ECE 300
Signals and Systems
Homework 4

Due Date: Friday March 31 at 2:30 PM

Reading: K & H, pp. 114-128.

Problems

1. K & H, Problem 3.16 parts **a**, **c**, and **e**. Do these graphically. You only need to plot the results for part **a**.

2. K & H, Problem 3.20.

3. K & H, Problem 3.22. You need to think again about causality. Use graphical convolution. You should get $y(t) = t^2 - 12t + 40$ for $4 \le t \le 6$.

4. K & H, Problem 3.26. (most of this one is pretty easy)

5. In this problem you will utilize the Matlab program **Fourier_Series.m** on the class website (download by ***right clicking***, ***select save target as***, and ***saving as a text document***). This function used a few of Matlab's built-in functions to numerically compute the Fourier series coefficients and evaluate the function, plus some properties of Fourier Series to compute the Fourier series. The arguments to this function are the initial and final times of a single period (the period starts at **Tlow** and ends at **Thigh**), **N** is the number of terms to use (in addition to the average value term), and **N_Periods,** which is the number of periods to plot. The first period is plotted is from **Tlow** to **Thigh**, and subsequent periods follow this one.

   The program **Fourier_Series.m** initially determines the Fourier series representation of

$$f(t) = \begin{cases} 0.135e^t & 0 \le t < 2 \\ 1 & 2 \le t < 3 \\ 4 - t & 3 \le t < 4 \\ 0 & 4 \le t < 5 \end{cases}$$

   The function **fcn** currently contains the representation of $f(t)$.You should examine the code to see how this function was represented in Matlab. This function has a period of 5. *You should type Fourier_Series(0,5,10,3) to see the Fourier series approximation to this function using 10 terms, plotted over 3 periods.*

a. Modify **Fourier_Series.m** to plot both the analytical function and the Fourier series approximation for the following functions.

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

You only need to modify the function **fcn.** Once you have determined how to represent each of these functions in Matlab, comment them out (put a % at the beginning of the line, or ***highlight*** them, ***select text***, and then ***select comment)*** before going on to the next function. We will be using these in what follows. Run the program and compare the plots of the true signals and the Fourier series representation using $N = 50$ terms to verify that everything is working Ok. Plot your results over 2 periods. **Turn in you plots.**

Note: You may want to move the legend around on the figure to get good plots. See the Matlab's **legend** function to see how to move the legend around. As an alternarive, you can often just drag the legend where you want it to be.

b. The average power in a periodic signal is defined as $P_{ave} = \frac{1}{T}\int_T |x(t)|^2 dt$ where T is the fundamental period of $x(t)$. Show that the average power in each of the periodic signals ($f_1(t)$, $f_2(t)$, and $f_3(t)$) in **a** is 0.166, 2.917, and 2.000, respectively.

c. We can also compute the **average power** in the Fourier series representation of a signal as

$$P_{ave} = |c_0|^2 + 2\sum_{k=1}^{N} |c_k|^2$$

You are to write *__a function__* (add it to the end of **Fourier_Series.m**) that computes the average power in a signal. The input to the function will be c_0 and the array $c = [c_1 \quad c_2 \quad ... \quad c_N]$. The output will be the average power. Matlab's built-in functions that may be helpful are **abs**, **sum**, and **.^**. ***You are not to use any loops***. You need to modify the title of the graph to print out the average power. You need to use the function **num2str** in the title (as was done for printing N). ***Do not hard code the value for the power***. If you use N = 5 for the functions in **a**, you should get average powers of 0.158, 2.820, and 1.886, respectively. Run your programs for $N = 5$ to verify all if working correctly. You do not need to turn anything in for this part.

d. **Parseval's Theorem** actually tells us that the average power in a signal is the same whether we utilize a time domain representation or a frequency representation, that is

$$P_{ave} = \frac{1}{T}\int_T |x(t)|^2 \, dt = |c_0|^2 + 2\sum_{k=1}^{\infty} |c_k|^2$$

Note that we must use all of the terms in the summation for the two sides to be exact. For each of the periodic signals in **a**, utilize **Fourier_Series.m** to determine the smallest number of terms N we need to use to before

$$|c_0|^2 + 2\sum_{k=1}^{N} |c_k|^2 \geq 0.95 P_{ave}$$

Be sure to also turn in your code for problem 5.