Name ____ CM __________

ECE 300
Signals and Systems

Exam 1
27 March, 2008

NAME Solutions

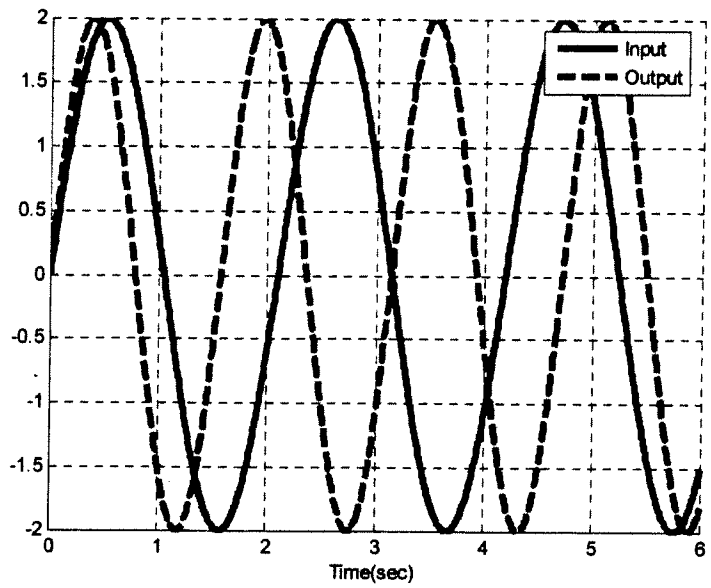
This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Credit will not be given for work not shown.

Problem 1-5	________	/ 20
Problem 6	________	/ 20
Problem 7	________	/ 30
Problems 8	________	/ 30
Exam 1 Total Score:	________	/ 100

Name Solutions CM

Multiple Choice Questions (20 points, 4 points each)

1. Consider a system with sinusoidal input and output shown below:



Which of the following statements is true:

input and output not at the same frequency

a) The system is linear. b) The system is not linear.
c) There is not enough information to determine whether the system is linear or not linear.

2. The average power in the signal $x(t) = ce^{j\omega t}$ is

a) 0 b) $\frac{|c|}{2}$ c) $|c|^2$ d) $\frac{|c|^2}{2}$ e) none of these

3. The average power in the signal $x(t) = A\cos(\omega t + \theta)$ is

a) $\frac{A}{2}$ b) A c) A^2 d) $\frac{A^2}{2}$ e) none of these

4. The signal $x(t) = e^{j(\pi t+1)} + e^{j\frac{\pi t}{4}}$ is

$\pi T_0 = \delta(2\pi)$ $\frac{\pi}{4} T_0 = r(2\pi)$

$T_0 = 2\delta = 8r$ $r=1, \delta = 4$

$T_0 = 8$

a) not periodic
b) periodic with fundamental period 2π seconds
c) periodic with fundamental period 4 seconds
d) periodic with fundamental period 8 seconds
e) none of the above

5. Is the system $y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda+1)\, d\lambda$ causal? a) yes b) no

y(t) depends on x(t+1)

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6. (20 points) Linearity and Time-Invariance

a) Using a formal test, such as was shown in class, determine if the following system is time-invariant. Be sure to show all your work.

$$y(t) = \int_{-\infty}^{t-1} e^{-(t-\lambda)} x(\lambda - 3) d\lambda$$

$z_1 = H\{x(t-t_0)\} = \int_{-\infty}^{t-1} e^{-(t-\lambda)} x(\lambda - t_0 - 3) d\lambda$ $\quad z_2 = \Big[H\{x(t)\} \Big]_{t=t-t_0} = \int_{-\infty}^{t-t_0-1} e^{-(t-t_0-\lambda)} x(\lambda-3) d\lambda$

In z_1, let $\sigma = \lambda - t_0$ $\quad d\sigma = d\lambda$ $\quad \lambda = \sigma + t_0$

$$z_1 = \int_{\sigma=-\infty}^{\sigma=t-1-t_0} e^{-(t-\sigma-t_0)} x(\sigma-3) d\sigma = z_2 \quad \text{so TI}$$

b) Using a formal test, such as was shown in class, determine if the following system is linear. Be sure to show all your work.

$$\dot{y}(t) + \sin(t) y(t) = t^2 x(t)$$

$\dot{y}_1 + \sin(t) y_1 = t^2 x_1 \qquad \dot{y}_2 + \sin(t) y_2 = t^2 x_2$

$\alpha_1 \dot{y}_1 + \alpha_1 \sin(t) y_1 = \alpha_1 t^2 x_1 \qquad \alpha_2 \dot{y}_2 + \alpha_2 \sin(t) y_2 = \alpha_2 t^2 x_2$

$(\alpha_1 \dot{y}_1 + \alpha_2 \dot{y}_2) + \sin(t)(\alpha_1 y_1 + \alpha_2 y_2) = t^2(\alpha_1 x_1 + \alpha_2 x_2)$

$\dot{Y} + \sin(t) Y = t^2 X$

linear

Name *Solutions* CM ____________

7. (30 points) Determining Impulse Responses

Be sure to include all necessary unit step functions in your answers!

a) Determine the impulse response for the system $y(t) = x(t) + \int_{-\infty}^{t} x(\lambda)d\lambda$

$$\boxed{h(t) = \delta(t) + u(t)}$$

b) Determine the impulse response for the system $\overset{h}{\cancel{y}}(t) = \int_{-\infty}^{t-1} e^{-(t-\lambda)} \overset{\delta}{\cancel{x}}(\lambda+3)d\lambda$

$t-1 > -3$

$t > -2$

$$\boxed{h(t) = e^{-(t+3)} u(t+2)}$$

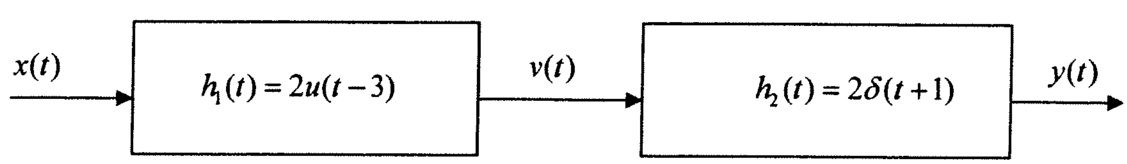
c) Determine the impulse response for the system $\dot{y}(t) - 3y(t) = 2x(t-1)$

$$\frac{d}{dt}\left(he^{-3t}\right) = 2e^{-3t}\delta(t-1) = 2e^{-3}\delta(t-1)$$

$$h(t)e^{-3t} = 2e^{-3}u(t-1)$$

$$\boxed{h(t) = 2e^{3(t-1)} u(t-1)}$$

d) Determine the impulse response for the system below



$$h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)h_2(\lambda)d\lambda$$

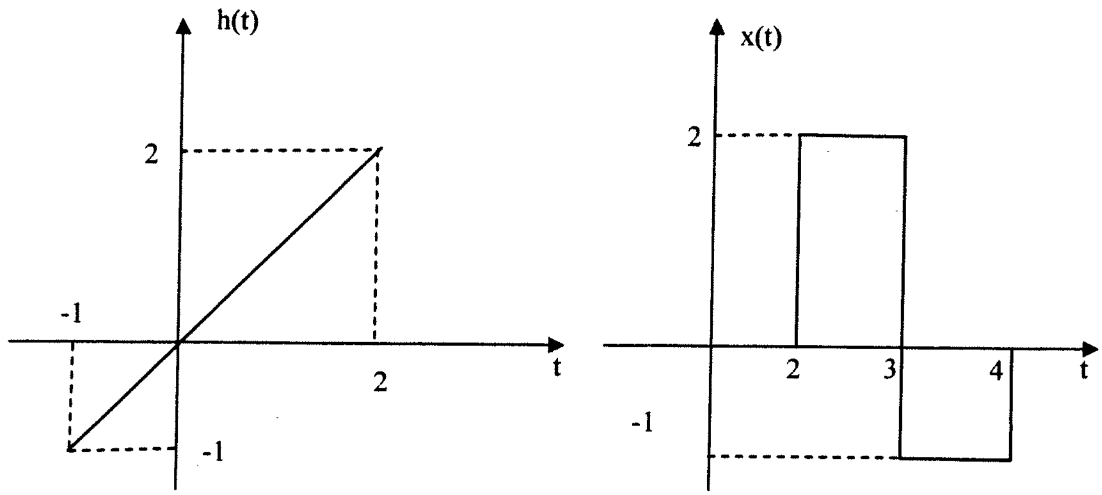
$$= \int_{-\infty}^{\infty} 2u(t-\lambda-3)\, 2\delta(\lambda+1)d\lambda = \boxed{4u(t-2) = h(t)}$$

Name Solutions CM

8. (30 points) Graphical Convolution

Consider a linear time invariant system with impulse response given by

$h(t) = t\,[u(t+1) - u(t-2)]$ and input $x(t) = 2u(t-2) - 3u(t-3) + u(t-4)$, shown below



Using ***graphical convolution***, determine the output $y(t) = h(t) * x(t)$

Specifically, you must

a) Flip and slide $h(t)$, ***NOT*** $x(t)$

b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest

c) Determine the range of t for which each part of your solution is valid

d) Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.

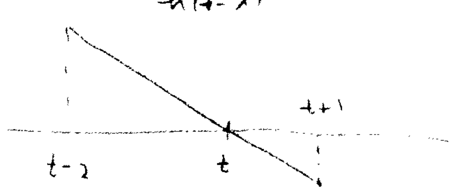
e) ***DO NOT EVALUATE THE INTEGRALS!!***

Hints: (1) Pay attention to the width of h(t)
(2) Made careful sketches

$h(2) = h(t-\lambda)$ $\quad 2 = t-\lambda \quad \lambda = t-2$

$h(-1) = h(t-\lambda)$ $\quad -1 = t-\lambda \quad \lambda = t+1$

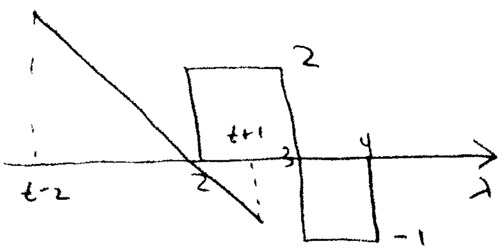
$h(t-\lambda)$



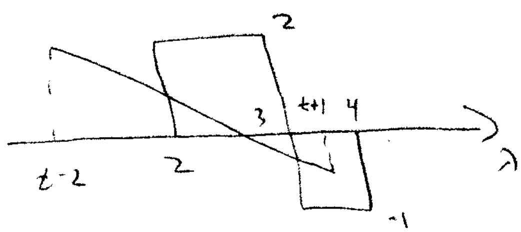
$h(t-\lambda) = t-\lambda$

Name Solutions CM

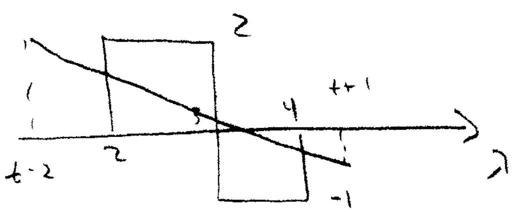
$$y(t) = 0 \quad t \le 1$$



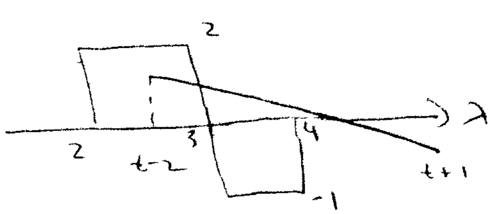
$$y(t) = \int_2^{t+1} (2)(t-\lambda)\,d\lambda \qquad 1 \le t \le 2$$



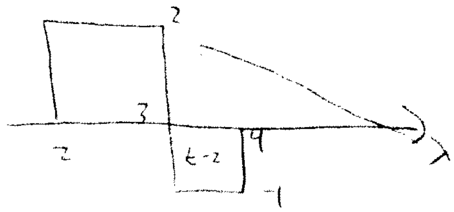
$$y(t) = \int_2^3 (2)(t-\lambda)\,d\lambda + \int_3^{t+1} (-1)(t-\lambda)\,d\lambda \qquad 2 \le t \le 3$$



$$y(t) = \int_2^3 (2)(t-\lambda)\,d\lambda + \int_3^4 (-1)(t-\lambda)\,d\lambda \qquad 3 \le t \le 4$$



$$y(t) = \int_{t-2}^3 (2)(t-\lambda)\,d\lambda + \int_3^4 (-1)(t-\lambda)\,d\lambda \qquad 4 \le t \le 5$$



$$y(t) = \int_{t-2}^4 (-1)(t-\lambda)\,d\lambda \qquad 5 \le t \le 6$$

$$y(t) = 0 \quad t \ge 6$$