ECE 300
Signals and Systems
Homework 6

Due Date: Friday April 21 at 2:30 PM **Exam 2, Tuesday April 25**

Reading: K & H, pp. 145-161, 202-215.

Problems:

1. Assume $g(t)$ is a periodic function with period $T_o = 2$, where $g(t) = 1 - t^2$ for $-1 \le t \le 1$. $g(t)$ has Fourier series representation

$$g(t) = \frac{2}{3} + \sum_{k \neq 0} -2\frac{(-1)^k}{k^2\pi^2} e^{jk\pi t}$$

a) Write (explicitly) the integral required to compute the average power in $g(t)$. (Go beyond just writing the power definition.) Do not solve the integral.

b) The power in g(t) is 8/15 W. What fraction of the average power in $g(t)$ is contained in the DC (average) term plus the first three harmonics?

c) Assume $g(t)$ is the input to a system that eliminates all signals except those with frequencies between 3.75 and 4.25 Hz. Determine an expression $y(t)$ for the output of this system. Since the input signal is real, your output signal must be real!

2. A periodic signal $x(t)$ is the input to an LTI system with output $y(t)$. The signal $x(t)$ has period 2 seconds, and is given over one period as

$$x(t) = e^{-t} \quad 0 < t < 2$$

$x(t)$ has the Fourier series representation

$$x(t) = \sum_k \frac{0.4323}{1 + jk\pi} e^{jk\pi t}$$

The system is an ideal lowpass filter that eliminates all signals with frequency content higher than 1.25 Hz.

a) Find the average power in $x(t)$.

b) Determine an expression for the output, $y(t)$. Your expression for $y(t)$ must be real.

c) Determine the average power in $y(t)$.

d) Plot the spectrum (magnitude and phase) for $x(t)$. Include the DC through second harmonic. Accurately label your plot.

3. Assume $x(t) = t^2 \quad -\pi \le t \le \pi$ with Fourier Series representation

$$x(t) = \sum_k a_k e^{jkt}$$

where

$$a_k = \begin{cases} \dfrac{\pi^2}{3} & k = 0 \\ \dfrac{2(-1)^k}{k^2} & k \neq 0 \end{cases}$$

a) Assume $x(t)$ is the input to a system that eliminates all signals with frequencies outside the range 0.5 to 0.7 Hz. What is the output of the system $y(t)$ and what fraction of the average power in $x(t)$ is in $y(t)$? (Note: your answers must be real, no e^{ja} terms.)

b) Assume $x(t)$ is the input to a system that eliminates all signals with frequencies in the range 0.5 to 0.7 Hz. What is the output of the system $y(t)$ and what fraction of the average power in $x(t)$ is in $y(t)$?

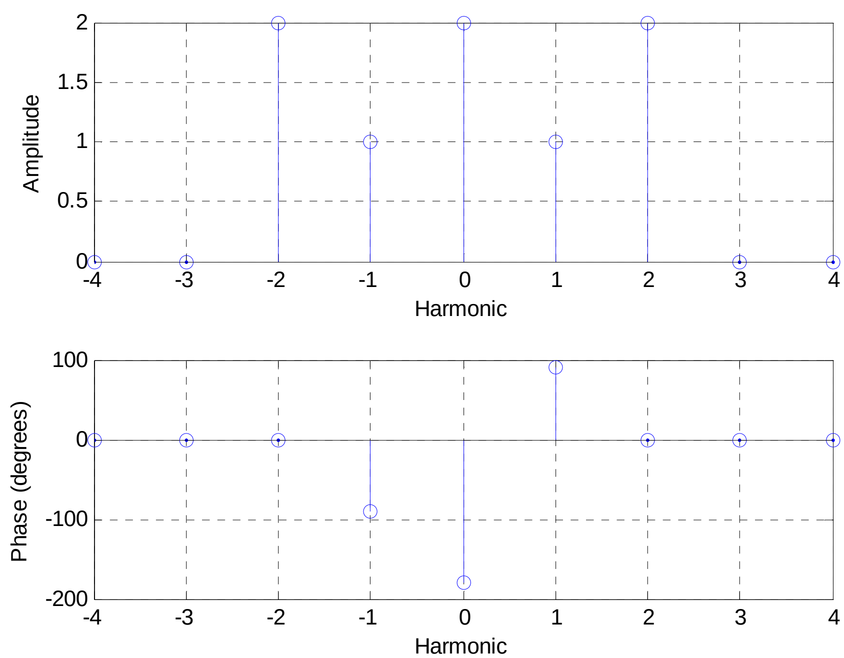
4. K & H, Problem 5.1. Use the example we did in class to get the Fourier series coefficients for part c.

5. K & H, Problem 5.3 (very easy)

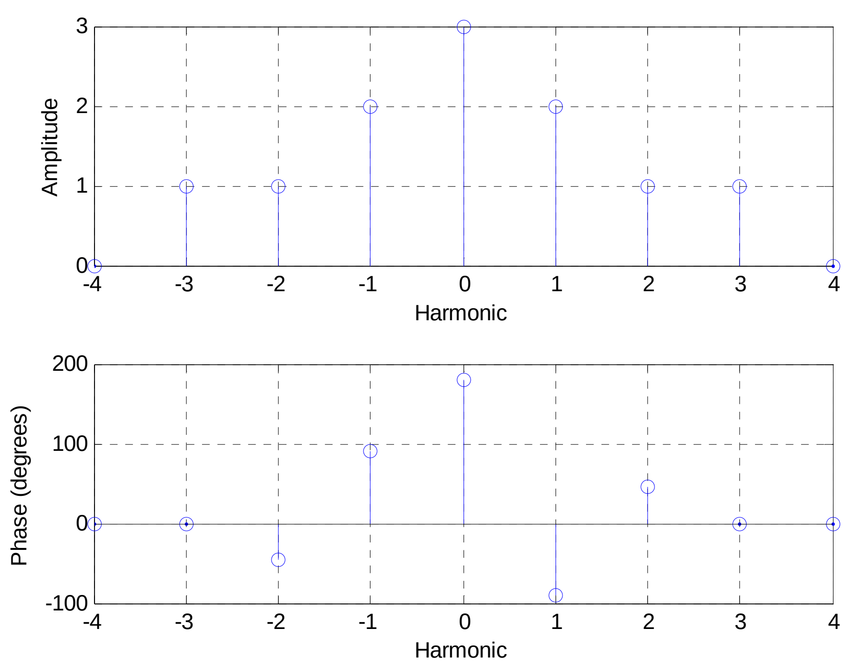
6. K & H, Problem 5.12. Note that $y(t) = x(t) - x(t-1)$. You need to write c_k^y in terms of c_k^x.

7. K & H, Problblme 5.13 (very easy)

8. The output of a LTI system, $y(t)$, has the following spectrum:



The system transfer function, $H(k\omega_o)$, has the following spectrum:



Assume all angles are multiples of 45 degrees.

a) Determine (sketch) the spectrum (magnitude and phase) of the input to the system, $x(t)$.
b) If $x(t)$ has the fundamental period $T = 2$ seconds, determine an analytical expression for $x(t)$ in terms of sine, cosines, and constants.