Name ______________________________ CM__________

ECE 300
Signals and Systems

Exam 2
24 April, 2008

NAME Solution

This exam is closed-book in nature. You may use a calculator for simple calculations, but not for things like integrals. You must show all of your work. Credit will not be given for work not shown.

Problem 1	________	/ 15
Problem 2	________	/ 15
Problem 3	________	/ 20
Problem 4	________	/ 25
Problem 5	________	/ 25

Exam 2 Total Score: _______ / 100

Name Solution CM

1. (15 points) Assume $x(t)$ and $y(t)$ are periodic signal with Fourier series representations, and

$$x(t) = \sum_k X_k e^{jk\omega_0 t} \qquad y(t) = \sum_k Y_k e^{jk\omega_0 t}$$

Assume also that $x(t)$ and $y(t)$ are related by the differential equation

$$y(t-a) + 2y(t) = \dot{x}(t)$$

a) Write the Y_k in terms of the X_k

b) If $x(t)$ is the input to an LTI system with transfer function $H(j\omega)$ with output $y(t)$, what is the transfer function $H(j\omega)$?

(a) $$\sum Y_k e^{jk\omega_0(t-a)} + \sum 2Y_k e^{j\omega_0 t} = \sum X_k (jk\omega_0) e^{jk\omega_0 t}$$

$$\sum \left[Y_k (e^{-jk\omega_0 a} + 2) - X_k (jk\omega_0) \right] e^{jk\omega_0 t} = 0$$

$$\boxed{Y_k = X_k \frac{jk\omega_0}{2 + e^{-jk\omega_0 a}}}$$

(b) $Y_k = X_k H(jk\omega_0)$ $$\boxed{H(j\omega) = \frac{j\omega}{2 + e^{-j\omega a}}}$$

Name Solution CM

2. (15 points) Assume we are computing the Fourier series coefficients, and after evaluating the integrals we end up with

$$X_k = \frac{e^{jk} - e^{j3k}}{jk}$$

Write X_k in terms of the **sinc** function.

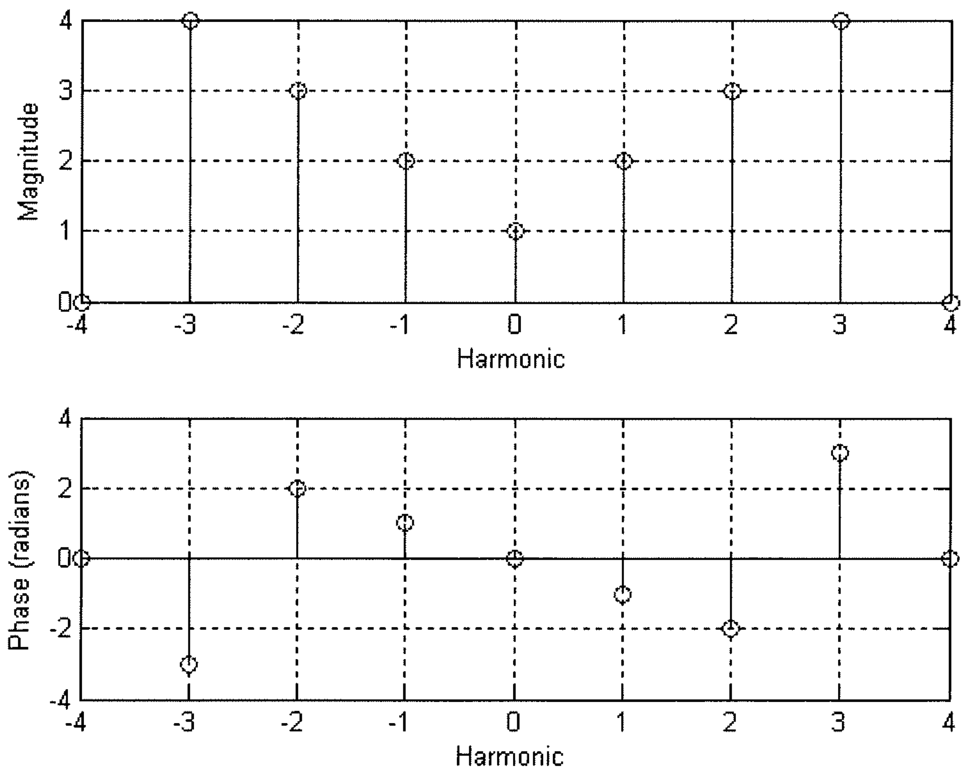
$$X_k = \frac{e^{jk} - e^{j3k}}{jk} = -e^{j2k} \frac{\left[e^{jk} - e^{-jk}\right]}{jk} \frac{2}{2}$$

$$= -2e^{j2k} \frac{\sin(k)}{k} = -2e^{j2k} \frac{\sin\left(\pi \cdot k/\pi\right)}{\left(\pi \cdot k/\pi\right)}$$

$$\boxed{X_k = -2e^{j2k} \operatorname{sinc}\left(\frac{k}{\pi}\right)}$$

Name Solution CM

3. (20 points) Assume $x(t)$ is a periodic signal with a Fourier series representation, and the following graph displays the spectrum of $x(t)$. Assume the fundamental frequency is $\omega_0 = 4$ rad/sec. Note that the phase is in radians, and all phases are multiples of 1 radian.



a) What is the average value of $x(t)$? $\overline{X} = c_0 = 1\angle 0^\circ = 1$

b) What is the average power in $x(t)$? $P_{ave} = \sum |c_k|^2 = 4^2+3^2+2^2+1^2+2^2+3^2+4^2 = 59$

c) What is the average power in the second harmonic of $x(t)$? $2|c_2|^2 = 2\cdot 9 = 18$

c) Write $x(t)$ in terms of sines and cosines.

$$\boxed{x(t) = 1 + 4\cos(4t-1) + 6\cos(8t-2) + 8\cos(12t+3)}$$

Name Solution CM

4. (25 points) Assume $x(t)$ is a periodic signal with Fourier series representation

$$x(t) = 2 + \sum_{k=-\infty}^{k=\infty} \frac{2}{1+jk} e^{jk4t}$$

Assume $x(t)$ is the input to an LTI system with transfer function

$$H(j\omega) = \begin{cases} 3 & |\omega| < 3 \\ 4e^{-j\frac{\omega}{10}} & 3 < |\omega| < 11 \\ 0 & |\omega| > 11 \end{cases}$$

Determine the steady state output of the system, $y(t)$. Your answer must be written in terms of sines and cosines, not complex exponentials. Your answer must also be in either degrees or radians, but not a mixture.

$\omega_0 = 4$

$c_0^y = c_0^x H(0) = (2+2)(3) = 12$

$c_1^y = c_1^x H(4) = \left(\frac{2}{2+j}\right)\left(4e^{-j0.4}\right) = \left(\frac{2}{\sqrt{2}} \measuredangle -\frac{\pi}{4}\right)\left(4 \measuredangle -0.4\right) = 5.657 \measuredangle -1.185 \text{ rad}$

$c_2^y = c_2^x H(8) = \left(\frac{2}{1+2j}\right)\left(4e^{-j0.8}\right) = \left(\frac{2}{\sqrt{5}} \measuredangle -1.107\right)\left(4 \measuredangle -0.8\right) = 3.578 \measuredangle -1.907 \text{ rad}$

$y(t) = c_0^y + 2|c_1^y| \cos(\omega_0 t + \measuredangle c_1^y) + 2|c_2^y| \cos(2\omega_0 t + \measuredangle c_2^y)$

$\boxed{y(t) = 12 + 11.31 \cos(4t - 1.185) + 7.16 \cos(8t - 1.907)}$

Name Solution CM

5. (25 points) Graphical Convolution and System Properties

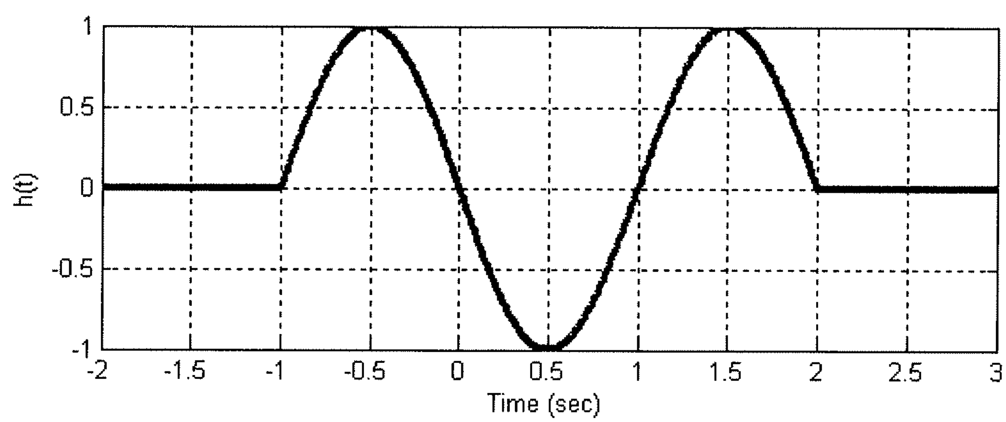
Consider a linear time invariant system with impulse response given by

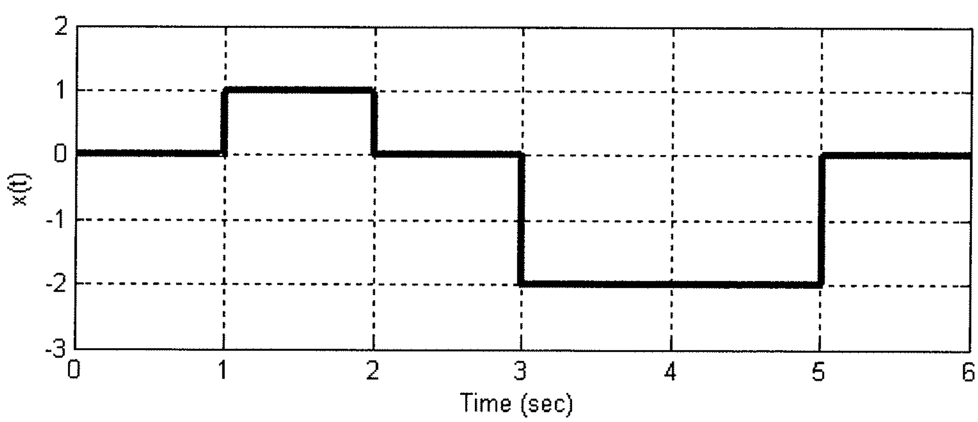
$$h(t) = -\sin(\pi t)\,[u(t+1) - u(t-2)]$$

and input

$$x(t) = u(t-1) - u(t-2) - 2u(t-3) + 2u(t-5)$$

shown below





a) Is this system causal? Why or why not? no $h(t) \neq 0$ for $t < 0$

b) Is this system BIBO stable? Why or why not? yes $\int_{-\infty}^{\infty} |h(t)|\, dt$ is finite

Name Solution CM

c) Using ***graphical convolution***, determine the output $y(t) = h(t) * x(t)$

Specifically, you must

a) Flip and slide $h(t)$, ***NOT*** $x(t)$
b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
c) Determine the range of t for which each part of your solution is valid
d) Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
e) ***DO NOT EVALUATE THE INTEGRALS!!***

Hints: (1) Pay attention to the width of h(t)
(2) It is the endpoints of h(t) that matter the most

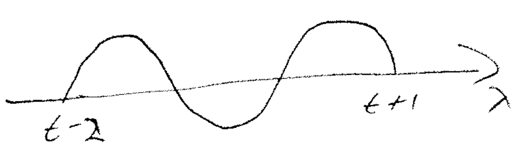
$h(2) = h(t-\lambda) \quad 2 = t-\lambda \quad \lambda = t-2$

$h(-1) = h(t-\lambda) \quad -1 = t-\lambda \quad \lambda = t+1$

$$y(t) = \int_{-\infty}^{\infty} h(t-\lambda)\, x(\lambda)\, d\lambda$$

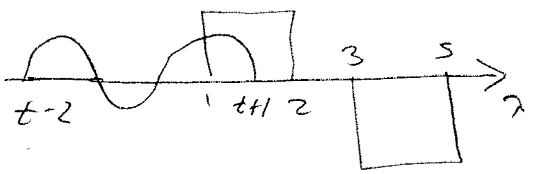
$h(t-\lambda) = -\sin(\pi(t-\lambda))$

$h(t-\lambda)$



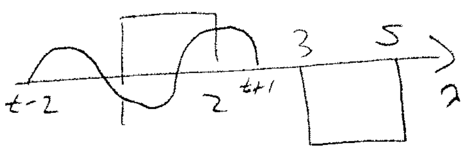
Name Solution CM

$y(t) = 0 \quad t < 0 \qquad h(t-\lambda) = -\sin(\pi(t-\lambda))$



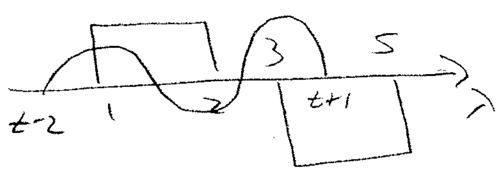
$0 \le t \le 1 \quad y(t) = \int_1^{t+1} -\sin(\pi(t-\lambda))(1)\,d\lambda$

$1 \le t \le 2 \quad y(t) = \int_1^{2} -\sin(\pi(t-\lambda))(1)\,d\lambda$



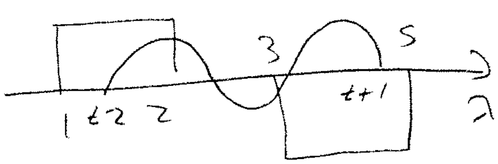
$2 \le t \le 3 \quad y(t) = \int_1^{2} -\sin(\pi(t-\lambda))(1)\,d\lambda$

$+ \int_3^{t+1} -\sin(\pi(t-\lambda))(-2)\,d\lambda$

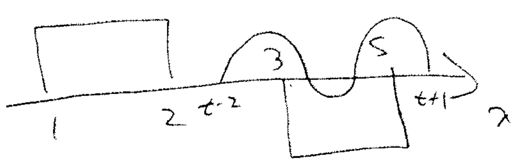


$3 \le t \le 4 \quad y(t) = \int_{t-2}^{2} -\sin(\pi(t-\lambda))(1)\,d\lambda$

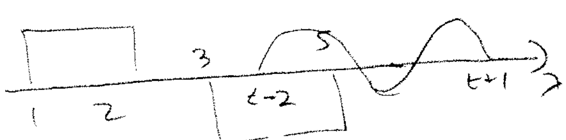
$+ \int_3^{t+1} -\sin(\pi(t-\lambda))(-2)\,d\lambda$



$4 \le t \le 5 \quad y(t) = \int_3^{5} -\sin(\pi(t-\lambda))(-2)\,d\lambda$



$5 \le t \le 7 \quad y(t) = \int_{t-2}^{5} -\sin(\pi(t-\lambda))(-2)\,d\lambda$



$t \ge 7 \quad y(t) = 0$