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ECE 300
Signals and Systems

Exam 2
26 October, 2009

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam. ***<u>You must show your work to receive credit for a problem.</u>***

Problem 1	_________	/ 30
Problem 2	_________	/25
Problem 3	_________	/ 20
Problem 4	_________	/ 10
Problem 5	_________	/ 15

Exam 2 Total Score: _______ / 100

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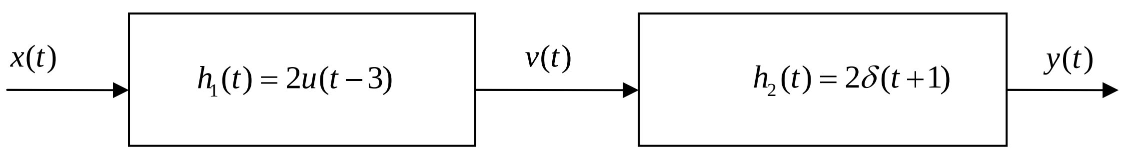
1. Impulse Response (30 points)

For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions. For full credit, simplify your answers as much as possible.*

a) $y(t) = \int_{-\infty}^{t-2} e^{-(t-\lambda)} x(\lambda - 2) d\lambda + e^{-t} x(t)$

b) $2\dot{y}(t) + y(t) = x(t-1)$

c) Determine the impulse response for the following system



d) If the response of a system to a step of amplitude *A* is given by

$$s(t) = A[1 + e^{-t/\tau}]u(t)$$

determine the **unit** impulse response of the system. (*Do not just guess the answer, you will probably be wrong, and besides, you need to show your work!)*

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2. Fourier Series (25 points)

The periodic function $x(t)$ is defined over one period ($T_0 = 5$ seconds) as

$$x(t) = \begin{cases} 2 & -2 \le t \le 1 \\ 0 & 1 \le t \le 3 \end{cases}$$

Determine the complex Fourier series coefficients, c_k by evaluating the appropriate integral.

Be sure to simplify your answer as much as possible and use a <u>sinc</u> function if appropriate.

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3. (20 points) A periodic signal has the Fourier series representation $x(t) = \sum_{k=-\infty}^{k=\infty} c_k^x e^{jk2t}$.

This signal is the input to an LTI system, and the (steady state) output of the system is

$$y(t) = 4 + 4\cos(4t + 30^o) + 6\cos(6t + 30^o)$$

Fill in the following table:

k	$\lvert c_k^x \rvert$	$\lvert H(jk\omega_0) \rvert$	$\measuredangle c_k^x$	$\measuredangle H(jk\omega_0)$
0	2		180^o	
1	3		-45^o	
2	1		45^o	
3	0.5		-30^o	

If you cannot determine a necessary value, leave the table entry blank.

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4. (10 points) Assuming the system input $x(t) = \sum_{k=-\infty}^{k=\infty} c_k^x e^{jk\omega_0 t}$ and output $y(t) = \sum_{k=-\infty}^{k=\infty} c_k^y e^{jk\omega_0 t}$ are related through the LTI system $\dot{y}(t) + 2y(t-2) = 6x(t-3)$

a) Determine the relationship between c_k^x and c_k^y.

b) Determine the *continuous* transfer function $H(j\omega)$ between the input and the output.

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5) (15 points) The periodic signal $x(t)$ has the Fourier series representation

$$x(t) = 2 + \sum_{k=-\infty}^{k=\infty} \frac{1}{1+kj} e^{jk3t}$$

$x(t)$ is the input to an LTI system (a high pass filter filter) with the transfer function

$$H(j\omega) = \begin{cases} 0 & |\omega| < 5 \\ 3e^{-j2\omega} & |\omega| > 5 \end{cases}$$

The steady state output of the system can be written as

$$y(t) = ax(t-b) + c + d\cos(e(t-b) + f)\ .$$

Determine numerical values for the parameters a, b, d, e and f

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Some Potentially Useful Relationships

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\mathrm{rect}\left(\frac{t-t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$