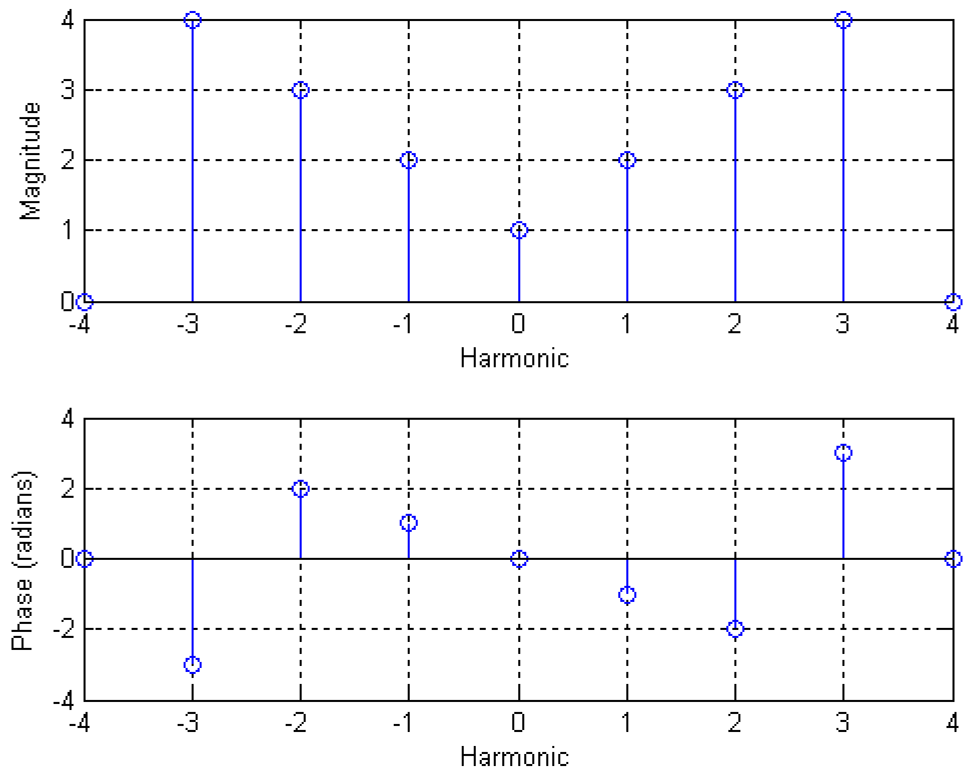
3. (20 points) Assume $x(t)$ is a periodic signal with a Fourier series representation, and the following graph displays the spectrum of $x(t)$. Assume the fundamental frequency is $\omega_0 = 4$ rad/sec. Note that the phase is in radians, and all phases are multiples of 1 radian.



a) What is the average value of $x(t)$?

b) What is the average power in $x(t)$?

c) What is the average power in the second harmonic of $x(t)$?

c) Write $x(t)$ in terms of sines and cosines.

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4. (25 points) Assume $x(t)$ is a periodic signal with Fourier series representation

$$x(t) = 2 + \sum_{k=-\infty}^{k=\infty} \frac{2}{1+jk} e^{jk4t}$$

Assume $x(t)$ is the input to an LTI system with transfer function

$$H(j\omega) = \begin{cases} 3 & |\omega| < 3 \\ 4e^{-j\frac{\omega}{10}} & 3 < |\omega| < 11 \\ 0 & |\omega| > 11 \end{cases}$$

Determine the steady state output of the system, $y(t)$. Your answer must be written in terms of sines and cosines, not complex exponentials. Your answer must also be in either degrees or radians, but not a mixture.

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5. (25 points) Graphical Convolution and System Properties

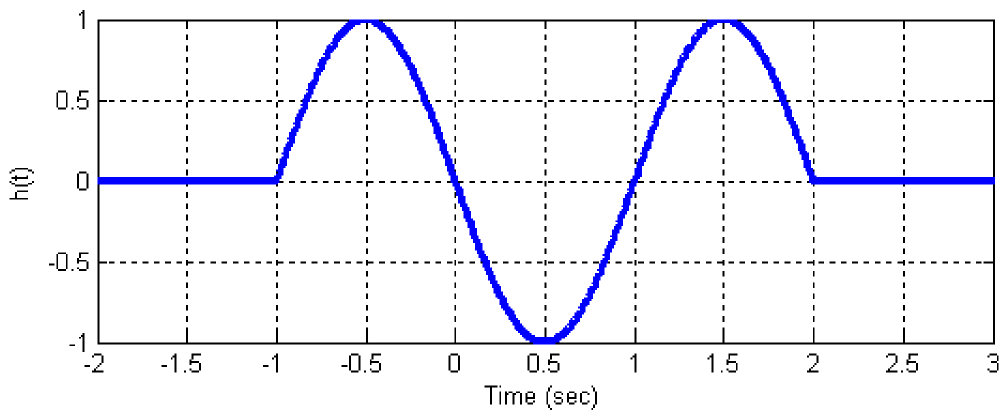
Consider a linear time invariant system with impulse response given by

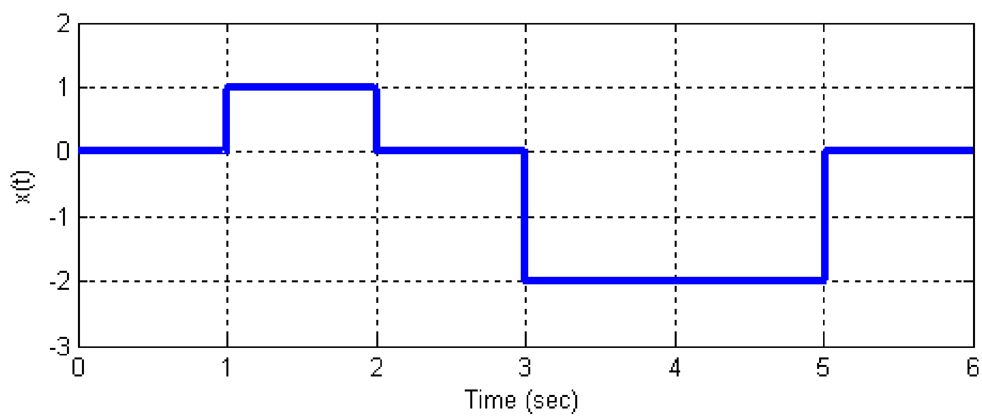
$$h(t) = -\sin(\pi t)\,[u(t+1) - u(t-2)]$$

and input

$$x(t) = u(t-1) - u(t-2) - 2u(t-3) + 2u(t-5)$$

shown below





a) Is this system causal? Why or why not?

b) Is this system BIBO stable? Why or why not?

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c) Using ***graphical convolution***, determine the output $y(t) = h(t) * x(t)$

Specifically, you must

a) Flip and slide $h(t)$, ***NOT*** $x(t)$
b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
c) Determine the range of t for which each part of your solution is valid
d) Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
e) ***DO NOT EVALUATE THE INTEGRALS!!***

Hints: (1) Pay attention to the width of h(t)
(2) It is the endpoints of h(t) that matter the most

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Some Potentially Useful Relationships

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\text{rect}\left(\frac{t-t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$