ECE 300
Signals and Systems
Homework 2

Due Date: Thursday September 17, 2009 *at the beginning of class*

Problems

1) Assume $x(t) = \text{rect}\left(\frac{t-2}{4}\right) + \text{rect}\left(\frac{t-1}{2}\right)$ and sketch the following:

a) $x_1(t) = x(2t)$ b) $x_2(t) = x\left(\frac{t}{2}\right)$

c) $x_3(t) = x(1-t)$ d) $x_4(t) = x(1+2t)$

2) Simplify the following as much as possible, giving numerical answers where possible. Use unit step functions as necessary to simplify your answers.

a) $\int_{-\infty}^{\infty} e^{-t} u(t-5)dt$ b) $\int_{-\infty}^{\infty} t^2 \left[u(t-6) - u(t-5)\right] dt$

c) $\int_{-\infty}^{\infty} t^2 \delta(t-2)dt$ d) $\int_{5}^{\infty} t^2 \delta(t-2)dt$

e) $\int_{-\infty}^{\infty} \delta(t-3)\delta(t-4)dt$ f) $\int_{-\infty}^{\infty} u(t-3)\delta(t-4)dt$

g) $\int_{-\infty}^{t} e^{-(t-\lambda-1)}\delta(\lambda-2)d\lambda$ h) $\int_{-\infty}^{t} e^{-2(t-\lambda)}\delta(\lambda+1)d\lambda$

i) $\int_{-\infty}^{t-1} e^{-3(t-\lambda)}\delta(\lambda-1)d\lambda$ j) $\int_{-t}^{\infty} e^{-(t-\lambda)}\delta(\lambda+2)d\lambda$

3) For each of the following signals, determine if the signal is periodic and, if so, the fundamental period.

a) $x(t) = \sin(2t) + \cos(3t + 30^\circ)$ b) $x(t) = \cos(2t) + \cos(\pi t)$

c) $x(t) = 1 + \cos(t)$ d) $x(t) = \cos\left(\frac{t}{2}\right) + e^{j\frac{3t}{4}}$

4) In this problem you will derive the following property of the delta function

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

using only the sifting property

$$\int_{-\infty}^{\infty} x(\lambda)\delta(\lambda)d\lambda = x(0)$$

a) By changing variables in the integral and using the sifting property, show that for $a > 0$

$$\int_{-\infty}^{\infty} x(\lambda)\delta(a\lambda)d\lambda = \frac{1}{a}x(0)$$

b) By changing variables in the integral and using the sifting property, show that for $a < 0$

$$\int_{-\infty}^{\infty} x(\lambda)\delta(a\lambda)d\lambda = \frac{-1}{a}x(0)$$

c) Combine parts **a** and **b** to show the result claimed. Basically, you need to show that $\delta(at)$ behaves like $\frac{1}{|a|}\delta(t)$.

d) Is $\delta(t)$ and even function, and odd function, or neither?

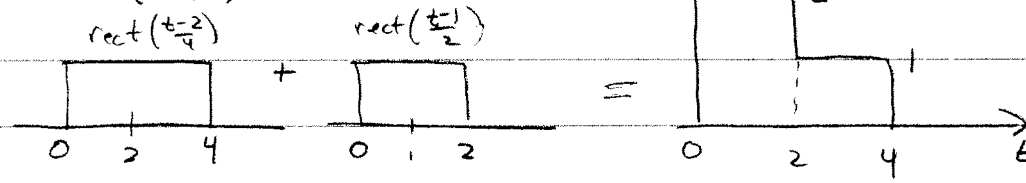
5) Determine if the following functions are energy signals, power signals, or neither.

a) $x(t) = e^{t}u(t)$ **b)** $x(t) = e^{-t}u(t)$ **c)** $x(t) = e^{-t}\cos(t)u(t)$ **d)** $x(t) = u(t)$
e) $x(t) = \cos(t) + e^{j2t}$ **f)** $x(t) = u(t+1) - u(t-3)$ **g)** $x(t) = u(t+1) - u(t-3) + u(t-5)$

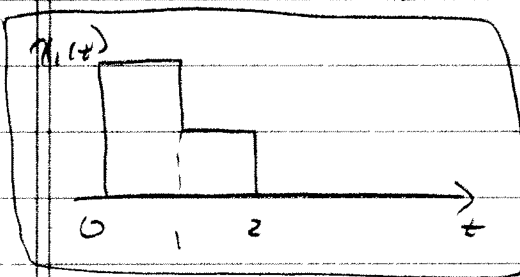
Matlab Problem

6) Using Matlab, plot each signal from Problem 3 for three *fundamental periods* if the signal is periodic, or three times the longest period in the signal if the signal is not periodic. Be sure there are at least 50 samples per period for each waveform and your graphs are neatly labeled. **Notes:** (1) Matlab works in radians, so all angles must be converted to radians, (2) use **exp** in Matlab to get an exponential, (3) **j** is Matlab's way of indicating the square root of -1, and if you want $x(t) = e^{j2t}$ you should type something like ***x = exp(j*2*t)***, and (4) if the waveform is complex, plot the real and imaginary parts separately. The Matlab commands **real** and **imag** are very useful for this. ***Turn in your plots.***

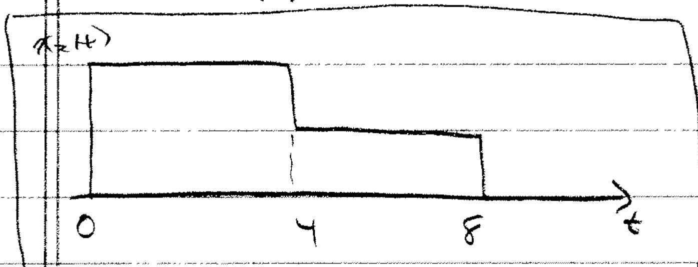
#1 $x(t) = \mathrm{rect}\left(\frac{t-2}{4}\right) + \mathrm{rect}\left(\frac{t-1}{2}\right)$



(a) $x_1(t) = x(2t)$ $\quad x_1(0) = x(0) \quad x_1(1) = x(2) \quad x_1(2) = x(4)$



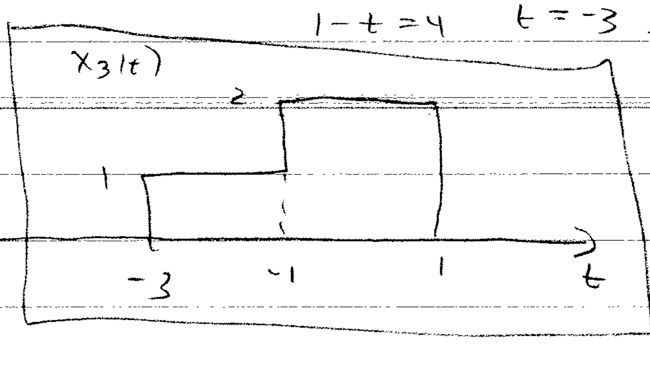
(b) $x_2(t) = x\left(\frac{t}{2}\right)$ $\quad x_2(0) = x(0) \quad x_2(4) = x(2) \quad x_2(8) = x(4)$



(c) $x_3(t) = x(1-t)$ $\quad 1-t=0 \quad t=1$ so $x_3(1) = x(0)$

$1-t=2 \quad t=-1$ so $x_3(-1) = x(2)$

$1-t=4 \quad t=-3$ so $x_3(-3) = x(4)$



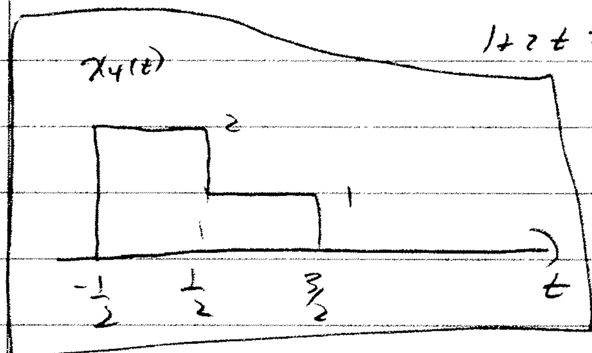
(continued)

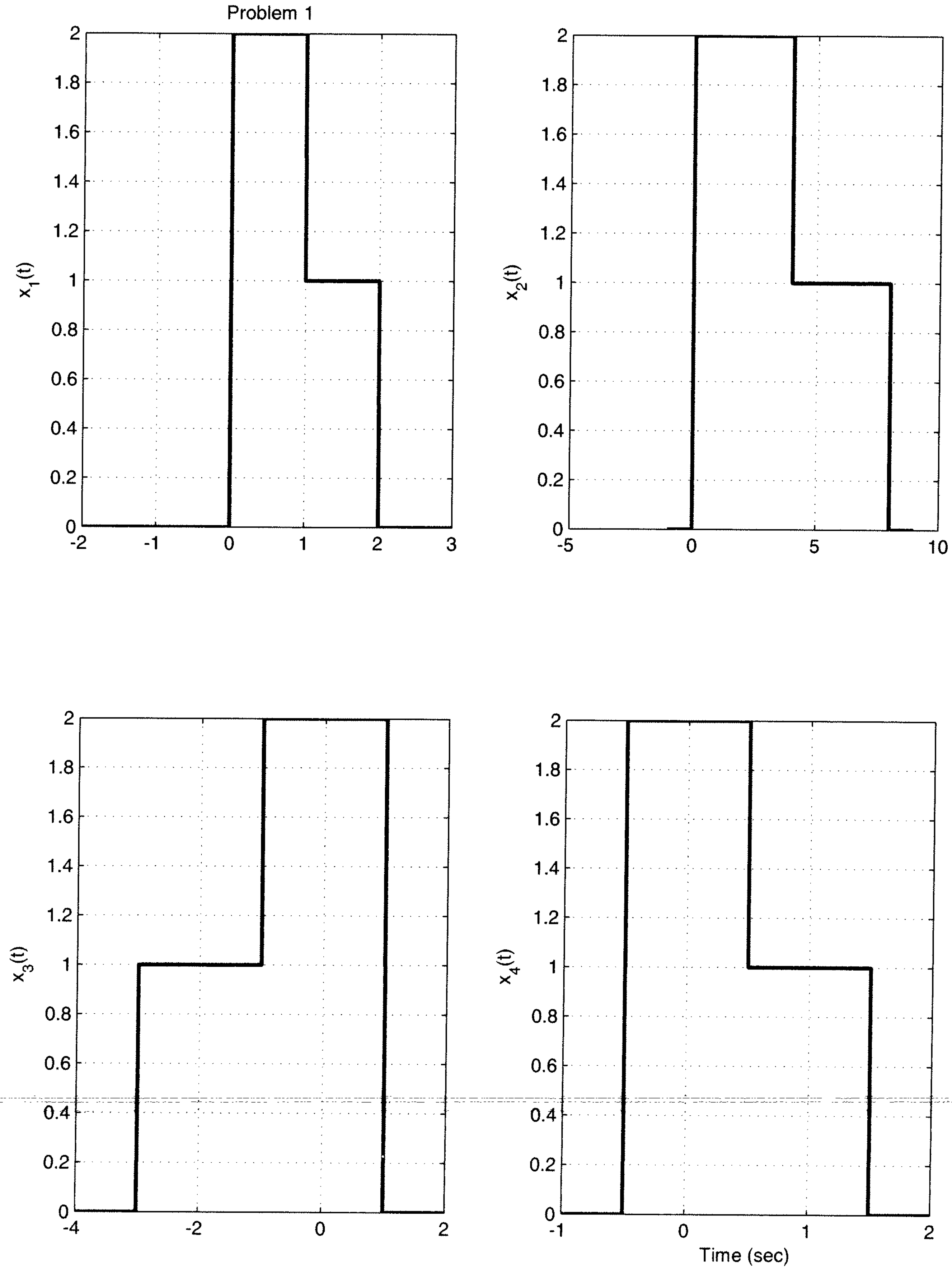
(#1) (continued)

(d) $x_4(t) = x(1+2t)$ $\quad 1+2t=0 \quad t=-\frac{1}{2} \quad$ so $x_4(-\frac{1}{2}) = x(0)$

$1+2t=2 \quad t=\frac{1}{2} \quad$ so $x_4(\frac{1}{2}) = x(2)$

$1+2t=4 \quad t=\frac{3}{2} \quad$ so $x_4(\frac{3}{2}) = x(4)$




Problem 1
x1(t)
x2(t)
x3(t)
x4(t)
Time (sec)

#2

(a) $\int_{-\infty}^{\infty} e^{-t} u(t-5)\,dt = \int_{5}^{\infty} e^{-t}\,dt = -e^{-t}\Big|_{5}^{\infty} = \boxed{e^{-5} = 0.006738}$

(b) $\int_{-\infty}^{\infty} t^2 [u(t-6) - u(t-5)]\,dt = -\int_{-\infty}^{\infty} t^2 [u(t-5) - u(t-6)]\,dt$

$= -\int_{5}^{6} t^2\,dt = -\frac{t^3}{3}\Big|_{5}^{6} = -\left[\frac{6^3}{3} - \frac{5^3}{3}\right] = \boxed{-30.33}$

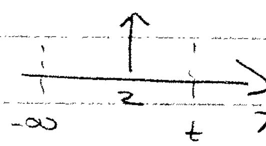
(c) $\int_{-\infty}^{\infty} t^2 \delta(t-2)\,dt = \boxed{4}$

(d) $\int_{5}^{\infty} t^2 \delta(t-2)\,dt = \boxed{0}$

(e) $\int_{-\infty}^{\infty} \delta(t-3)\,\delta(t-4)\,dt = \boxed{0}$

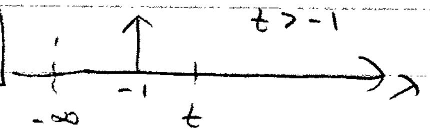
(f) $\int_{-\infty}^{\infty} u(t-3)\,\delta(t-4)\,dt = u(4-3) = u(1) = \boxed{1}$

(g) $\int_{-\infty}^{t} e^{-(t-\lambda-1)}\,\delta(\lambda-2)\,d\lambda = \boxed{e^{-(t-3)}\,u(t-2)}$



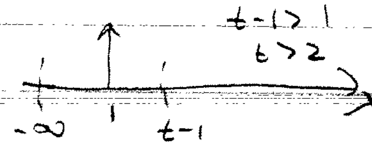
$t > 2$

(h) $\int_{-\infty}^{t} e^{-2(t-\lambda)}\,\delta(\lambda+1)\,d\lambda = \boxed{e^{-2(t+1)}\,u(t+1)}$



$t > -1$

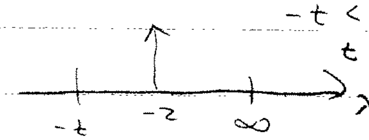
(i) $\int_{-\infty}^{t-1} e^{-3(t-\lambda)}\,\delta(\lambda-1)\,d\lambda = \boxed{e^{-3(t-1)}\,u(t-2)}$



$t-1 > 1$

$t > 2$

(j) $\int_{-t}^{\infty} e^{-(t-\lambda)}\,\delta(\lambda+2)\,d\lambda = \boxed{e^{-(t+2)}\,u(t-2)}$



$-t < -2$

$t > 2$

#3

a) $x(t) = \sin(2t) + \cos(3t + 30^\circ)$

$x(t+T) = \sin(2t + 2T) + \cos(3t + 3T + 30^\circ)$

$2T = q(2\pi) \qquad 3T = r(2\pi)$

$T = q\pi = r\frac{2}{3}\pi \qquad r = 3 \quad q = 2$ $\boxed{\text{periodic } T = 2\pi}$

b) $x(t) = \cos(2t) + \cos(\pi t)$

$x(t+T) = \cos(2t + 2T) + \cos(\pi t + \pi T)$

$2T = q(2\pi) \qquad \pi T = r(2\pi)$

$T = q\pi = r\,2$ no integers q, r will solve this

$\boxed{\text{not periodic}}$

c) $x(t) = 1 + \cos(t)$

$x(t+T) = 1 + \cos(t + T)$

$T = q(2\pi) \qquad q = 1$ $\boxed{\text{periodic, } T = 2\pi}$

d) $x(t) = \cos(\frac{t}{2}) + e^{j\frac{3}{4}t}$

$x(t+T) = \cos(\frac{t}{2} + \frac{T}{2}) + e^{j\frac{3}{4}t} e^{j\frac{3}{4}T}$

$\frac{T}{2} = q(2\pi) \qquad \frac{3}{4}T = r(2\pi)$

$T = q(4\pi) = r\frac{8}{3}\pi \qquad r = 3 \quad q = 2$ $\boxed{\text{periodic, } T = 8\pi}$

#4

a) $\int_{-\infty}^{\infty} x(\lambda)\,\delta(a\lambda)\,d\lambda \qquad a>0$

let $p = a\lambda \quad \frac{p}{a} = \lambda \quad \frac{dp}{a} = d\lambda$

$$\int_{-\infty}^{\infty} x(\lambda)\,\delta(a\lambda)\,d\lambda = \frac{1}{a}\int_{-\infty}^{\infty} x\left(\tfrac{p}{a}\right)\delta(p)\,dp = \frac{1}{a}x(0)$$

b) $\int_{-\infty}^{\infty} x(\lambda)\,\delta(a\lambda)\,d\lambda \qquad a<0$

let $p = a\lambda \quad \frac{p}{a} = \lambda \quad \frac{dp}{a} = d\lambda$

$$\int_{-\infty}^{\infty} x(\lambda)\,\delta(a\lambda)\,d\lambda = \frac{1}{a}\int_{+\infty}^{-\infty} x\left(\tfrac{p}{a}\right)\delta(p)\,dp = -\frac{1}{a}\int_{-\infty}^{\infty} x\left(\tfrac{p}{a}\right)\delta(p)\,dp$$

$$= -\frac{1}{a}\,x(0)$$

c) for $a>0$ $\delta(at)$ acts like $\frac{1}{|a|}\delta(t)$

for $a<0$ $-\frac{1}{a} = \frac{1}{|a|}$ $\delta(at)$ acts like $\frac{1}{|a|}\delta(t)$

so $\boxed{\delta(at) = \frac{1}{|a|}\delta(t)}$

d) for $a=-1$ $\delta(-t) = \delta(t)$ δ is an <u>even</u> function

#5

(a) $x(t) = e^{+t}u(t)$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_0^T e^{2t}\,dt = \lim_{T\to\infty} \frac{1}{4T} e^{2T} = \infty \quad \boxed{\text{neither}}$$

(b) $x(t) = e^{-t}u(t)$

$$E_\infty = \lim_{T\to\infty} \int_0^T e^{-2t}\,dt = \lim_{T\to\infty} \left.\frac{e^{-2t}}{-2}\right|_0^T = \frac{1}{2} \quad \boxed{\text{Energy}}$$

(c) $x(t) = e^{-t}\cos(t)u(t) \le e^{-t}u(t)$ so $\boxed{\text{Energy}}$

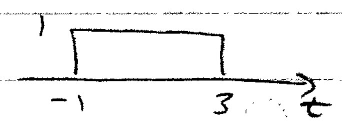
(d) $x(t) = u(t)$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_0^T dt = \frac{1}{2} \quad \boxed{\text{power}}$$

(e) $x(t) = \cos(t) + e^{j2t}$ $\quad x(t) \le 2$

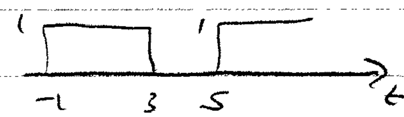
For $x(t) = 2$, $P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 4\,dt = 4$ $\boxed{\text{Power signal}}$

(f) $x(t) = u(t+1) - u(t-3)$



$$E_\infty = \int_{-1}^{3} 1^2\,dt = 4 \quad \boxed{\text{Energy}}$$

(g) $x(t) = u(t+1) - u(t-3) + u(t-5)$



$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\left[\int_{-1}^{3} 1^2\,dt + \int_5^T 1^2\,dt\right] = \frac{1}{2} \quad \boxed{\text{Power}}$$

Problem 6