ECE 300
Signals and Systems
Homework 7

Due Date: Friday April 28, 2006 at 2:30 PM

Reading: K & H, pp. 161-192

Problems

1. By evaluating the integral by hand, show that the Fourier transform of $x(t) = e^{-t}u(t)$ is given by

$$X(\omega) = \frac{1}{1+j\omega} = \frac{1}{\sqrt{1+\omega^2}} \measuredangle -\tan^{-1}(j\omega) \left[\frac{180}{\pi}\right] \text{ degrees}$$

2. By evaluating the integral by hand, show that the Fourier transform of $x(t) = e^{-|t|}$ is

$$X(\omega) = \frac{2}{1+\omega^2}$$

In this problem you will utilize the Matlab program **Fourier_Series.m** on the class website (download by ***right clicking***, ***select save target as***, and ***saving as a text document***). The arguments to this function are

- the initial and final times of a single period (the period starts at **Tlow** and ends at **Thigh**)
- **N**, the number of terms (or *harmonics*) to use (in addition to the average value term)
- **N_Periods,** which is the number of periods to plot the Fourier series

Throughout this assignment we will only be looking at one period of the Fourier series representation.

a. Write a function that takes as arguments c_0, c, and T (or T_0, the fundamental frequency) and plots the amplitude $T|c_k|$ versus kω_o and the phase $\measuredangle c_k$ (in degrees) versus kω_o. You should use the subplot command and plot both on one page. You should use the command **orient tall** before any plotting to use more of the page. Some Matlab commands you might find useful are **angle**, **length**, and **abs**. Instead of using the stem command, you should use the plot command and plot discrete points, like dots ('.').

b. By using the axis command, limit the axes to the range -8 to 8 and from 0 to the maximum value of $T|c_k|$. The **max** command may prove useful here.

c. Change the function **fcn** so we are plotting the spectrum of $x(t) = e^{-t}u(t)$. You should be sure to look at the Fourier series representation to verify everything is correct.

d. Add plots of the magnitude and phase of $X(\omega)$ on the existing plots. You may need the functions **sqrt** and **./** or **.*** Use a solid line type and be sure to add legends. If you have done everything correctly, and you type Fourier_Series(-4,4,100,1), you should get the plot shown in Figure 1. Be sure to modify the title and axes so they look like those in this figure (to get subscripts type c_k , to get T0 type T_0, and to get w0 type \omega_0)

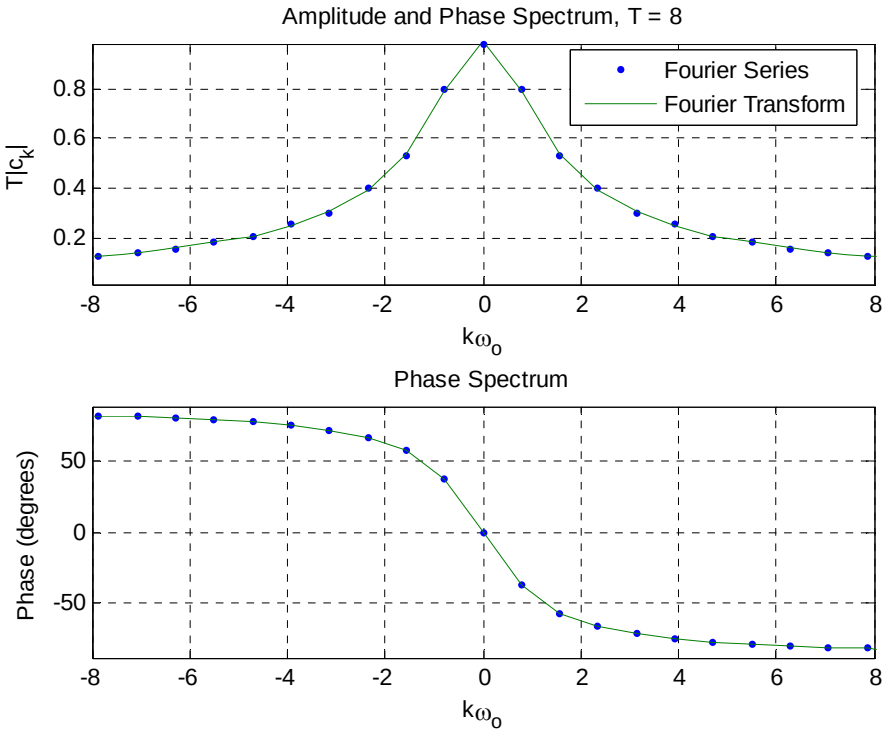

Figure 1: Example plots for part **d**.

e. Change [**Tlow,Thigh**] to [-8,8] and [-16,16] and rerun your code. Turn in your plot (these are just two of the three inputs to your function). Keep the number of points at $N = 100$. Do not change $x(t)$. Here we are increasing the period of the function $x(t)$ to demonstrate that the Fourier transform is just a Fourier series in the limit as $T \to \infty$, $k\left(\frac{2\pi}{T}\right) = k\omega_0 \to \omega$, and $Tc_k \to X(\omega)$

f) Redo the above for $x(t) = e^{-|t|}$. Turn in 3 plots for this part.