ECE 300
Signals and Systems
Homework 7

Due Date: Tuesday April 29 at the beginning of class

Problems

1) Show that if $x(t)$ is real and even, then $X(\omega)$ is real and even, and if $x(t)$ is real and odd, then $X(\omega)$ is imaginary and odd.

2) Show that if $x(t)$ is real, then $|X(\omega)| = |X(-\omega)|$ (the magnitude is even) and $\measuredangle X(-\omega) = -\measuredangle X(\omega)$ (the phase is odd).

3) By evaluating the Fourier transform integral directly, and using Euler's identiy, show that the Fourier transform of $x(t) = \cos(t)\left[u(t+\frac{\pi}{2}) - u(t-\frac{\pi}{2}) \right]$

is $X(\omega) = \dfrac{2\cos\left(\omega\dfrac{\pi}{2}\right)}{1-\omega^2}$

4) By evaluating the integral by hand, show that the Fourier transform of $x(t) = e^{-t}u(t)$ is given by

$$X(\omega) = \frac{1}{1+j\omega} = \frac{1}{\sqrt{1+\omega^2}} \measuredangle - \tan^{-1}(\omega)\left[\frac{180}{\pi}\right] \text{degrees}$$

5) By evaluating the integral by hand, show that the Fourier transform of $x(t) = e^{-|t|}$ is

$$X(\omega) = \frac{2}{1+\omega^2}$$

6 In this problem you will utilize your Matlab program **Complex_Fourier_Series.m** to demonstrate that as the period of a periodic function increases, the Fourier series approximates the Fourier transform. Use your answers for Problems 4 and 5 to plot the Fourier transform results.
   a) Use your program to determine the Fourier series for $x(t) = e^{-t}u(t)$ over the time interval [-4,4]. Hence the period $T = 8$ in this case. You should be sure to look at the Fourier series representation to verify everything is correct.
   b) Construct a vector $W = [-N:N]\omega_0$ just as you did in lab. This will make plotting and evaluation functions much easier.

Spring 2008

c) Construct the vector C = [fliplr(conj(c)) c0 c]. This will make plotting easier.
d) Modify your code to plot the amplitude $T|c_k|$ versus $k\omega_o$ and the phase $\angle c_k$ (in degrees) versus $k\omega_o$. You should use the **subplot** command and plot both on one page. You should use the command **orient tall** before any plotting to use more of the page. Some Matlab commands you might find useful are **angle**, **length**, and **abs**. Instead of using the stem command, you should use the plot command and plot discrete points, like dots ('.').
e) By using the **axis** command, limit the axes for magnitude plot to the range -8 to 8 radians/sec and from 0 to the maximum value of $T|c_k|$. The **max** command may prove useful here. Limit the axes for the phase plot to the range -8 to 8 radians/sec and from -90 to 90 degrees.
f) Add plots of the magnitude and phase of $X(\omega)$ on the existing plots. It might be easiest if you define an anonymous function for this (just as you did for ($x(t)$)). You may need the functions **sqrt** and **./** or **.*** Use a solid line type and be sure to add legends. If you have done everything correctly, and you use N=100 points in the Fourier series, you should get the plot shown in Figure 1. Be sure to modify the title and axes so they look like those in this figure (to get c_k type c_k , and to get ω_0 type \omega_0)
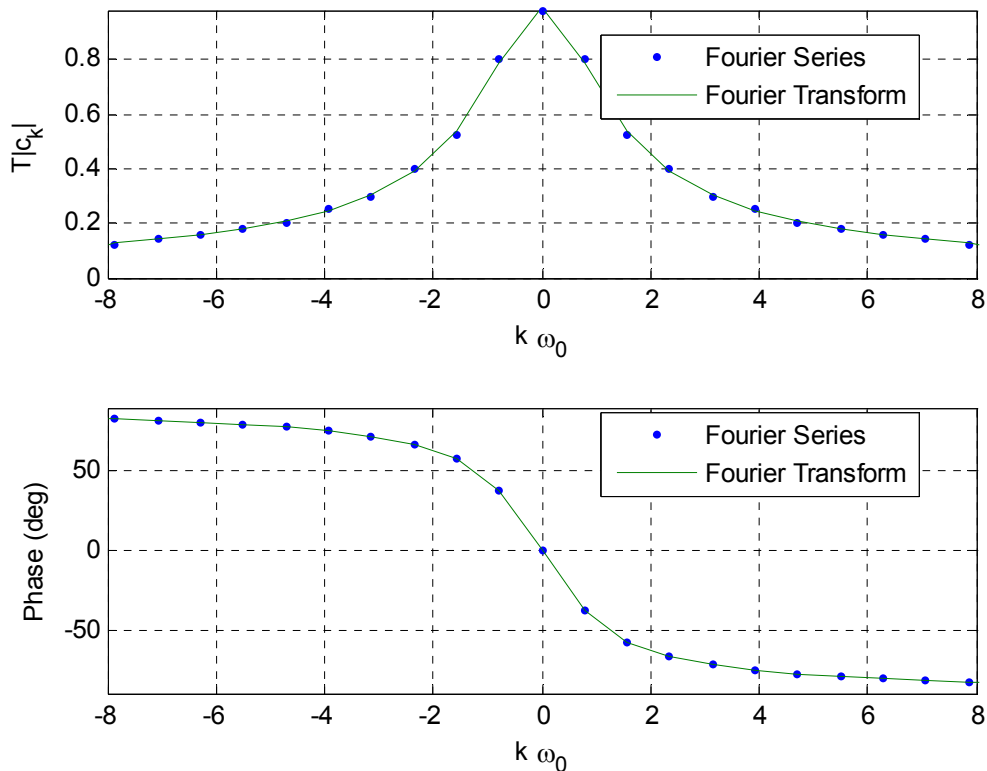

Figure 1: Example plots for part **d**.

g) Change the duration of the periodic signal to [-8,8] (so the period is T = 16) and [-16,16] (so the period is T = 32) and rerun your code. Try and run a duration of [-32,32] (so the period T = 64), though this may not work well. Turn in your plots. Keep the number of points at $N = 100$. Do not change $x(t)$. Here we are increasing the period of the function $x(t)$ to demonstrate that the Fourier transform is just a Fourier series in the limit as $T \to \infty$, $k \left(\dfrac{2\pi}{T} \right) = k\omega_0 \to \omega$, and $Tc_k \to X(\omega)$

h) Redo the above for $x(t) = e^{-|t|}$. Turn in 3 plots for this part (4 plots if you can get T=64 to go...).