ECE 300
Signals and Systems
Homework 9

Due Date: **Friday** November 6 at 5:15 PM, 2009

Problems

1. Determine the transfer function $H(\omega)$ that would produce the following input /output relationships. Simplify your answers as much as possible.

a) $y(t) = a\dot{x}(t-b)$

b) $y(t) = ax(t+b) + ax(t-b)$

c) $\dot{y}(t) = x(t) * e^{-t}u(t-b)$

2. Using the ***duality property***, find the corresponding Fourier transform for the following:

a) $g(t) = \text{sinc}^2(Bt)$

b) $g(t) = \text{sinc}(Wt)$

c) $g(t) = \delta(t)$

d) $g(t) = \cos(\omega_0 t)$

Do not just look up the pairs from the table (though you can use any other pairs except the one you are trying to find).

3. Consider a linear time invariant system with transfer function given by

$$H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \le 2 \\ 0 & else \end{cases}$$

with input $x(t) = \frac{8}{\pi}\text{sinc}^2\left(\frac{2(t-1)}{\pi}\right)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

d) Determine $y(t)$, the output of the system.

Answer $y(t) = \frac{20}{\pi}\text{sinc}\left[\frac{2}{\pi}(t-3)\right] + \frac{10}{\pi}\text{sinc}^2\left[\frac{1}{\pi}(t-3)\right]$

4. Consider a linear time invariant system with impulse response given by $h(t)=\frac{1}{2\pi}\mathrm{sinc}\left(\frac{\text{t-2}}{2\pi}\right)$ with input $x(t)=\frac{4}{\pi}\mathrm{sinc}\left(\frac{2\text{t}}{\pi}\right)\cos(t)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Determine the energy in $x(t)$

d) Determine $H(\omega)$.

e) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

f) Determine $y(t)$, the output of the system.

g) Determine the energy in $y(t)$.

5. Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t)=5\,\mathrm{sinc}\left(\frac{t}{0.002}\right)+5\,\mathrm{sinc}\left(\frac{t}{0.001}\right)$$ *Answer* 56%

6. Consider the signal $x(t)=\cos(2t)+\cos(3t)$

a) Sketch the spectrum of $X(\omega)$

b) $x(t)$ is the input to an ideal sampler sampling at rate $f_s=\frac{2}{\pi}=\frac{1}{T}$ seconds. Sketch the spectrum of the sampled signal $X_s(\omega)$.

c) Assume $x_s(t)$ is the input to an ideal lowpass filter with a cutoff frequency of 4 rad/sec and passband gain of T. Determine the output signal $x_r(t)$ and write it in terms of the original signal $x(t)$ plus any aliased terms.

7. In this problem we will go over some of the Fourier series and Fourier transform results we need to understand impulse sampling.

a) Assume we have periodic function $p(t)$ with Fourier series representation

$$p(t) = \sum_{k=-\infty}^{k=\infty} c_k e^{jk\omega_0 t}$$

and we construct the function $x_s(t) = x(t)p(t)$. Show that in the frequency domain we have

$$X_s(\omega) = \sum_{k=-\infty}^{k=\infty} c_k X(\omega - k\omega_0)$$

b) Show that the Fourier series for the (periodic) impulse train

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$

Is given by

$$p(t) = \sum_{k=-\infty}^{k=\infty} \frac{1}{T_0} e^{jk\omega_0 t} \ , \quad \omega_0 = \frac{2\pi}{T_0}$$

c) Combine the previous two parts to show

$$X_s(\omega) = \sum_{k=-\infty}^{k=\infty} \frac{1}{T_0} X(\omega - k\omega_0)$$

8. (Matlab) This problem is a continuation of Problem 7, but you only need to use the results of that problem. There are three important things you should know about impulse sampling

- If the original signal $x(t)$ is sampled with an impulse train with period T_0 (the time between samples is T_0) then the spectrum of the original signal, $X(f)$ (or $X(\omega)$), will be replicated every $f_0 = \frac{1}{T_0}$ Hz (or ω_0 radians/sec) in the spectrum of the sampled signal, $X_s(f)$ (or $X_s(\omega)$)
- The replicated spectra $X(f)$ (or $X(\omega)$) will be scaled by $\frac{1}{T_0}$
- If we want to recover the original signal from the sampled signal, we need to be able to isolate one instance of the original signal's spectrum by lowpass filtering.

The Matlab routine **impulse_sampling.m** illustrates the effects of sampling the signal

$$x(t) = \text{sinc}^2\left(\frac{t}{0.0002}\right)$$

using a sampling rate of $f_0 = 20,000Hz$ and then filtering to try and reconstruct the original signal. You do not need to understand what this routine is doing, you will only be modifying it a bit and trying to understand what is happening. *Run the routine as it is for parts b, c, and d and turn in this plot.*

a) Show that for this function $X(\omega) = 0.0002\Lambda\left(\frac{\omega}{(2\pi)5000}\right)$

b) Verify that the spectrum of the replicated signal $X(\omega)$ has the correct bandwidth

c) Verify that the replicated signal $X_s(\omega)$ has maximum amplitude equal to $\frac{1}{T_0}\max\{|X(\omega)|\}$

d) Verify that the signal is replicated every $f_0 = 20,000Hz$

The bottom panel (and the second figure) shows the original signal and the signal we tried to reconstruct after sampling. If our sampling is effective we should be able to reconstruct the original signal.

e) Modify the lowpass filter (lines 59 and 60) so that we get a reasonably good approximation of the original signal. *Turn in your plots.*

f) Modify the sampling rate (line 31) so you are sampling at 15 kHz, 10 KHz, and then 7.5 kHz. *Turn in your plots.* Notice where the replicated signals are located. Are you able to reconstruct the original signal for these sampling rates?