ECE 300
Signals and Systems
Homework 1

Due Date: Thursday March 8 at the beginning of class

Reading: K & H, Ch 1.1.1-1.1.7 and your course notes.

Problems

1. K & H, Problem 1.1, figures **a-d**. Use the **rect** function and the **triangle** (Λ) function. See Problem **4** for plotting.

The **rect** function represents a rectangular pulse. To represent a pulse x(t) having amplitude 3, located (centered) at t=5, and width 4, we would write:

$$x(t) = 3 \operatorname{rect}\left(\frac{t-5}{4}\right)$$

The **triangle** function represents a triangular pulse. To represent a triangle x(t) having amplitude 3, located (centered) at t = 5, and width 4, we would write

$$x(t) = 3\Lambda\left(\frac{t-5}{4}\right)$$

2. K & H, Problem 1.4, parts **a-c** only. Rewrite each of the functions from parts **a-c** in the form

$$x(t) = x_1(t)[u(t-t_1)-u(t-t_2)] + x_2(t)[u(t-t_2)-u(t-t_3)] + \dots$$

In this problem we are trying to determine an expression for a function within different time intervals

3. Simplify or solve the following as much as possible, giving numerical answers where possible:

a) $\int_{-\infty}^{\infty} e^{-t} u(t-5) dt$

b) $\int_{-8}^{10} t^2 [u(t-6)-u(t-5)] dt$

c) $\int_{-\infty}^{\infty} t^2 \delta(t-2) dt$

d) $\sin(ty\pi)\delta(t-2)$

e) $\int_{-\infty}^{\infty} \delta(t-3)\delta(t-4) dt$

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f) $\int\limits_{-\infty}^{5} u(t-x+5)\delta(t-4)dt$

g) $H(\omega)\delta(\omega-1)+A(\omega-x+1)\delta(\omega)$

h) $\int\limits_{-9}^{10} u(t+3)u(t-2)dt$

4. Read the **Appendix** to this assignment, and look at the Matlab tutorials on the class website if you need to. Using the Matlab functions **unit_step.m**, **unit_rect.m**, and **unit_triangle.m** available on the class website, plot your functions for **1.1 a-d, 1.4 a-c, and problem 2**. Note that to generate an exponential, such as e^{-3t} in Matlab we would type exp(-3*t). To generate sin(2πt) it would be sin(2*pi*t). Remember that all arguments to sinusoids must be expressed as radians. Also, for **1.4 b** and **c** you may need to use the .* (element by element) multiplication. **Turn in all your plots, neatly labeled.**

Appendix

We will be using Matlab extensively in this class, particularly during labs **and during the lab practical**. The sooner you start learning Matlab the better off you will be. This quarter I am going to try and use Matlab on every homework, so you have every opportunity to learn. *You will also receive a separate grade on your Matlab part of the assignment, so it is important to do these! They are fair game for exams!*

First of all, you need to go to *tibia* and download Matlab (if you haven't already)

Next, make a folder for ECE-300. You want to put all of your software in this folder, **not** in the **work** folder Matlab defaults to.

Next start Matlab, and you should get something like the following window:
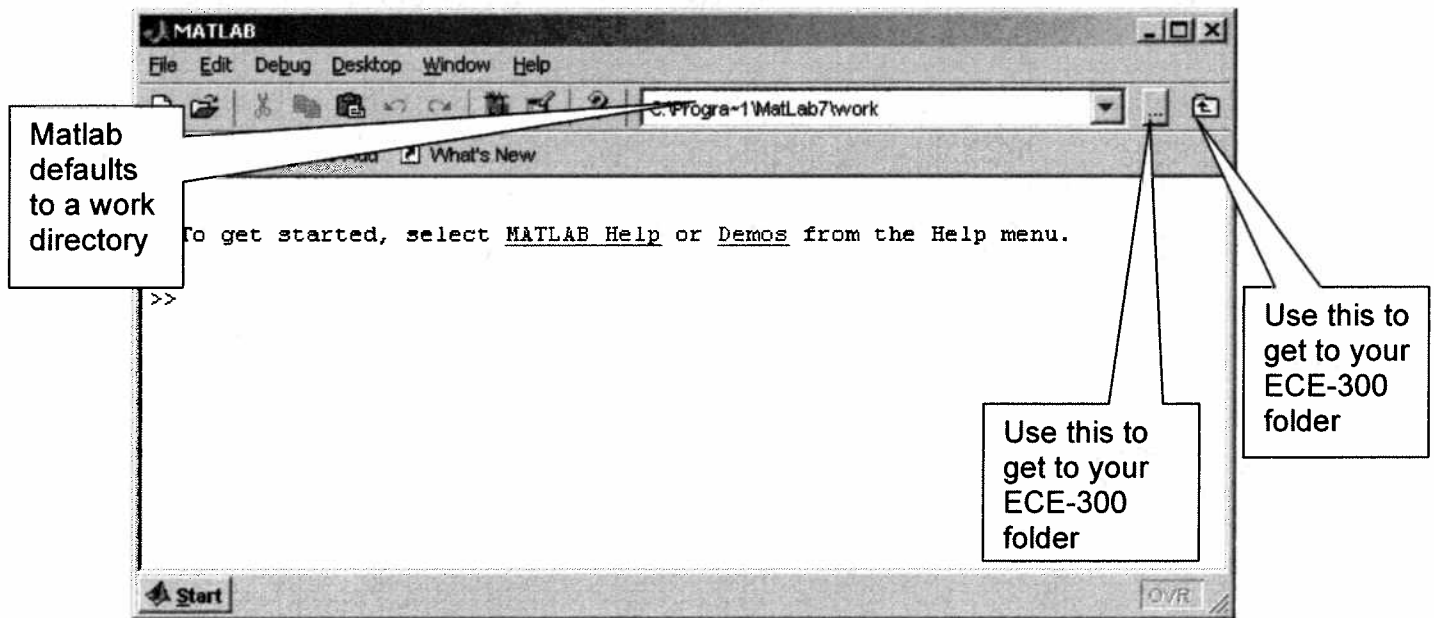
You next need to make sure Matlab is set to the appropriate folder, using one of the two methods shown above. (Use the button with the ellipses (...) the first time.)

In this first assignment, we are going to use some simple functions that I have written to implement a few standard signal models. You need to go to the class website and download the function **unit_step.m**, **unit_rect.m**, and **unit_triangle.m** to your ECE-300 folder.

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When you type **unit_step**, **unit_rect**, or **unit_triangle** in Matlab, it will first look in the current folder to see if these functions are defined, and then use them. This is why it is important that you set Matlab to the ECE-300 folder that you download these functions into this directory.

unit_step.m implements the function $u(t - t_0) = \begin{cases} 1 & t - t_0 > 0 \\ 0 & elsewhere \end{cases}$.

unit_rect.m implements the function $\text{rect}\left(\dfrac{t - t_0}{T}\right) = \Pi\left(\dfrac{t - t_0}{T}\right) = \begin{cases} 1 & |t - t_0| \le \dfrac{T}{2} \\ 0 & elsewhere \end{cases}$.

unit_triangle.m implements the function $\Lambda\left(\dfrac{t - t_0}{W}\right) = \begin{cases} 1 - \dfrac{2|t - t_0|}{W} & |t - t_0| \le \dfrac{W}{2} \\ 0 & elsewhere \end{cases}$

Note that each of these functions is defined in terms of an offset, t0. If we want there to be a **delay** in the signal, the offset t0 will be **positive**, while if the want an **advance** in the signal, the offset t0 will be **negative**.

To use these simple functions we first need to define a time vector, t. There are many ways to do this in Matlab, but we'll just learn one way now. At the command prompt, type

t = linspace(-1,2,10000);

This command tells Matlab to make an array filled with values from -1 to 2, with 10000 points between -1 and 2. This array is assigned to the variable t.

Each of our new functions has input arguments, which is how we tell the functions what the parameters we are using are. Each of the input arguments will be defined, and then I will show how they are used.

The arguments to **unit_step** are the time vector (t), and the offset (t0), *in that order*.

To plot the function $x(t) = u(t)$ from *t = -2* to *t = 3* we should type the following commands

t = linspace(-2,3,10000);
x = unit_step(t,0);
plot(t,x); grid; xlabel('Time (sec)');

To plot the function $x(t) = u(t - 2)$ from t = -3 to t = 4 we should type the following commands

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```
t = linspace(-3,4,10000);
x = unit_step(t,2);
plot(t,x); grid; xlabel('Time (sec)');
```

To plot the function $x(t) = 5u(t+3) - 3u(t-1)$ from t = -4 to t =2 we should type the following commands

```
t = linspace(-4,2,10000);
x =5* unit_step(t,-3)-3*unit_step(t,1);
plot(t,x); grid; xlabel('Time (sec)');
```

To plot a unit ramp function, $x(t) = tu(t)$ from t = -1 to t = 5, we should type the following commands

```
t = linspace(-1,5,10000);
x =t.*unit_step(t,0);
plot(t,x); grid; xlabel('Time (sec)');
```

Note that in the second line we used `.*`. We need to do this whenever we multiple two vectors *element by element*. We need to do this since t is an array, and **unit_step** returns and array, and we want the first element of t multiplied by the first element of **unit_step**, the second element of t multiplied by the second element of **unit_step**, etc. This is different from doing array multiplication, which would result from typing t*unit_step(t,0).

The arguments to **unit_rect** are the time vector(t), the offset (t0), and the width (T) _in that order._

To plot the function $x(t) = \text{rect}(t-5)$ from t = 3 to t = 7 we should type the following commands (note that T = 1 in this rect function example)

```
t = linspace(3,7,10000);
x = unit_rect(t,5,1);
plot(t,x); grid; xlabel('Time (sec)');
```

To plot the function $x(t) = \text{rect}\left(\dfrac{t+1}{3}\right)$ from -3 to 2 we should type the following

```
t = linspace(-3,2,10000);
x = unit_rect(t,-1,3);
plot(t,x); grid; xlabel('Time (sec)');
```

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To plot the function $x(t) = 2\mathrm{rect}\left(\dfrac{t+1}{2}\right) - 3\mathrm{rect}\left(\dfrac{t-3}{5}\right)$ from -3 to 6 we should type the following

```
t = linspace(-3,6,10000);
x = 2*unit_rect(t,-1,2)-3*unit_rect(t,3,5);
plot(t,x); grid; xlabel('Time (sec)');
```

The arguments to **unit_triangle** are the time vector(t), the offset (t0), and the width (W) _in that order._

To plot the function $x(t) = \Lambda\left(\dfrac{t+1}{3}\right)$ from -5 to 3 we should type the following

```
t = linspace(-5,3,10000);
x = unit_triangle(t,-1,3);
plot(t,x); grid; xlabel('Time (sec)');
```

To plot the function $x(t) = 2\Lambda\left(\dfrac{t+1}{2}\right) - 3\Lambda\left(\dfrac{t-3}{5}\right)$ from -3 to 6 we should type the following

```
t = linspace(-3,6,10000);
x = 2*unit_triangle(t,-1,2)-3*unit_triangle(t,3,5);
plot(t,x); grid; xlabel('Time (sec)');
```

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#1

(K+H 1.1)

(a)

$$x(t) = \text{rect}\left(\frac{t}{4}\right) + \text{rect}\left(\frac{t}{2}\right)$$

(b)
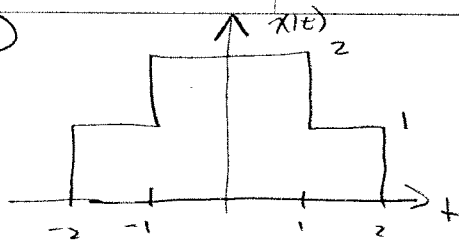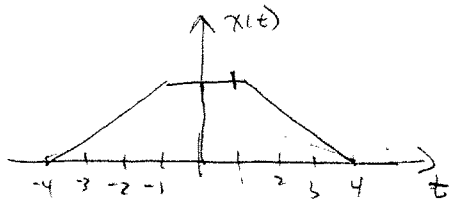
For $x_1(t) = \Lambda\left(\frac{t}{8}\right)$ we have

we need $x_1(t)$ to have a value of 1 when $t = \pm 1$

$0 \le t \le 4$, $\quad x_1(t) = 1 - \frac{t}{4}$ $\quad x_1(t) = 1 - \frac{1}{4} = \frac{3}{4}$

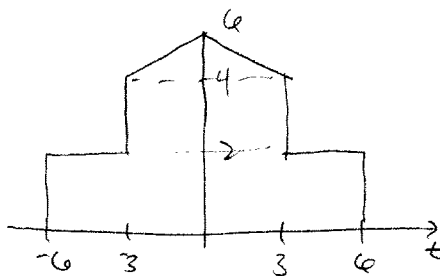so we want $x_1(t) = \frac{4}{3}\Lambda\left(\frac{t}{8}\right)$

$x_2(t)$ is the small triangle at the top we need to remove.

we need to remove this!

$$x_2(t) = \frac{1}{3}\Lambda\left(\frac{t}{2}\right)$$

$$\boxed{x(t) = \frac{4}{3}\Lambda\left(\frac{t}{8}\right) - \frac{1}{3}\Lambda\left(\frac{t}{2}\right)}$$
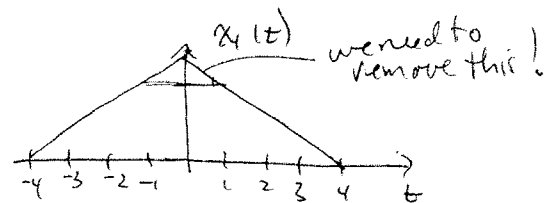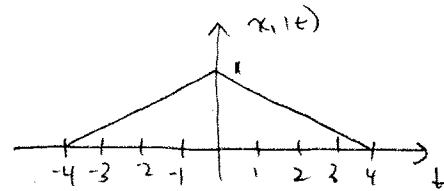
(c)

$$x(t) = 2\,\text{rect}\left(\frac{t}{12}\right) + 2\,\text{rect}\left(\frac{t}{6}\right) + 2\,\Lambda\left(\frac{t}{6}\right)$$

(d)
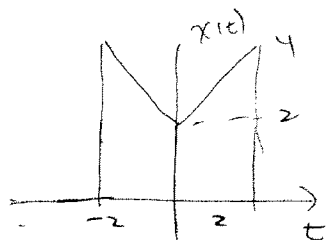
$$x(t) = 4\,\text{rect}\left(\frac{t}{4}\right) - 2\,\Lambda\left(\frac{t}{4}\right)$$
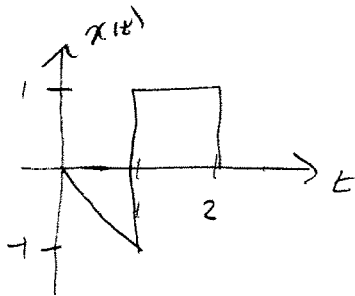
#2

K+H 1.4

a) $x(t) = u(t+1) - 2u(t-1) + u(t-3)$

$$x(t) = (1)\left[u(t+1) - u(t-1)\right] + (-1)\left[u(t-1) - u(t-3)\right]$$
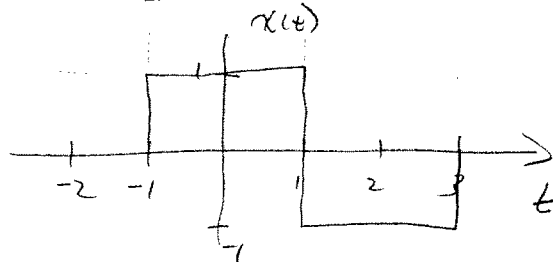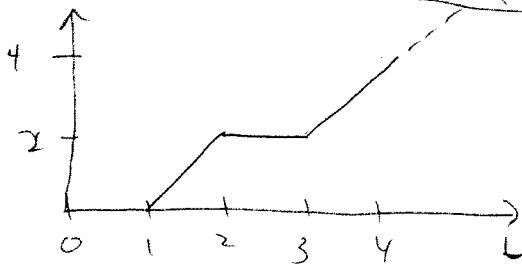

b) $x(t) = (t+1)u(t-1) - tu(t) - u(t-2)$

$$x(t) = -t\left[u(t) - u(t-1)\right] + \left[u(t-1) - u(t-2)\right]$$

c) $x(t) = 2(t-1)u(t-1) - 2(t-2)u(t-2) + 2(t-3)u(t-3)$

$$= 2(t-1)u(t-1) - \left[2(t-1) - 2\right]u(t-2) + (2t-6)u(t-3)$$

$$x(t) = 2(t-1)\left[u(t-1) - u(t-2)\right] + 2\left[u(t-2) - u(t-3)\right]$$

$$+ (2t-4)u(t-3)$$

#3

a) $\int_{-\infty}^{\infty} e^{-t} u(t-5) dt$ $u(t-5) = 1$ for $t-5 \geq 0$ or $t \geq 5$

$$= \int_{5}^{\infty} e^{-t} dt = -e^{-t}\Big|_{5}^{\infty} = \boxed{e^{-5} = 0.00674}$$

b) $\int_{-8}^{10} t^2 \left[u(t-6) - u(t-5) \right] dt$
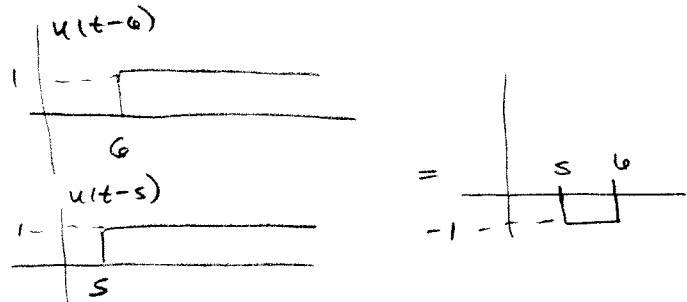
$$= \int_{5}^{6} (-1) t^2 dt = -\frac{t^3}{3}\Big|_{5}^{6} = -\left[\frac{6^3 - 5^3}{3}\right] = \boxed{-30.33}$$

c) $\int_{-\infty}^{\infty} t^2 \delta(t-2) dt = t^2\Big|_{t=2} = \boxed{4}$

d) $\sin(t y \pi) \delta(t-2) = \boxed{\sin(2\pi y) \delta(t-2)}$

e) $\int_{-\infty}^{\infty} \delta(t-3) \delta(t-4) dt = \delta(4-3) = \delta(1) = \boxed{0}$

f) $\int_{-\infty}^{5} u(t-x+5) \delta(t-4) dt = u(4-x+5) = \boxed{u(9-x)}$

g) $H(w) \delta(w-1) + A(w-x+1) \delta(w) = \boxed{H(1) \delta(w-1) + A(1-x) \delta(w)}$

h) $\int_{-9}^{10} u(t+3) u(t-2) dt$

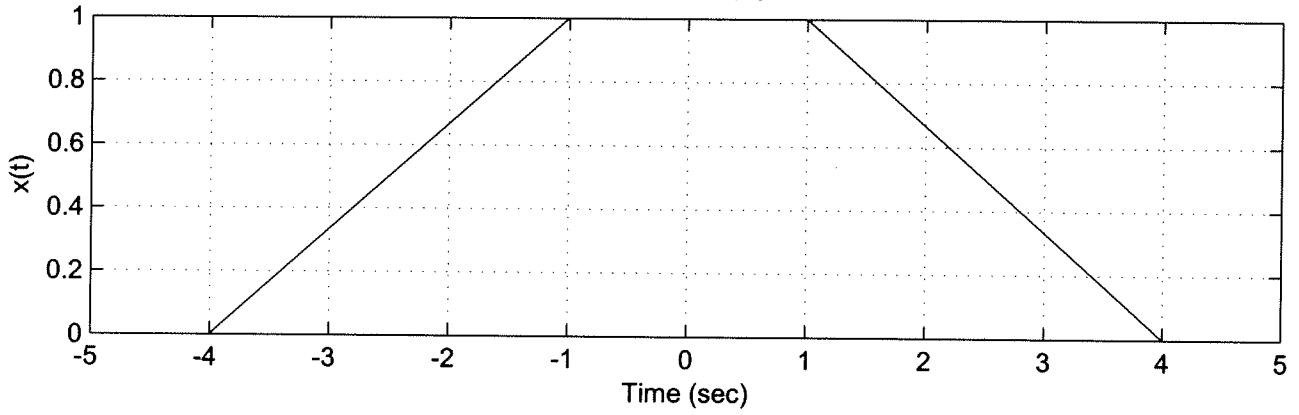$u(t+3) = 1$ for $t+3 \geq 0$ $t \geq -3$
$u(t-2) = 1$ for $t-2 \geq 0$ $t \geq 2$

$$= \int_{2}^{10} 1 \, dt = \boxed{8}$$

Problem 1.1a
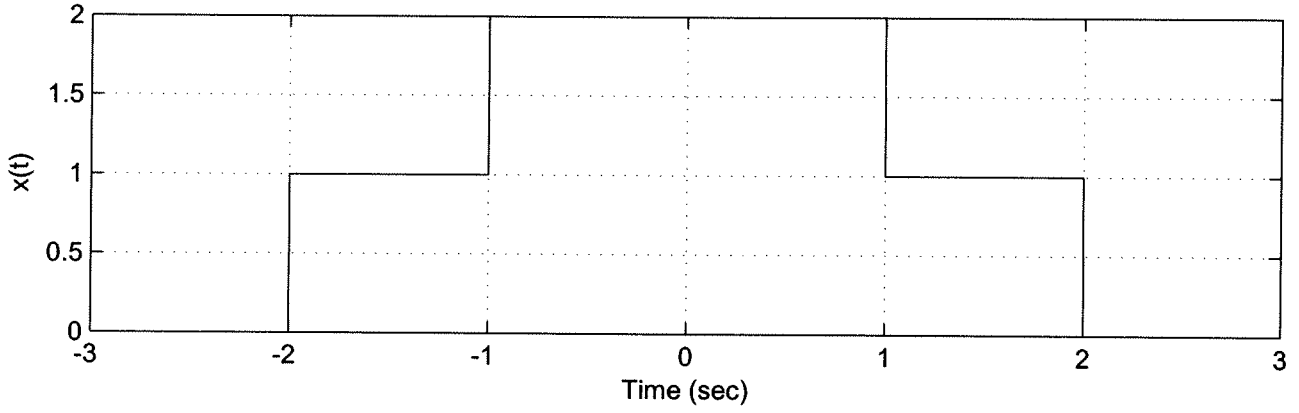
Problem 1.1b

Problem 1.1c
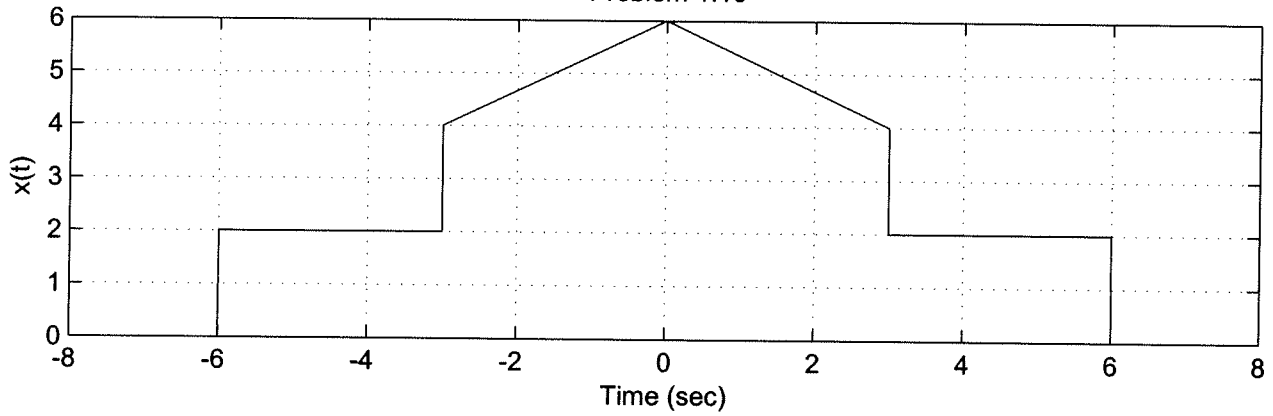
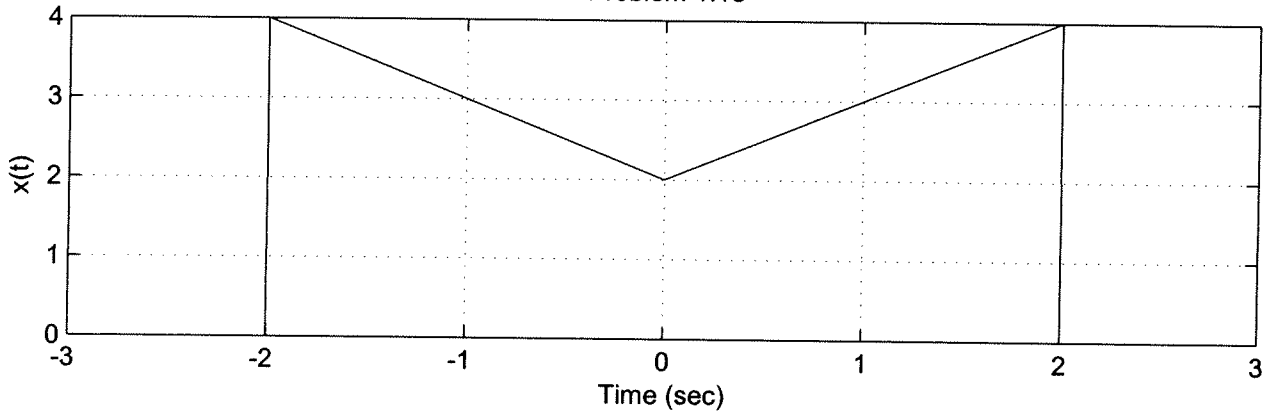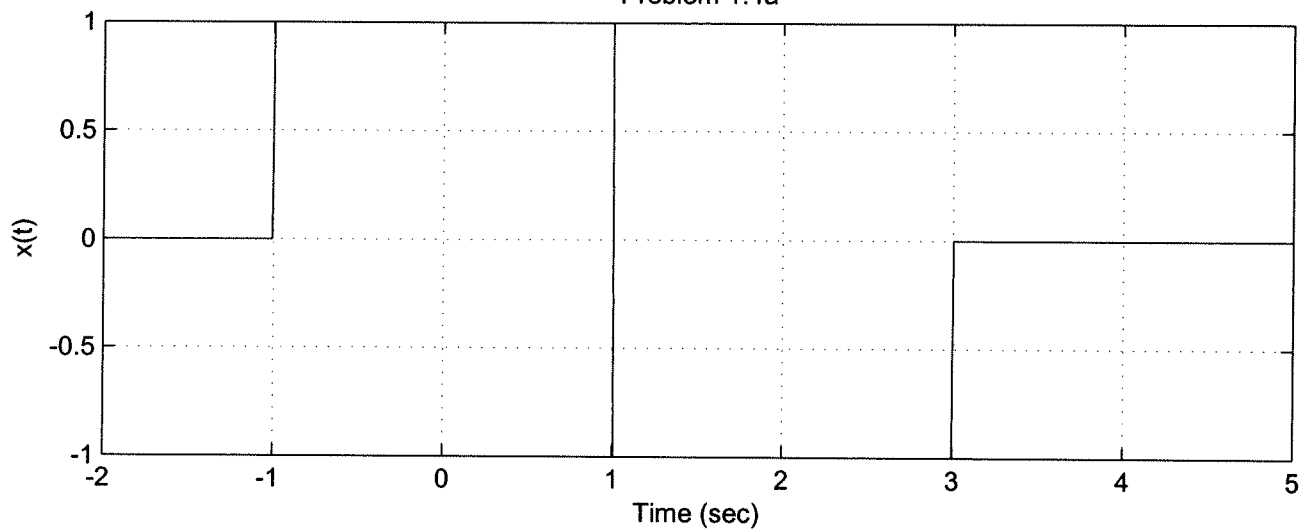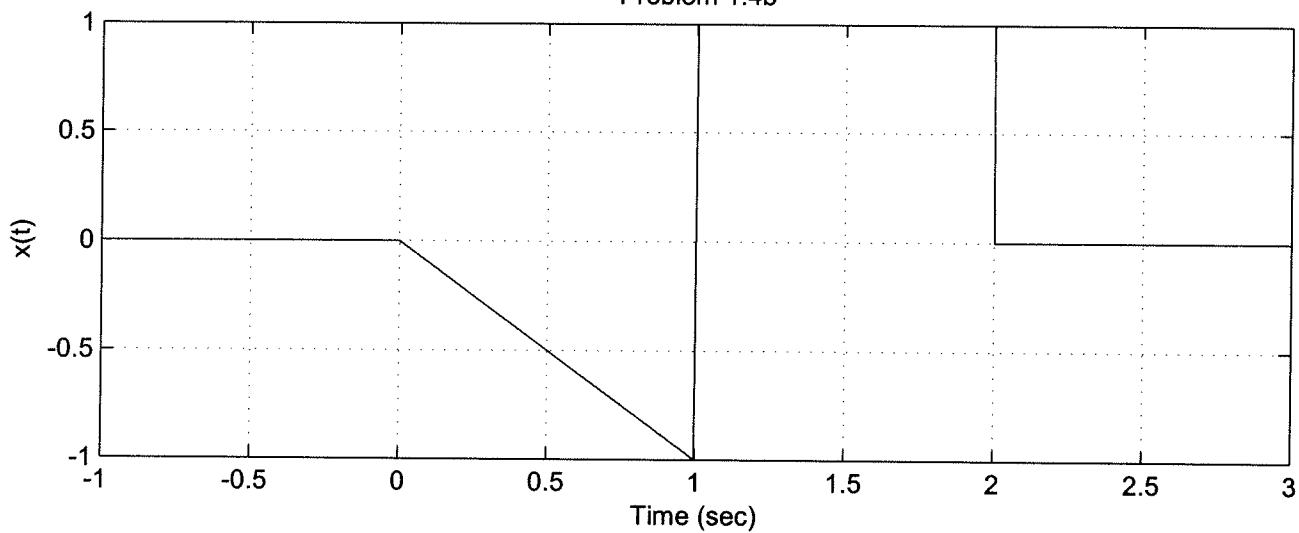## Problem 1.1e

Problem 1.4a

Problem 1.4b

Problem 1.4b