ECE 300
Signals and Systems
Homework 5

Due Date: Friday April 7 at 2:30 PM

Reading: K & H, pp. 145-161.

Problems:

1. Simplify each of the following into the form $c_k = \alpha(k)e^{-j\beta(k)}\text{sinc}(\lambda k)$

a) $c_k = \dfrac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

b) $c_k = \dfrac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

c) $c_k = \dfrac{e^{j5k} - e^{j2k}}{k}$

Scrambled Answers $c_k = 3\pi e^{-j\frac{7\pi k}{2}}\text{sinc}\left(\dfrac{3k}{2}\right)$, $c_k = 3e^{j(\frac{7}{2}k+\frac{\pi}{2})}\text{sinc}\left(\dfrac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi}\text{sinc}\left(k\dfrac{9}{2}\right)$

2. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of ω_0 and the c_k. Hint: Draw the signal, and then use the sifting property to calculate the c_k . *Hint: If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - 3p)$$

3. For the periodic square wave $x(t)$ with period $T_o = 0.5$ and

$$x(t)\begin{cases} 1 & 0 \le t < 0.25 \\ -1 & 0.25 \le t < 0.5 \end{cases}$$

show that the Fourier series coefficients are given by

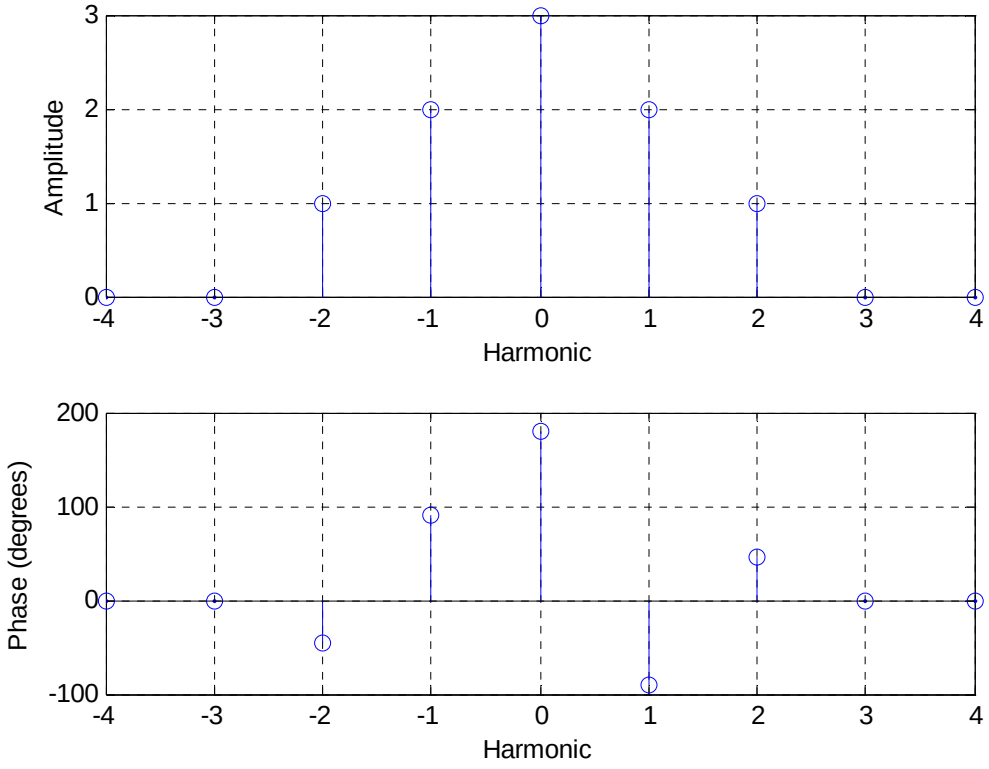
$$c_k = \begin{cases} \dfrac{-2j}{k\pi} & k \quad odd \\ 0 & k \quad even \end{cases}$$

where $x(t) = \sum_k c_k e^{jk4\pi t}$

5. K & H, Problem 4.9. For part **c** you should get $c_k^v = c_{k-1}^x$, use Euler's identity for part **d**.

6. K & H, Problem 4.12 parts **a** and **b** only. Write the integral as the sum of two integrals (with zero as the midpoint). Change variables to make the limits on the integrals the same.

7. A signal $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45 degrees)

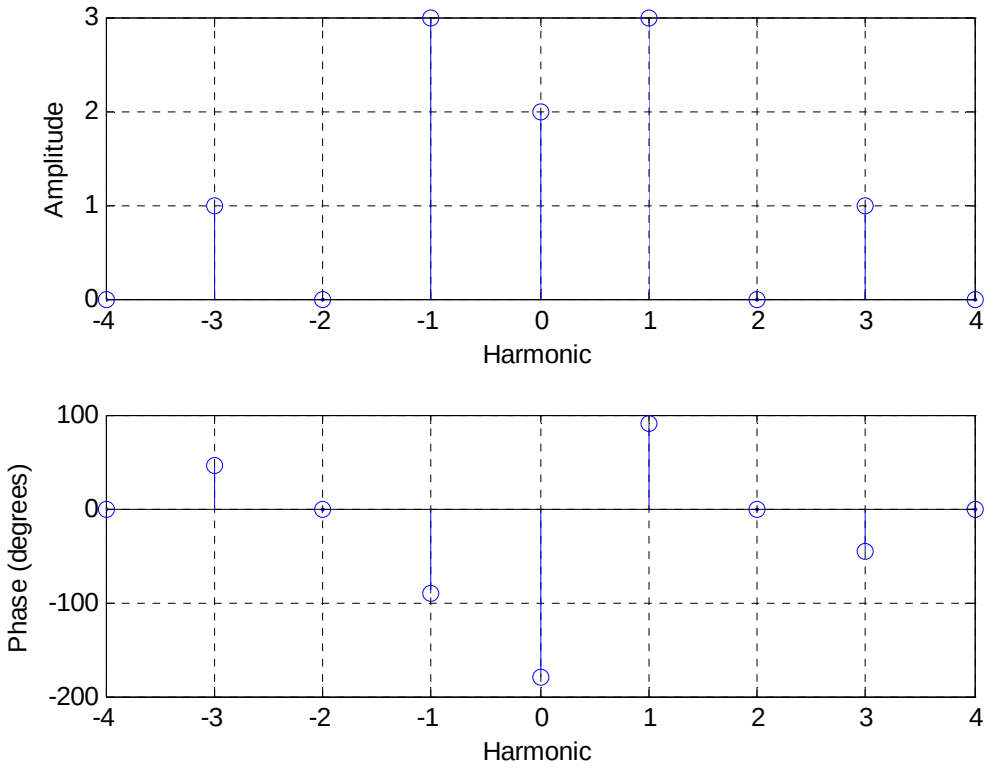


a) What is $x(t)$? Your expression must be real.

b) What is the average power in $x(t)$?

c) Sketch the single sided power spectrum for $x(t)$.

8. A signal $x(t)$, which has a fundamental period of 3 seconds, has the following spectrum (all phases are multiples of 45 degrees)



a) What is $x(t)$? Your expression must be real.

b) What is the average power in $x(t)$?

c) Sketch the single sided power spectrum for $x(t)$.

9. (Matlab Problem) A useful way of presenting information about the Fourier series representation of a signal is a **single sided power spectrum**, which tells us how the signal is distributed in frequency. To plot the single sided power spectrum, we just plot the power terms $|c_0|^2 \quad 2|c_1|^2 \quad 2|c_2|^2 \quad \ldots \quad 2|c_N|^2$ versus the corresponding frequency $0 \quad \omega_0 \quad 2\omega_0 \quad \ldots \quad N\omega_0$. Since the fundamental frequency ω_0 is common to all of the frequency terms, we often just plot $|c_0|^2 \quad 2|c_1|^2 \quad 2|c_2|^2 \quad \ldots \quad 2|c_N|^2$ versus $0 \quad 1 \quad 2 \quad \ldots \quad N$. You are to write a function in **Fourier_Series.m** to plot the single sided power spectrum of the signal. The arguments to the function should again be c_0 and the array

$c = [c_1 c_2 ... c_N]$. Utilize the **stem** command in Matlab to do the plotting. You may want to use the Matlab function **length** to determine the length of c. You may need to use the **figure** function so you can plot both the Fouier series (time-domain) plot and the power spectrum plot in two different windows. Plot the single sided power spectrum for each of the following signals utilizing N = 10 terms. The y-axis should be labeled *Average Power*, the x-axis labeled *Harmonic* and the graph should be titled *One Sided Power Spectrum*.

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$