

ECE 300
Signals and Systems
Homework 10

Due Date: Wednesday May 16 at 7 PM (beginning of Q/A)
Note: Exam 3 Thursday May 17 , Lab Practical Friday May 18

Note: Use the Fourier transform table given out in class.

Problems

1. Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t) = 5 \operatorname{sinc}\left(\frac{t}{0.002}\right) + 5 \operatorname{sinc}\left(\frac{t}{0.001}\right) \quad \text{Answer } 56\%$$

2. Using the **duality property**, find the corresponding Fourier transform for the following: **a)** $g(t) = \operatorname{sinc}^2(Bt)$ **b)** $g(t) = \operatorname{sinc}(Wt)$ **c)** $g(t) = \delta(t)$ **d)** $g(t) = \cos(\omega_0 t)$ **Do not** just look up the pairs from the table (though you can use any other pairs except the one you are trying to find).

3. K & H, Problem 5.16 (**a, b, c** only)

4. Consider a linear time invariant system with transfer function given by

$$H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \leq 2 \\ 0 & \text{else} \end{cases}$$

with input $x(t) = \frac{8}{\pi} \operatorname{sinc}^2\left(\frac{2(t-1)}{\pi}\right)$. The output of the system is $y(t)$.

- a) Determine $X(\omega)$.
- b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.
- c) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.
- d) Determine $y(t)$, the output of the system.

$$\text{Answer } y(t) = \frac{20}{\pi} \operatorname{sinc}\left[\frac{2}{\pi}(t-3)\right] + \frac{10}{\pi} \operatorname{sinc}^2\left[\frac{1}{\pi}(t-3)\right]$$

5. Determine the transfer function $H(\omega)$ that would produce the following input/output relationships. Simplify your answers as much as possible.

a) $y(t) = a\dot{x}(t-b)$

b) $y(t) = ax(t+b) + ax(t-b)$

c) $\dot{y}(t) = x(t) * e^{-t}u(t-b)$