

ECE 300
Signals and Systems
Homework 2

Due Date: Tuesday March 18, 2008 *at the beginning of class*

Problems

1) (**Matlab Problem**) The **average value** of a function $x(t)$ is defined as

$$\bar{x} = \frac{1}{b-a} \int_a^b x(t) dt$$

and the **root-mean-square (rms)** value of a function is defined as

$$x_{rms} = \sqrt{\frac{1}{b-a} \int_a^b x^2(t) dt}$$

Read the **Appendix**, then

a) use Matlab to find the average and rms values of the function $x(t) = t^2$ for $-1 < t < 1$

b) use Matlab to find the average and rms values of the following functions

$$x(t) = \cos(t) \quad 0 < t < \pi$$

$$x(t) = \cos(t) \quad 0 < t < 2\pi$$

$$x(t) = |t| \quad -1 < t < 1$$

$$x(t) = t \cos(t) \quad -2 < t < 4$$

Hint: You will probably find the sqrt function useful.

For this problem you can just copy down the answers from the Matlab screen. We will assume you are doing this in Matlab because if you are not it will soon become obvious....

2) ZTF, Problem 1-38 (parts **a,b,c,d**, and **f**).

3) ZTF, Problem 1-42.

4) Consider the following mathematical models of systems:

a $y(t) = e^{-(t+1)}x(t)$ **b** $y(t) = \begin{cases} x(t) & |x(t)| \leq 10 \\ 10 & |x(t)| > 10 \end{cases}$ **c** $y(t) = x\left(1 - \frac{t}{2}\right)$

d $y(t) = x\left(\frac{t}{3}\right) + 2$ **e** $y(t) = \sin(x(t))$ **f** $y(t) = 1 - e^{-x(t)}$

g $\frac{dy(t)}{dt} = y(t)x(t)$ **h** $\frac{dy(t)}{dt} = 3y(t) + 2x(t)$ **i** $\frac{dy(t)}{dt} = 3ty(t) + 2x(t)$

j $y(t) = \int_{-\infty}^t (t - \lambda)x(\lambda + 1)d\lambda$

Fill in the following table (Y or N for each question) for each system. You must justify your answers to receive credit. Assume t can be any possible value.

| Part | Causal? | Memoryless? | Linear? | Time Invariant? |
|-----------------|---------|-------------|---------|-----------------|
| a | | | | |
| b | | | | |
| c | | | | |
| d | | | | |
| e- $x(t)$ large | | | | |
| e- $x(t)$ small | | | | |
| f- $x(t)$ large | | | | |
| f- $x(t)$ small | | | | |
| g | | | | |
| h | | | | |
| i | | | | |
| j | | | | |

For parts **e** and **f**, look at the case when $x(t)$ is large, and then when $x(t)$ is assumed to be sufficiently small that you can use a small value (Taylor series) approximation. Even though $x(t)$ is a function, you can still use the same approximations you would use if it was just a small number.

For part **g**, you should show $y(t) = y(t_0)e^{\int_{t_0}^t x(\lambda)d\lambda}$ in order to determine whether the system is or is not causal and has memory or is memoryless.

For part h you should show $y(t) = y(t_0)e^{3(t-t_0)} + \int_{t_0}^t 2e^{3(t-\lambda)}x(\lambda)d\lambda$ in order to

determine the system is or is not causal and has memory or is memoryless.

For part i you should solve the DE first (see handout about integrating factors) and then determine whether the system is or is not causal and has memory or is memoryless. Your solution to the DE will have a similar form to that given for part g.

5) For the system $y(t) = e^t \int_c^t e^{-\lambda}x(\lambda)d\lambda$, determine the value of “c” that will make the system time-invariant. Use a formal technique such as you used in problem 4 to justify your answer.

Appendix

Maple is often used for symbolically integrating a function. Sometimes, though, what we really care about is the numerical value of the integral. Rather than integrating symbolically, we might want to just use numerical integration to evaluate the integral. Since we are going to be using Matlab a great deal in this course, in this appendix we will learn to use one of Matlab’s built-in functions for numerical integration. In order to efficiently use this function, we will learn how to construct what are called *anonymous* functions. We will then use this information to determine the average and rms value of a function. Some of this is going to seem a bit strange at first, so just try and learn from the examples.

Numerical Integration in Matlab Let’s assume we want to numerically integrate the following:

$$I = \int_0^{2\pi} (t^2 + 2)dt$$

In order to do numerical integration in Matlab, we will use the built-in command **quadl**. The **arguments** to quadl, e.g., the information passed to quadl, are

- A function which represents the integrand (the function which is being integrated). Let’s call the integrand $x(t)$. This function must be written in such a way that it returns the value of $x(t)$ at each time t . Clearly here $x(t) = t^2 + 2$
- The lower limit of integration, here that would be 0
- The upper limit of integration, here that would be 2π

Note that an optional fourth argument is the tolerance, which defaults to 10^{-6} . When the function value is very small, or the integration time is very small, you will have to change this.

Anonymous Functions Let's assume we wanted to use Matlab to construct the function $x(t) = t^2 + 2$. We can do this by creating what Matlab calls an **anonymous function**. To do this, we type into Matlab

```
x = @(t) t.*t+2;
```

If we want the value of $x(t)$ at $t = 2$, we just type `x(2)`

Hence, to evaluate the integral $I = \int_0^{2\pi} (t^2 + 2)dt$ in Matlab we would type

```
x = @(t) t.*t+2;
I = quadl(x,0,2*pi)
```

Note that it is important to define x **before** it is used by (passed to) `quadl`

Example 1 To numerically evaluate $I = \int_{-1}^1 e^{-t} \cos(2t)dt$ we could type

```
x = @(t) exp(-t).*cos(2*t);
I = quadl(x,-1,1);
```

Example 2 To numerically evaluate $I = \int_{-2}^1 |t| e^{-|t|} dt$ we could type

```
y = @(t) abs(t).*exp(-abs(t));
I = quadl(y,-2,1);
```

Integrating Products of Functions Sometimes we are going to want to integrate the product of functions. While we could just multiply the functions together, it is usually easier to let Matlab do it for us.

Let's assume we want to evaluate the integral $I = \int_0^1 x(t)y(t)dt$, and let's assume that we already have anonymous functions x and y . The function **quadl** needs to be passed a function which is the product of x and y . To do this, we make a new anonymous function z , using the following:

```
z = @(t) x(t).*y(t);
```

and then perform the integration

```
I = quadl(z,0,1)
```

An alternative is to write

```
I = quadl(@(t) x(t).*y(t),0,1);
```

Example 3 To numerically evaluate $I = \int_{-1}^1 e^{-t} \cos(2t) dt$ we could type

```
x = @(t) exp(-t);  
y = @(t) cos(2*t);  
z = @(t) x(t).*y(t);  
I = quadl(z,-1,1);
```

or

```
I = quadl(@(t) x(t).*y(t),-1,1);
```

Example 4 To numerically evaluate $I = \int_{-2}^1 |t| e^{-|t|} dt$ we could type

```
x = @(t) abs(t);  
y = @(t) exp(-abs(t));  
z = @(t) x(t).*y(t);  
I = quadl(z,-2,1);
```

or

```
I = quadl(@(t) x(t).*y(t),-2,1);
```

```
%
% script for homework 2
%
% define the function for part a
%
x = @(t) t.*t;
y = @(t) x(t).*x(t);
%
% compute the average
%
ave = (1/2)*quadl(x,-1,1)
%
% compute the rms
%
rms = sqrt((1/2)*quadl(y,-1,1))
%
% now define the functions for part b
%
x = @(t) cos(t);
y = @(t) x(t).*x(t);
%
ave = (1/pi)*quadl(x,0,pi)
rms = sqrt((1/pi)*quadl(y,0,pi))
%
ave = (1/(2*pi))*quadl(x,0,2*pi)
rms = sqrt(1/(2*pi)*quadl(y,0,2*pi))
%
x = @(t) abs(t);
y = @(t) x(t).*x(t);
%
ave = (1/2)*quadl(x,-1,1)
rms = sqrt((1/2)*quadl(y,-1,1))
%
x = @(t) t.*cos(t);
y = @(t) x(t).*x(t);
%
ave = (1/6)*quadl(x,-2,4)
rms = sqrt((1/6)*quadl(y,-2,4))
```

To get started, select MATLAB Help or Demos from the Help menu.

ave =

0.3333 \bar{x} for $x(t) = t^2$

rms =

0.4472 x_{rms}

ave =

4.9335e-017 \bar{x} for $x(t) = \cos(t)$ $0 < t < \pi$

rms =

0.7071 x_{rms}

ave =

7.8024e-013 \bar{x} for $x(t) = \cos(t)$ $0 < t < 2\pi$

rms =

0.7071 x_{rms}

ave =

\bar{x} for $x(t) = |t|$ $-1 < t < 1$

0.5000

rms =

0.5774 x_{rms}

ave =

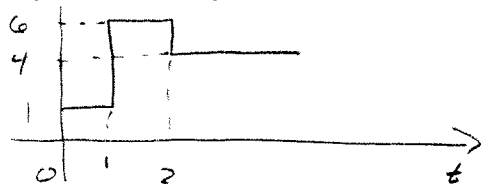
-0.8472 \bar{x} for $x(t) = t \cos(t)$ $-2 < t < 2$

rms =

1.5652

#2 1-38 $E_{\infty} = \int_{-\infty}^{\infty} |x(t)|^2 dt$ $P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$

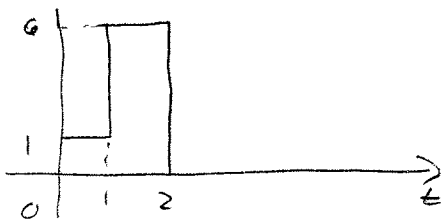
(a) $x(t) = u(t) + 5u(t-1) - 2u(t-2)$



$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\int_0^1 1^2 dt + \int_1^2 6^2 dt + \int_2^T 4^2 dt \right] = \lim_{T \rightarrow \infty} \frac{16(T-2)}{2T} = \boxed{8 = P_{\infty}}$$

Power signal

(b) $x(t) = u(t) + 5u(t-1) - 6u(t-2)$



$$E_{\infty} = \int_0^1 1^2 dt + \int_1^2 6^2 dt = 1 + 36 = \boxed{37 = E_{\infty}}$$

Energy signal

(c) $x(t) = e^{-st} u(t)$

$$E_{\infty} = \int_{-\infty}^{\infty} e^{-10t} u(t) dt = \int_0^{\infty} e^{-10t} dt = \left. \frac{-e^{-10t}}{10} \right|_0^{\infty} = \frac{1}{10} = \boxed{0.1 = E_{\infty}}$$

Energy signal

(d) $x(t) = (e^{-st} + 1) u(t)$

$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T (1 + e^{-st})^2 u(t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T [1 + 2e^{-st} + e^{-10t}] dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\int_0^T 1 dt + \int_0^T 2e^{-st} dt + \int_0^T e^{-10t} dt \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \left[T + \frac{2}{s}(1 - e^{-sT}) + \frac{1}{10}(1 - e^{-10T}) \right] = \boxed{\frac{1}{2} = P_{\infty}}$$

Power signal

(f) $x(t) = r(t) = t u(t)$

$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T t^2 u(t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T t^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \frac{T^3}{3} = \infty$$

neither power or energy

#3

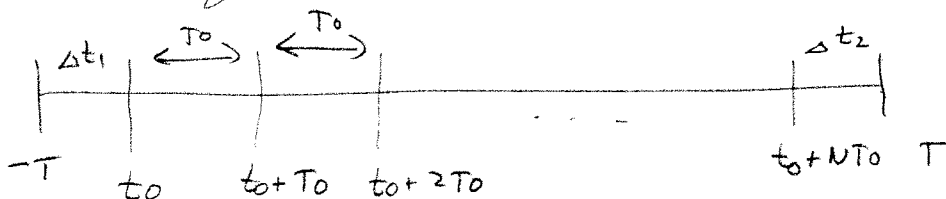
ZTF 1-42,

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad \text{eg. 1-76}$$

$$P_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |x(t)|^2 dt \quad \text{eg. 1-84}$$

Use eg. 1-76 to prove eg. 1-84

Consider breaking up the time interval as follows



$$\begin{aligned} \text{So } \int_{-T}^T |x(t)|^2 dt &= \int_{-T}^{t_0} |x(t)|^2 dt + N \int_{t_0}^{t_0+T_0} |x(t)|^2 dt + \int_{t_0+NT_0}^T |x(t)|^2 dt \\ &= \epsilon - N + N \int_{t_0}^{t_0+T_0} |x(t)|^2 dt + \epsilon N \end{aligned}$$

$$2T = NT_0 + \Delta t_1 + \Delta t_2 \quad \text{where } \Delta t_1 < T_0 \quad \Delta t_2 < T_0$$

$$\lim_{T \rightarrow \infty} 2T = \lim_{N \rightarrow \infty} (NT_0 + \Delta t_1 + \Delta t_2)$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt = \lim_{N \rightarrow \infty} \frac{1}{NT_0 + \Delta t_1 + \Delta t_2} \left[\epsilon - N + \epsilon N + N \int_{t_0}^{t_0+T_0} |x(t)|^2 dt \right]$$

$$= \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |x(t)|^2 dt = P$$

#4 System Properties

a) $y(t) = e^{-(t+1)} x(t)$

Causal? $y(t)$ only depends on x at the same time, so causal
 memoryless? and memoryless

Linear? $z_1 = \mathcal{H}\{d_1 x_1 + d_2 x_2\} = e^{-(t+1)} [d_1 x_1(t) + d_2 x_2(t)]$

$$z_2 = d_1 \mathcal{H}\{x_1\} + d_2 \mathcal{H}\{x_2\} = d_1 [e^{-(t+1)} x_1(t)] + d_2 [e^{-(t+1)} x_2(t)]$$

$$= e^{-(t+1)} [d_1 x_1(t) + d_2 x_2(t)] = z_1 \quad \underline{\text{linear}}$$

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = e^{-(t+1)} x(t-t_0)$

$$z_2 = \mathcal{H}\{x(t)\} \Big|_{t=t-t_0} = e^{-(t-t_0+1)} x(t-t_0) \quad z_1 \neq z_2$$

not time-invariant

b) $y(t) = \begin{cases} x(t) & |x(t)| < 10 \\ 10 & |x(t)| > 10 \end{cases}$

Causal? memoryless? $y(t)$ only depends on x at the same time,
 so causal and memoryless

Linear? Assume $x(t) = 5$ then $y(t) = 5$
 Next assume $x(t) = 20 = 4 \cdot 5$ (scaled original input by 4)
 then $y(t) = 10$ (scaled original input by 2)
not linear

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = \begin{cases} x(t-t_0) & |x(t-t_0)| < 10 \\ 10 & |x(t-t_0)| > 10 \end{cases}$

$$z_2 = \mathcal{H}\{x(t)\} \Big|_{t=t-t_0} = \begin{cases} x(t-t_0) & |x(t-t_0)| < 10 \\ 10 & |x(t-t_0)| > 10 \end{cases}$$

so time-invariant

$$\textcircled{c} y(t) = x\left(1 - \frac{t}{2}\right)$$

causal? $y(0) = x(1)$ not causal

memoryless? $y(0) = x(1)$ not memoryless

Linear? $z_1 = \mathcal{H}\{d_1 x_1 + d_2 x_2\} = d_1 x_1\left(1 - \frac{t}{2}\right) + d_2 x_2\left(1 - \frac{t}{2}\right)$

$$z_2 = d_1 \mathcal{H}\{x_1\} + d_2 \mathcal{H}\{x_2\} = d_1 x_1\left(1 - \frac{t}{2}\right) + d_2 x_2\left(1 - \frac{t}{2}\right)$$

$z_1 = z_2$ so linear

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = x\left(1 - \frac{t-t_0}{2} - t_0\right)$

$$z_2 = \mathcal{H}\{x(t)\} \Big|_{t=t-t_0} = x\left(1 - \frac{(t-t_0)}{2}\right)$$

$z_1 \neq z_2$ so not time-invariant

$$\textcircled{d} y(t) = x\left(\frac{t}{3}\right) + 2$$

causal? $y(-1) = x\left(\frac{-1}{3}\right) + 2$ not causal

memoryless? not memoryless

Linear? $z_1 = \mathcal{H}\{d_1 x_1(t) + d_2 x_2(t)\}$
 $= d_1 x_1\left(\frac{t}{3}\right) + d_2 x_2\left(\frac{t}{3}\right) + 2$

$$z_2 = d_1 \mathcal{H}\{x_1(t)\} + d_2 \mathcal{H}\{x_2(t)\}$$

$$= d_1 \left[x_1\left(\frac{t}{3}\right) + 2 \right] + d_2 \left[x_2\left(\frac{t}{3}\right) + 2 \right]$$

$$= d_1 x_1\left(\frac{t}{3}\right) + d_2 x_2\left(\frac{t}{3}\right) + 2(d_1 + d_2) \neq z_1 \text{ for all } d_1, d_2$$

not linear

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = x\left(\frac{t-t_0}{3}\right) + 2$

$$z_2 = \mathcal{H}\{x(t)\} \Big|_{t=t-t_0} = x\left(\frac{t-t_0}{3}\right) + 2$$

$z_1 = z_2$ not time-invariant

$$\textcircled{e} y(t) = \sin(x(t))$$

x(t) - Large

Causal, memoryless? $y(t)$ only depends on x at time t ,
so the system is causal and memoryless

Linear? $z_1 = \mathcal{H}\{\alpha_1 x_1 + \alpha_2 x_2\} = \sin(\alpha_1 x_1(t) + \alpha_2 x_2(t))$
 $z_2 = \alpha_1 \mathcal{H}\{x_1\} + \alpha_2 \mathcal{H}\{x_2\} = \alpha_1 \sin(x_1(t)) + \alpha_2 \sin(x_2(t)) \neq z_1$
not linear

Time-Invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = \sin(x(t-t_0))$
 $z_2 = \mathcal{H}\{x(t)\} \Big|_{t=t-t_0} = \sin(x(t-t_0)) = z_1$ so time-invariant

x(t) - small so $y(t) = \sin(x(t)) \approx x(t)$ ($\sin \theta \approx \theta$ for θ small)

$$y(t) = x(t)$$

clearly causal, memoryless, linear, and time-invariant

$$\textcircled{f} y(t) = 1 - e^{-x(t)}$$

x(t) large

Causal, memoryless? The system is clearly causal and memoryless

Linear? $z_1 = \mathcal{H}\{\alpha_1 x_1(t) + \alpha_2 x_2(t)\} = 1 - e^{-(\alpha_1 x_1(t) + \alpha_2 x_2(t))}$
 $z_2 = \alpha_1 \mathcal{H}\{x_1\} + \alpha_2 \mathcal{H}\{x_2\} = \alpha_1 [1 - e^{-x_1(t)}] + \alpha_2 [1 - e^{-x_2(t)}]$
 $z_1 \neq z_2$ not linear

Time-invariant? $z_1 = \mathcal{H}\{x(t-t_0)\} = 1 - e^{-x(t-t_0)}$
 $z_2 = \mathcal{H}\{x(t)\} \Big|_{t=t-t_0} = 1 - e^{-x(t-t_0)} = z_1$ time-invariant

x(t) small so $y(t) = 1 - [1 - x(t)] = x(t)$ since $e^{-a} \approx 1 - a$

This system is causal, memoryless, linear, and time-invariant.

$$g) \frac{dy}{dt} = y(t) x(t)$$

$$\frac{dy}{y(t)} = x(t) dt \Rightarrow \int_{t_0}^t \frac{dy}{y(\lambda)} = \int_{t_0}^t x(\lambda) d\lambda$$

$$y(t) = y(t_0) e^{\int_{t_0}^t x(\lambda) d\lambda} \quad \ln(y(t)) - \ln(y(t_0)) = \ln\left(\frac{y(t)}{y(t_0)}\right) = \int_{t_0}^t x(\lambda) d\lambda$$

causal? yes, $y(t)$ depends on the values of the input $x(t)$ only up till time t

memoryless? no, clearly has memory since there is an integral

$$\text{Linear? } z_1 = \{d_1 x_1 + d_2 x_2\} = y(t_0) e^{\int_{t_0}^t [d_1 x_1(\lambda) + d_2 x_2(\lambda)] d\lambda}$$

$$z_2 = d_1 \{x_1\} + d_2 \{x_2\} = d_1 y(t_0) e^{\int_{t_0}^t x_1(\lambda) d\lambda} + d_2 y(t_0) e^{\int_{t_0}^t x_2(\lambda) d\lambda}$$

$z_1 \neq z_2$ so not linear

$$\underline{DC} \quad \dot{y}_1 = y_1 x_1 \quad \dot{y}_2 = y_2 x_2$$

$$d_1 \dot{y}_1 = d_1 y_1 x_1 \quad d_2 \dot{y}_2 = d_2 y_2 x_2$$

$$d_1 \dot{y}_1 + d_2 \dot{y}_2 = d_1 y_1 x_1 + d_2 y_2 x_2$$

cannot write DE in terms of $Y = d_1 y_1 + d_2 y_2$ and $X = d_1 x_1 + d_2 x_2$

so not linear

$$\text{Time-invariant? } z_1 = \{x(t-t_0)\} \rightarrow \dot{y}(t-t_0) = y(t-t_0) x(t-t_0)$$

$$z_2 = \{x(t)\} \Big|_{t=t-t_0} \rightarrow \left[\dot{y}(t) = y(t) x(t) \right]_{t=t-t_0} = z_1$$

so Time-Invariant

$$\textcircled{2} \dot{y} = 3y + 2x$$

$$\dot{y} - 3y = 2x \quad \frac{d}{dt} (y(t) e^{-3t}) = 2x(t) e^{-3t}$$

$$\int_{t_0}^t \frac{d}{d\lambda} (y(\lambda) e^{-3\lambda}) d\lambda = y(t) e^{-3t} - y(t_0) e^{-3t_0} = \int_{t_0}^t 2x(\lambda) e^{-3\lambda} d\lambda$$

$$y(t) = y(t_0) e^{3(t-t_0)} + \int_{t_0}^t 2 e^{3(t-\lambda)} x(\lambda) d\lambda$$

Causal? yes, $y(t)$ only depends on the input up until time t
 memoryless? no, $y(t)$ depends on the input at more than just the current time

$$\text{Linear? } \dot{y}_1 = 3y_1 + 2x_1 \quad \dot{y}_2 = 3y_2 + 2x_2$$

$$d_1 \dot{y}_1 = 3d_1 y_1 + 2d_1 x_1 \quad d_2 \dot{y}_2 = 3d_2 y_2 + 2d_2 x_2$$

$$(d_1 \dot{y}_1 + d_2 \dot{y}_2) = 3(d_1 y_1 + d_2 y_2) + 2(d_1 x_1 + d_2 x_2)$$

$$\dot{Y} = 3Y + 2X \quad Y = d_1 y_1 + d_2 y_2 \quad X = d_1 x_1 + d_2 x_2$$

so linear

$$\text{Time-invariant? } z_1 = \{x(t-t_0)\} \rightarrow \dot{y}(t-t_0) = 3y(t-t_0) + 2x(t-t_0)$$

$$z_2 = \{x(t)\} \Big|_{t=t-t_0} \rightarrow \left[\dot{y}(t) = 3y(t) + 2x(t) \right]_{t=t-t_0} = z_1$$

so time-invariant

$$(i) \frac{dy}{dt} = 3ty(t) + 2x(t)$$

$$\dot{y} - 3ty = 2x \quad \frac{d}{dt} \left(e^{-\frac{3}{2}t^2} y(t) \right) = e^{-\frac{3}{2}t^2} x(t)$$

$$\int_{t_0}^t \frac{d}{d\lambda} \left(e^{-\frac{3}{2}\lambda^2} y(\lambda) \right) d\lambda = e^{-\frac{3}{2}t^2} y(t) - e^{-\frac{3}{2}t_0^2} y(t_0) = \int_{t_0}^t e^{-\frac{3}{2}\lambda^2} x(\lambda) d\lambda$$

$$y(t) = y(t_0) e^{\frac{3}{2}(t^2 - t_0^2)} + \int_{t_0}^t e^{-\frac{3}{2}(t^2 - \lambda^2)} x(\lambda) d\lambda$$

Causal? yes, $y(t)$ only depends on x up until time t

Memoryless? no, $y(t)$ depends on the input at more times than t

Linear? $\dot{y}_1 = 3ty_1 + 2x_1 \quad \dot{y}_2 = 3ty_2 + 2x_2$

$$\alpha_1 \dot{y}_1 = 3t \alpha_1 y_1 + 2 \alpha_1 x_1 \quad \alpha_2 \dot{y}_2 = 3t \alpha_2 y_2 + 2 \alpha_2 x_2$$

$$(\alpha_1 \dot{y}_1 + \alpha_2 \dot{y}_2) = 3t(\alpha_1 y_1 + \alpha_2 y_2) + 2(\alpha_1 x_1 + \alpha_2 x_2)$$

$$\dot{Y} = 3tY + 2X \quad Y = \alpha_1 y_1 + \alpha_2 y_2 \quad X = \alpha_1 x_1 + \alpha_2 x_2$$

so linear

Time-invariant? $Z_1 = \mathcal{F}\{x(t-t_0)\} \rightarrow \dot{y}(t-t_0) = 3ty(t-t_0) + 2x(t-t_0)$

$$Z_2 = \mathcal{F}\{x(t)\} \Big|_{t=t-t_0} \rightarrow \left[\dot{y}(t) = 3ty(t) + 2x(t) \right]_{t=t-t_0} \neq Z_1$$

not time-invariant

$$\textcircled{1} y(t) = \int_{-\infty}^t (t-\lambda) x(\lambda+1) d\lambda$$

causal? no, $y(t)$ depends on $x(t+1)$

memoryless? no, $y(t)$ depends on $x(t+1)$

$$\text{Linear? } z_1 = \mathcal{H}\{\alpha_1 x_1 + \alpha_2 x_2\} = \int_{-\infty}^t (t-\lambda) [\alpha_1 x_1(\lambda+1) + \alpha_2 x_2(\lambda+1)] d\lambda$$

$$z_2 = \alpha_1 \mathcal{H}\{x_1\} + \alpha_2 \mathcal{H}\{x_2\}$$

$$= \alpha_1 \int_{-\infty}^t (t-\lambda) x_1(\lambda+1) d\lambda + \alpha_2 \int_{-\infty}^t (t-\lambda) x_2(\lambda+1) d\lambda$$

$$= \int_{-\infty}^t (t-\lambda) [\alpha_1 x_1(\lambda+1) + \alpha_2 x_2(\lambda+1)] d\lambda = z_1$$

linear

Time-invariant?

$$z_1 = \mathcal{H}\{x(t-t_0)\} = \int_{-\infty}^t (t-\lambda) x(\lambda-t_0+1) d\lambda$$

$$z_2 = \mathcal{H}\{x(t)\} \Big|_{t \rightarrow t-t_0} = \int_{-\infty}^{t-t_0} (t-t_0-\lambda) x(\lambda+1) d\lambda$$

$$z_1 = \int_{-\infty}^t (t-\lambda) x(\lambda-t_0+1) d\lambda \quad \text{let } \sigma = \lambda - t_0 \quad d\sigma = d\lambda$$

$$\quad \quad \quad \sigma + t_0 = \lambda$$

$$= \int_{-\infty}^{t-t_0} (t - (\sigma+t_0)) x(\sigma+1) d\sigma = \int_{-\infty}^{t-t_0} (t-t_0-\sigma) x(\sigma+1) d\sigma = z_2$$

so time-invariant!

(#5) $g(t) = e^t \int_c^t e^{-\lambda} x(\lambda) d\lambda$ what value of c will make this time-invariant?

$$z_1 = \mathcal{H}\{x(t-t_0)\} = e^{+t} \int_c^t e^{-\lambda} x(\lambda-t_0) d\lambda$$

$$z_2 = \mathcal{H}\{x(t)\} \Big|_{t=t-t_0} = e^{t-t_0} \int_c^{t-t_0} e^{-\lambda} x(\lambda) d\lambda$$

$$\text{in } z_1 = e^t \int_c^t e^{-\lambda} x(\lambda-t_0) d\lambda, \text{ let } \sigma = \lambda - t_0 \\ \sigma + t_0 = \lambda \quad d\sigma = d\lambda \\ = e^t \int_{c-t_0}^{t-t_0} e^{-(\sigma+t_0)} x(\sigma) d\sigma = e^{t-t_0} \int_{c-t_0}^{t-t_0} e^{-\sigma} x(\sigma) d\sigma$$

for $z_1 = z_2$, we need $c = c - t_0$ or $c = \pm \infty$

The only answer that really makes sense here is $\boxed{c = -\infty}$