ECE 300
Signals and Systems
Homework 7

Due Date: Tuesday April 24 *at the beginning of class*

Exam 2, Thursday April 26

Problems:

1. A periodic signal $x(t)$ is the input to an LTI system with output $y(t)$. The signal $x(t)$ has period 2 seconds, and is given over one period as

$$x(t) = e^{-t} \quad 0 < t < 2$$

$x(t)$ has the Fourier series representation

$$x(t) = \sum_k \frac{0.4323}{1 + jk\pi} e^{jk\pi t}$$

The system is an ideal lowpass filter that eliminates all signals with frequency content higher than 1.25 Hz.

a) Find the average power in $x(t)$.

b) Determine an expression for the output, $y(t)$. Your expression for $y(t)$ must be real.

c) Determine the average power in $y(t)$.

d) Plot the spectrum (magnitude and phase) for $x(t)$. Include the DC through second harmonic. Accurately label your plot.

2. Assume $x(t) = t^2 \quad -\pi \le t \le \pi$ with Fourier Series representation

$$x(t) = \sum_k a_k e^{jkt}$$

where

$$a_k = \begin{cases} \dfrac{\pi^2}{3} & k = 0 \\ \dfrac{2(-1)^k}{k^2} & k \neq 0 \end{cases}$$

a) Assume $x(t)$ is the input to a system that eliminates all signals with frequencies outside the range 0.5 to 0.7 Hz. What is the output of the system $y(t)$ and what fraction of the average power in $x(t)$ is in $y(t)$? (Note: your answers must be real, no e^{ja} terms.)

b) Assume $x(t)$ is the input to a system that eliminates all signals with frequencies in the range 0.5 to 0.7 Hz. What is the output of the system $y(t)$ and what fraction of the average power in $x(t)$ is in $y(t)$?

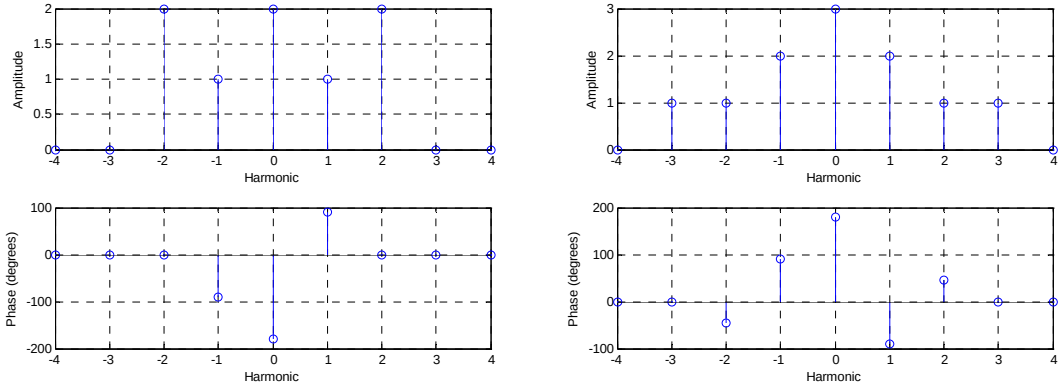
3. K & H, Problem 5.1. Use the example we did in class to get the Fourier series coefficients for part c.

4. K & H, Problem 5.3.

5. K & H, Problem 5.12. Note that $y(t) = x(t) - x(t-1)$. You need to write c_k^y in terms of c_k^x.

6. K & H, Problme 5.13.

7. The output of a LTI system, $y(t)$, has the following spectrum shown on the left, while the system transfer function, $H(k\omega_o)$, has the spectrum shown on the right. Assume all angles are multiples of 45 degrees.



a) Determine (sketch) the spectrum (magnitude and phase) of the input to the system, $x(t)$.

b) If $x(t)$ has the fundamental period $T = 2$ seconds, determine an analytical expression for $x(t)$ in terms of sine, cosines, and constants.