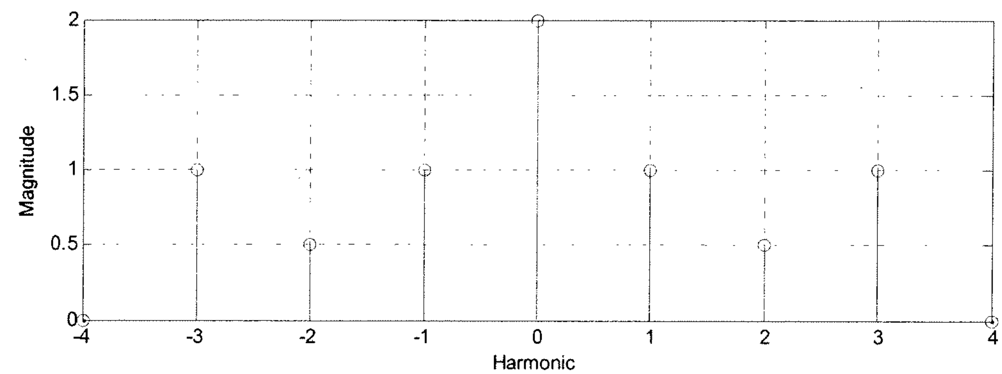
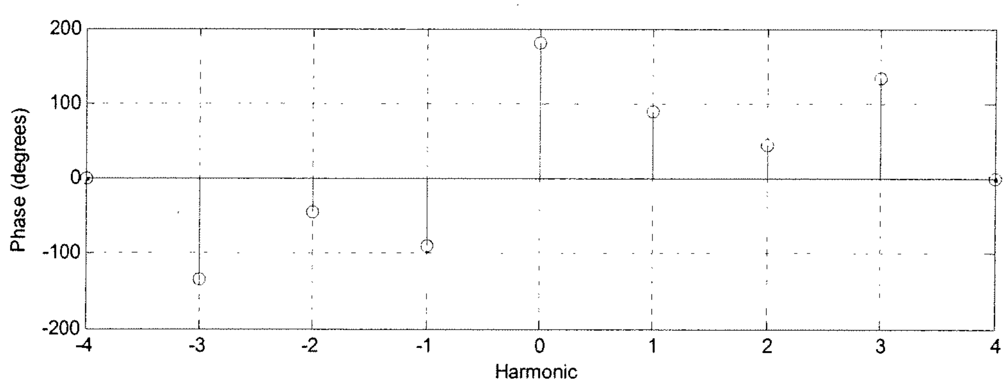
ECE 300
Signals and Systems
Homework 6

Due Date: ***Tuesday*** October 13, 2009 ***at 5:15 PM***

Problems:

1. Assume $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45 degrees)



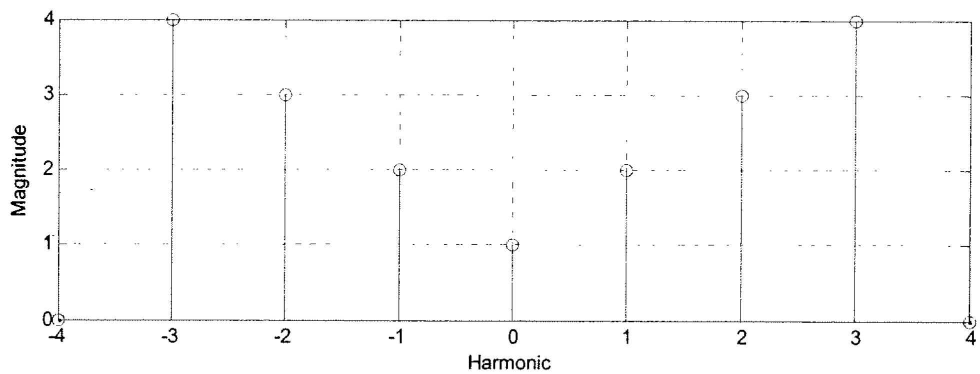


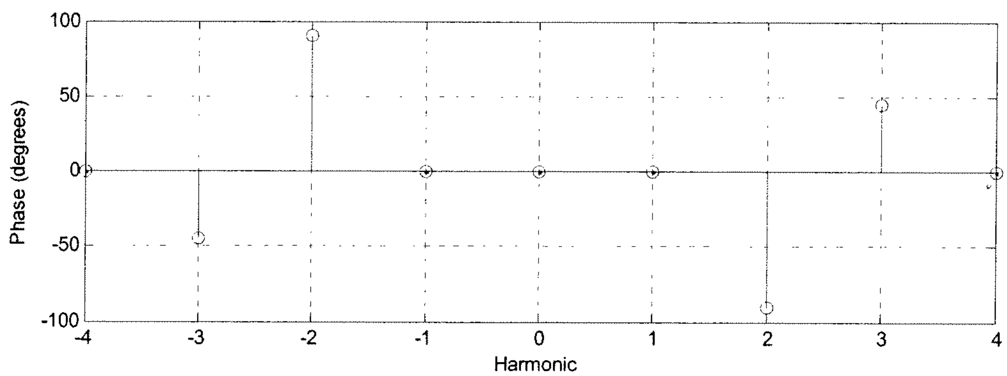
a) What is $x(t)$? Your expression must be real.

b) What is the average value of $x(t)$?

c) What is the average power in $x(t)$?

2. Assume $x(t)$ has the spectrum shown below (the phase is shown in radians) and a fundamental frequency $\omega_o = 2 \text{ rad/sec}$:





a) What is $x(t)$? Your expression must be real.

b) What is the average value of $x(t)$?

c) What is the average power in $x(t)$?

d) What is the average power in the second harmonic of $x(t)$?

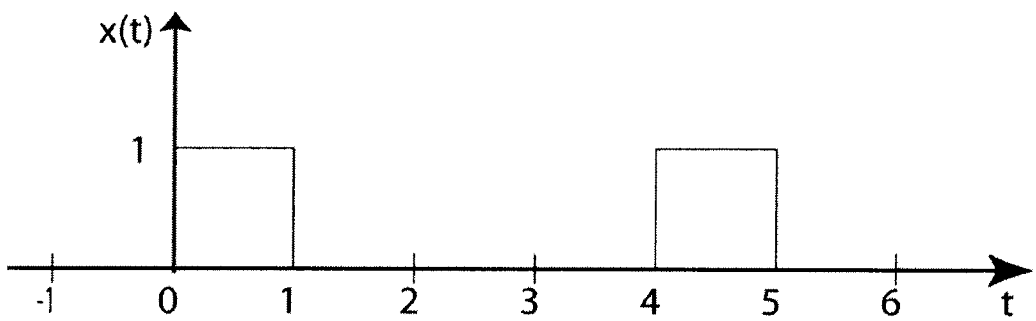
3. Simplify each of the following into the form $c_k = \alpha(k)e^{-j\beta(k)}\text{sinc}(\lambda k)$

a) $c_k = \dfrac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

b) $c_k = \dfrac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

c) $c_k = \dfrac{e^{j5k} - e^{j2k}}{k}$

Scrambled Answers $c_k = 3\pi e^{-j\frac{7\pi k}{2}}\text{sinc}\left(\frac{3\text{k}}{2}\right)$, $c_k = 3e^{j(\frac{7}{2}k+\frac{\pi}{2})}\text{sinc}\left(\frac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi}\text{sinc}\left(k\frac{9}{2}\right)$



4. For the periodic signal shown above, with period $T = 4$

a) Determine the fundamental frequency ω_0.

b) Determine the average value.

c) Determine the average power in the DC component of the signal.

d) Determine an expression for the expansion coefficients, c_k. You must write your expression in terms of the **sinc** function, and possibly a leading phase term.

(#1) $x(t) = c_0 + \sum_{k=1}^{\infty} 2|c_k| \cos(k\omega_0 t + \angle c_k)$ $\omega_0 = \frac{2\pi}{2} = \pi$

$c_0 = -2$ $c_2 = 0.5 \angle 45°$ $c_4 = 0$

$c_1 = 1 \angle 90°$ $c_3 = 1 \angle 135°$

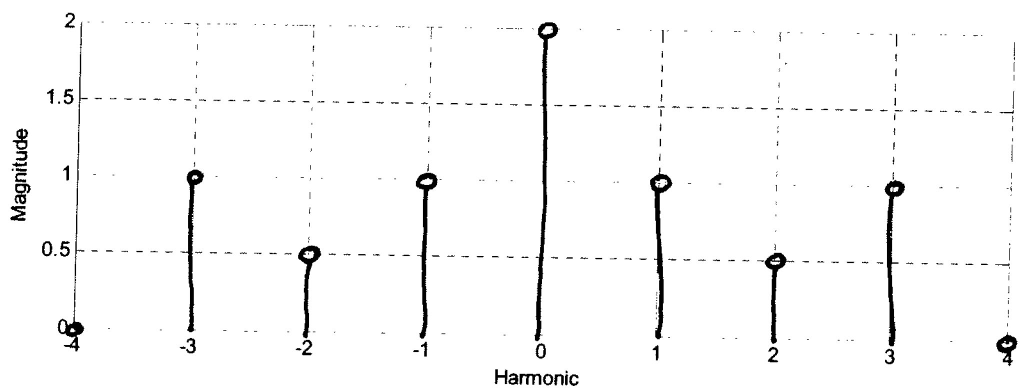
(a) $x(t) = -2 + 2\cos(\pi t + 90°) + \cos(2\pi t + 45°) + 2\cos(3\pi t + 135°)$

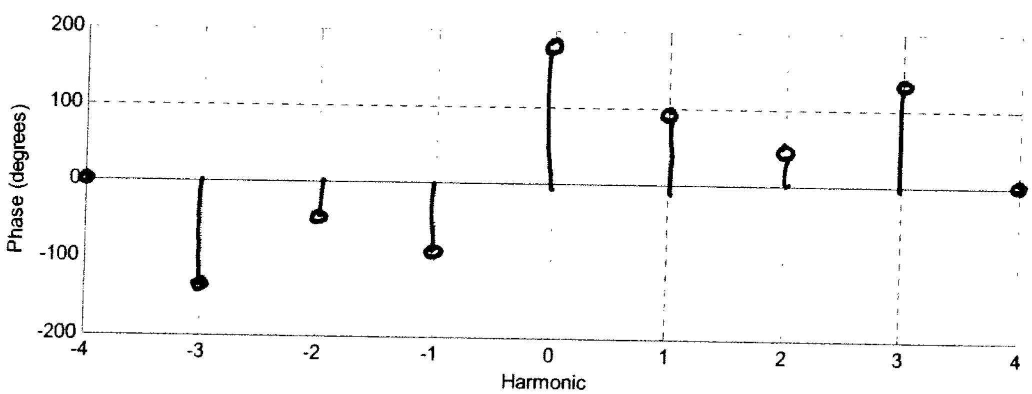
(b) $\bar{x} = -2$

(c) $P_{ave} = c_0^2 + \sum_{k=1}^{\infty} 2|c_k|^2 = (-2)^2 + 2(1^2) + 2(0.5^2) + 2(1^2)$

$= 8.5 = P_{ave}$

1. Assume $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45 degrees)





(#2) $x(t) = C_0 + \sum_{k=1}^{\infty} 2|C_k| \cos(k\omega_0 t + \angle C_k) \quad \omega_0 = 2$

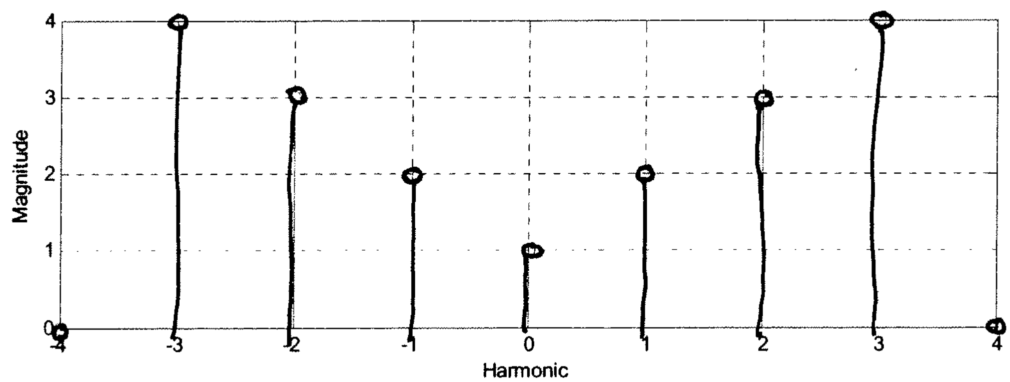
$C_0 = 1 \quad C_2 = 3\angle -90° \quad C_4 = 0$

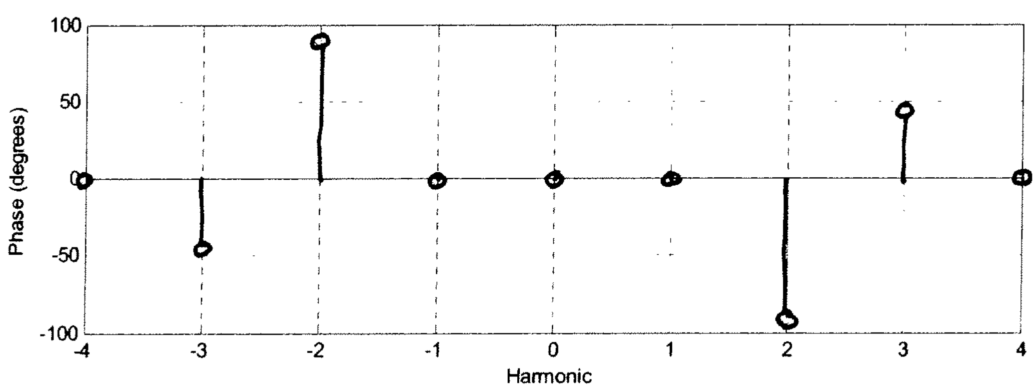
$C_1 = 2\angle 0° \quad C_3 = 4\angle 45°$

(a) $x(t) = 1 + 4\cos(2t) + 6\cos(4t - 90°) + 8\cos(6t + 45°)$

(b) $\bar{x} = 1$ (c) $P_{ave} = 1^2 + 2(2^2) + 2(3^2) + 2(4^2) = 59 = P_{ave}$

(d) $P_{ave}^2 = 2|C_2|^2 = 2|3^2| = 18 = P_{ave}^2$





#3

(a) $$c_k = \frac{e^{j7k\pi} - e^{-j2k\pi}}{jk\pi} = \frac{e^{j\frac{5}{2}\pi k}\left[e^{j\frac{9}{2}\pi k} - e^{-j\frac{9}{2}\pi k}\right]}{jk\pi}$$

$$= \frac{e^{j\frac{5}{2}\pi k}}{k\pi}\, 2 \left[\frac{e^{j\frac{9}{2}\pi k} - e^{-j\frac{9}{2}\pi k}}{2j}\right]$$

$$= \frac{2e^{j\frac{5}{2}\pi k}}{k\pi} \sin\left(\frac{9}{2}\pi k\right)$$

$$= 2e^{j\frac{5}{2}\pi k} \frac{\sin\left(\frac{9}{2}\pi k\right)}{\pi k\left(\frac{9}{2}\right)\left(\frac{2}{9}\right)} = \boxed{9e^{j\frac{5}{2}\pi k}\operatorname{sinc}\left(k\frac{9}{2}\right) = c_k}$$

(b) $$c_k = \frac{e^{-j2k\pi} - e^{-j5\pi k}}{jk} = \frac{e^{-j\frac{7}{2}\pi k}\left[e^{+j\frac{3}{2}\pi k} - e^{-j\frac{3}{2}\pi k}\right]}{jk}$$

$$= \frac{2e^{-j\frac{7}{2}\pi k}}{k}\sin\left(\frac{3}{2}\pi k\right) = 2e^{-j\frac{7}{2}\pi k}\frac{\sin\left(\frac{3}{2}\pi k\right)}{k\cdot\left(\frac{3}{2}\pi\right)\left(\frac{2}{3\pi}\right)}$$

$$= 2e^{-j\frac{7}{2}\pi k}\operatorname{sinc}\left(\frac{3k}{2}\right)\cdot\frac{3\pi}{2}$$

$$\boxed{c_k = 3\pi e^{-j\frac{7}{2}\pi k}\operatorname{sinc}\left(\frac{3k}{2}\right)}$$

(c) $$c_k = \frac{e^{j5k} - e^{j2k}}{k} = \frac{e^{j\frac{7}{2}k}\left(e^{j\frac{3}{2}k} - e^{-j\frac{3}{2}k}\right)}{k}$$

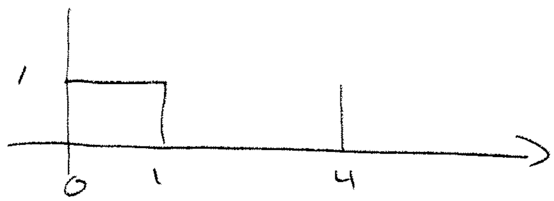
$$= \frac{2j}{k}e^{j\frac{7}{2}k}\left(\frac{e^{j\frac{3}{2}k} - e^{-j\frac{3}{2}k}}{2j}\right) = \frac{2j}{k}e^{j\frac{7}{2}k}\sin\left(\frac{3}{2}k\right)$$

$$= \frac{2e^{j\left(\frac{7}{2}k+\frac{\pi}{2}\right)}}{k}\sin\left(\frac{3}{2}k\frac{\pi}{\pi}\right) = \frac{2}{k}e^{j\left(\frac{7}{2}k+\frac{\pi}{2}\right)}\sin\left(\pi\cdot\frac{3k}{2\pi}\right)$$

$$= \frac{2e^{j\left(\frac{7}{2}k+\frac{\pi}{2}\right)}\sin\left(\pi\cdot\frac{3k}{2\pi}\right)}{k\cdot\frac{\pi}{\pi}\cdot\frac{3}{3}\cdot\frac{2\pi}{2\pi}} = \frac{2e^{j\left(\frac{7}{2}k+\frac{\pi}{2}\right)}\sin\left(\pi\frac{3k}{2\pi}\right)}{\frac{\pi 3k}{2\pi}\cdot\frac{2\pi}{3\pi}}$$

$$\boxed{c_k = 3e^{j\left(\frac{7}{2}k+\frac{\pi}{2}\right)}\operatorname{sinc}\left(\frac{3k}{2\pi}\right)}$$

(#4)



(a) $T_0 = 4$ so $\omega_0 = 2\pi/4 = \boxed{\pi/2 = \omega_0}$

(b) $c_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{4}\int_0^1 1\,dt = \boxed{1/4 = c_0}$

(c) $P_0 = c_0^2 = \boxed{1/16 = P_0}$

(d) $c_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\,dt$

$$= \frac{1}{T_0}\int_0^1 1\, e^{-jk\omega_0 t}\,dt = \frac{1}{T_0}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Bigg|_0^1\right]$$

$$= \frac{e^{-jk\omega_0} - 1}{-jk\omega_0 T_0} = \frac{1 - e^{-jk\omega_0}}{2\pi k j} = \frac{e^{-jk\frac{\omega_0}{2}}}{\pi k (2j)}\left[e^{jk\frac{\omega_0}{2}} - e^{-jk\frac{\omega_0}{2}}\right]$$

$$= \frac{e^{-jk\frac{\omega_0}{2}}}{\pi k}\sin\left(k\frac{\omega_0}{2}\right) = e^{-jk\pi/4}\,\frac{\sin\left(k\,\pi/4\right)}{\pi k\,(4/4)}$$

$$\boxed{c_k = \frac{1}{4}\, e^{-jk\pi/4}\,\mathrm{sinc}(k/4)}$$

```
%
%  This routine implements a Complex Fourier series
%
% Inputs: N is the number of terms to be used in the series
%
function Complex_Fourier_series(N)
%
%  one period of the function goes from low to high
%
  low = 0;
  high = 1/60;
%
%  the difference between low and high is one period
%
  T = high-low;
  w0 = 2*pi/T;
%
%  the periodic function
%
  x  = @(t)abs(sin(w0*t));
%  x =  @(t) sin(w0*t).*((0<=t)&(t<=T/2))+0*((t>T/2)&(t<T));
%  x =  @(t) exp(-t/T);
%
%   find c(1) to c(N)
%
  for k = 1:N
      arg = @(t) x(t).*exp(-j*k*w0*t);
      c(k) = (1/T)*quadl(arg,low,high);
  end;
%
  c0 = (1/T)*quadl(x,low,high);
%


%
%  determine a time vector
%
  t = linspace(low,T,1000);
%
%  Find the Fourier series representation
%
  est_c = c0;
  for k = 1:N
      est_c = est_c + 2*abs(c(k))*cos(k*w0*t+angle(c(k)));
  end;
%
%  determine the average power
%
  P_ave = (1/T)*quadl(@(t)x(t).*x(t), low, high)
  P = c0*c0;
  for k=1:N
      P = P+2*abs(c(k))*abs(c(k));
  end;
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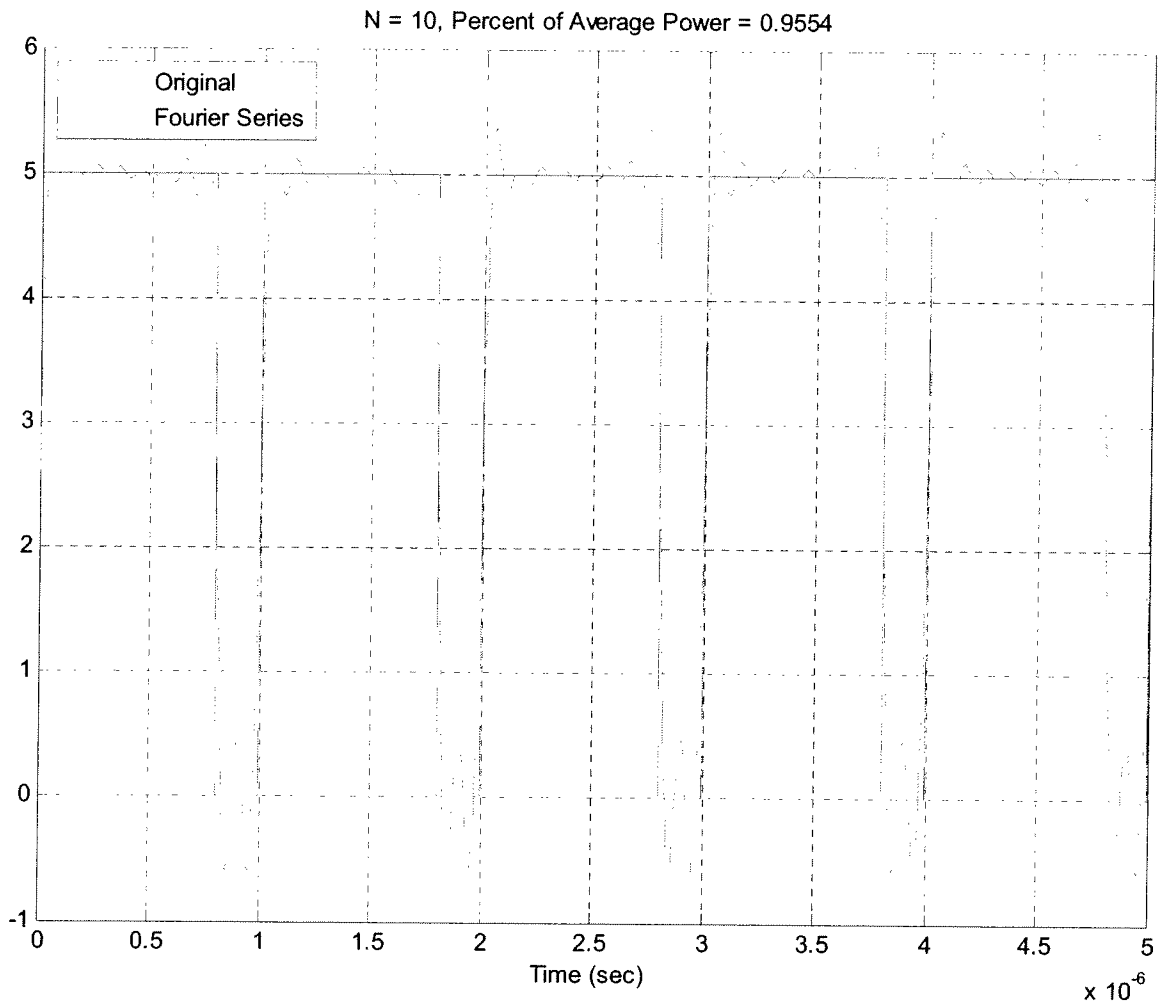

Figure 1: Pulse Width Modulated (PWM) signal with 80% duty cycle.