ECE 300
Signals and Systems
Homework 6

Due Date: Thursday April 19 at the beginning of class

Problems:

1. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of ω_0 and the c_k. *Hints:* (1) *Draw the signal, and then use the sifting property to calculate the* c_k. (2) *If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t-3p)$$

2. For the periodic square wave $x(t)$ with period $T_o = 0.5$ and

$$x(t) \begin{cases} 1 & 0 \le t < 0.25 \\ -1 & 0.25 \le t < 0.5 \end{cases}$$

show that the Fourier series coefficients are given by

$$c_k = \begin{cases} \dfrac{-2j}{k\pi} & k \quad odd \\ 0 & k \quad even \end{cases}$$

where $x(t) = \sum_k c_k e^{jk4\pi t}$

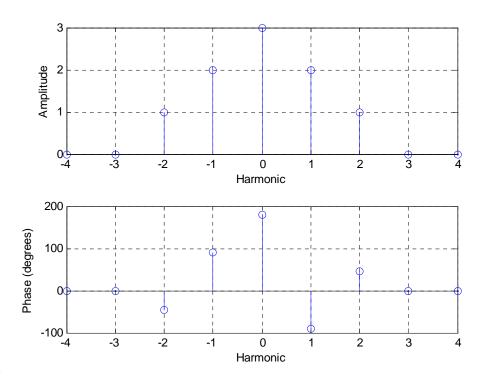3. Simplify each of the following into the form $c_k = \alpha(k)e^{-j\beta(k)}\text{sinc}(\lambda k)$

a) $c_k = \dfrac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

b) $c_k = \dfrac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

c) $c_k = \dfrac{e^{j5k} - e^{j2k}}{k}$

Scrambled Answers $c_k = 3\pi e^{-j\frac{7\pi k}{2}}\text{sinc}\left(\dfrac{3k}{2}\right)$, $c_k = 3e^{j(\frac{7}{2}k+\frac{\pi}{2})}\text{sinc}\left(\dfrac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi}\text{sinc}\left(k\dfrac{9}{2}\right)$
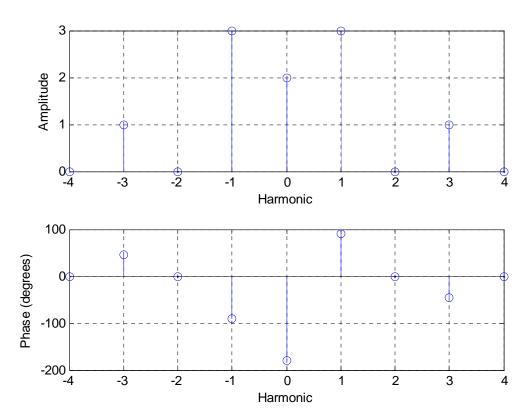
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4. K & H, Problem 3.13. For part **c** you should get $c_k^y = c_{k-1}^x$, use Euler's identity for part **d**.

5. A signal $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45

degrees)

a) What is $x(t)$? Your expression must be real.

b) What is the average value of $x(t)$?

c) What is the average power in $x(t)$?

6. A signal $x(t)$, which has a fundamental period of 3 seconds, has the following spectrum (all phases are multiples of 45 degrees)

a) What is $x(t)$? Your expression must be real.

b) What is the average power in $x(t)$?

c) Sketch the single sided power spectrum for $x(t)$.

7. Pre-Lab/Matlab Exercises (to be done by all students. Turn this in with your homework and bring a copy of this with you to lab!

Note: The function we use for numerical integration, **quadl**, has a default tolerance. This tolerance level is used so the function can determine when its estimate of the integral is "good enough". When dealing with either small valued functions, or very short intervals, you may need to change this tolerance. In the following problems, you will need to change the tolerance to get the proper results (use help **quadl**).
Read the **Appendix** and then do the following:

a) Plot
  - the Fourier series representation,
  - the spectrum,
  - the single sided power spectrum in dBmV

for each of the waveforms listed below. Use 9 terms for each function. It is probably easiest to modify your program **Complex_Fourier_Series.m**.

Some useful hints:

i) It is probably easiest for you to construct two arrays in Matlab

C_single = [c0 c];
C_double = [fliplr(conj(c)) c0 c]; % conj takes the complex conjugate, fliplr does what?

ii) You should use the **log10** command to compute the base 10 logarithm. You may want to use the Matlab function **length** to determine the length of c. The functions **angle** and **abs** will also be useful.

You should also use the **subplot** command so you can plot both a signal and its Fourier series representation in one graph, its spectrum in the next two graphs, and its single-sided power spectrum in a subsequent graph within the same window. Utilize the **stem** command in Matlab to do the plotting.
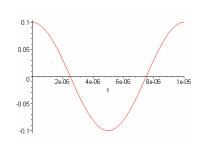
For the power spectrum plot, the y-axis should be labeled *Power(dBmV)*, the x-axis labeled *Harmonic* .Your plots for function $x_2(t)$ should look like those in Figure 1. (Note the axis on the top graph is limited using the command **axis('tight')**).
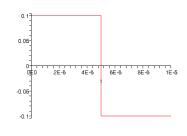
The waveforms, each having zero DC offset, are:

i) $x_1(t) = 0.1\cos\left(2\pi 100 \times 10^3 t\right)$ V

II) $x_2(t)$ is a square wave of period 10 μs and peak-to-peak amplitude 0.2 V.

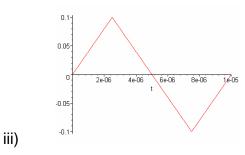III) $x_3(t)$ is a triangle wave of period 10 μs and peak-to-peak amplitude 0.2 V.

Just to be sure there is no confusion regarding the waveforms, they are displayed below.
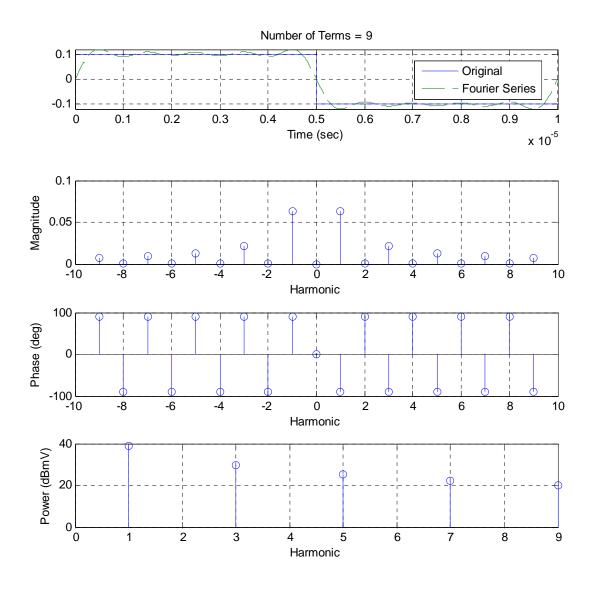
i)

ii)

iii)

Figure 1: Results for problem 7a for function $x_2(t)$.

b) For each waveform, create a table containing a column of harmonics, corresponding frequency (in Hz), and values of predicted decibel levels (dBmV) for the corresponding c_k . Do not read these from your plots, but get them directly from you program. Note that if you do not put a semicolon after a statement in Matlab, the values in the array will be printed to the screen. (For $x_2(t)$ you should get numbers like –inf, 38.0, -157, 29.5, -161, 25…)

Appendix

There are a number of different ways of graphically presenting the information in the Fourier series representation of a periodic function. The most common methods of presenting this information are by plotting the spectrum of the signal, or the power spectrum of the signal. In the following examples we will use the function

$$f(t) = \begin{cases} 0 & -2 \leq t < -1 \\ t & -1 \leq t < 1 \\ 1 & 1 \leq t < 2 \end{cases}$$

Spectrum of a Periodic Signal The spectrum of a periodic signal is a plot of the magnitude $|c_k|$ against the corresponding frequency $k\omega_o$, and the phase $\angle c_k$ against the corresponding frequency. Since our signal is periodic we only have values at *discrete* frequencies ($k\omega_o$), we do not know what happens in between these discrete frequencies so we **_do not_** "connect the dots". Since ω_o is common to all of the discrete frequencies, we often plot against the *harmonic* k, rather than the frequency $k\omega_o$. Figure 2 shows the spectrum of $f(t)$.

Power Spectrum of a Periodic Signal The power spectrum of a signal tells us how the power in the signal is distributed in frequency. To plot the power spectrum of a signal, we consider each term of the complex Fourier series, $c_k e^{jk\omega_o t}$. The average power in this term is $|c_k|^2$. The power spectrum indicates the power associated with each frequency, and is thus a plot of $|c_k|^2$ against frequency $k\omega_o$ or just the harmonic *k*. In this plot we need to include both positive and negative frequencies. Figure 3 shows the power spectrum of $f(t)$.

Single-Sided Power Spectrum of a Periodic Signal Because the magnitudes of the coefficients are even ($|c_k| = |c_{-k}|$), the powers associated with negative frequencies are the same as those associated with positive frequencies. Since only positive frequencies are realizable, we often just want to plot the power against positive frequencies. This gives us a single sided power spectrum, which is a plot of $|c_0|^2 \quad 2|c_1|^2 \quad 2|c_2|^2 \quad ... \quad 2|c_N|^2$ versus the corresponding frequency $0 \quad \omega_0 \quad 2\omega_0 \quad ... \quad N\omega_0$. Since the fundamental frequency ω_0 is common to all of the frequency terms, we often just plot against the harmonics $0 \quad 1 \quad 2 \quad ... \quad N$. Figure 4 shows the single sided power spectrum of $f(t)$.

Spectrum Analyzer (SA) Display The SA displays a one-sided spectrum, but instead of showing the value of $2|c_k|^2$ at each frequency, the spectrum analyzer shows average

power in decibels with respect to a one millivolt RMS reference. For the sinusoid at frequency kf_0, the average power in decibels is given by

$$P_k\big|_{dB} = 10\log_{10}\frac{P_k}{P_{ref}},$$

where the power P_k represents the power spectrum coefficient $2|c_k|^2$, and the power P_{ref} is the average power delivered to a one-ohm resistor by a one millivolt RMS sinusoid. We have

$$P_k\big|_{dBmV} = 10\log_{10}\left(\frac{2|c_k|^2}{(0.001)^2}\right) \text{ dBmV} \quad .$$

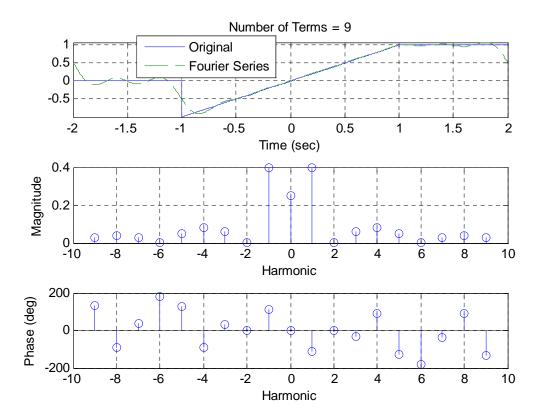The units "dBmV" indicate that the reference for the decibels is a one millivolt RMSsinusoid.

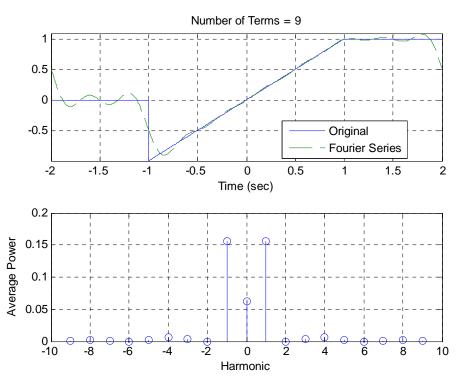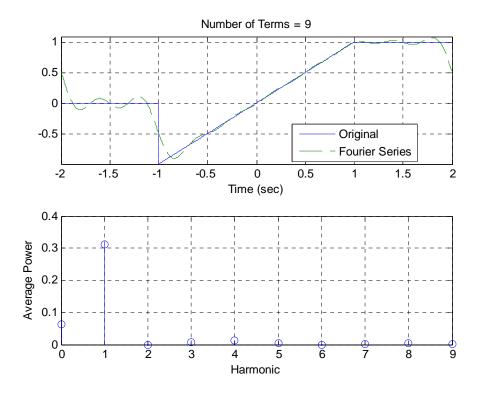Figure 2: The spectrum of $f(t)$.

Figure 3: The power spectrum of $f(t)$.

Figure 4: The single-sided power spectrum of $f(t)$.