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ECE 300
Signals and Systems

Exam 1
1 October, 2009

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam. ***<u>You must show your work to receive credit for a problem.</u>***

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Problem 2	________	/5
Problem 3	________	/ 30
Problem 4	________	/ 30
Problem 5	________	/ 25

Exam 1 Total Score: _______ / 100

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1. (10 points) For the following signal, determine if the signal is periodic and if it is perioidic determine its fundamental period.

$$x(t) = \cos\left(\frac{\pi}{2}t\right) + e^{j(\frac{\pi}{5}t+\sqrt{2})}$$

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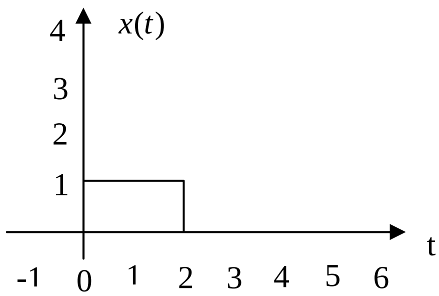
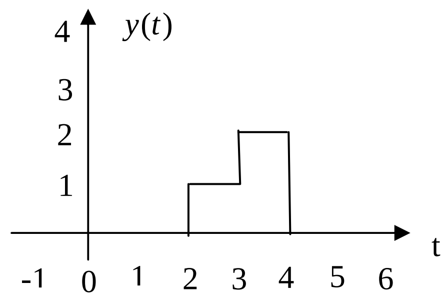
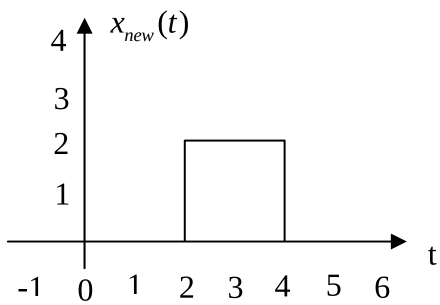
2. (5 points) Assume we know a system is a linear time invariant (LTI) system. We also know the following input $x(t)$ – output $y(t)$ pair:



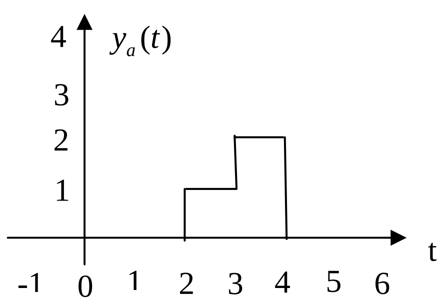


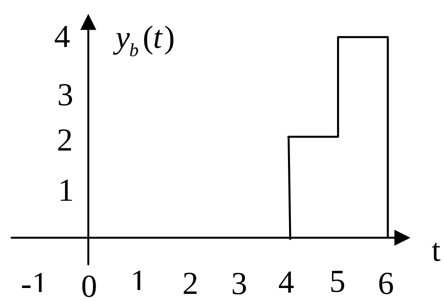
If the input to the system is now $x_{new}(t)$

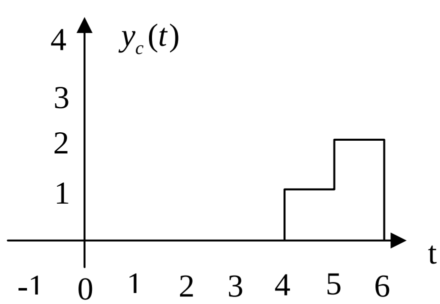


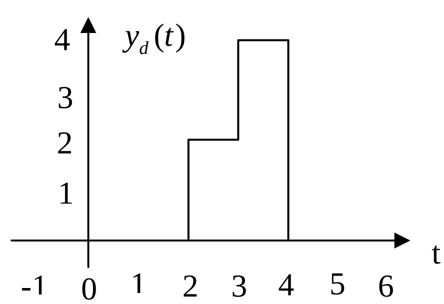
Which of the following best represents the output of the system?

a) $y_a(t)$ b) $y_b(t)$ c) $y_c(t)$ d) $y_d(t)$









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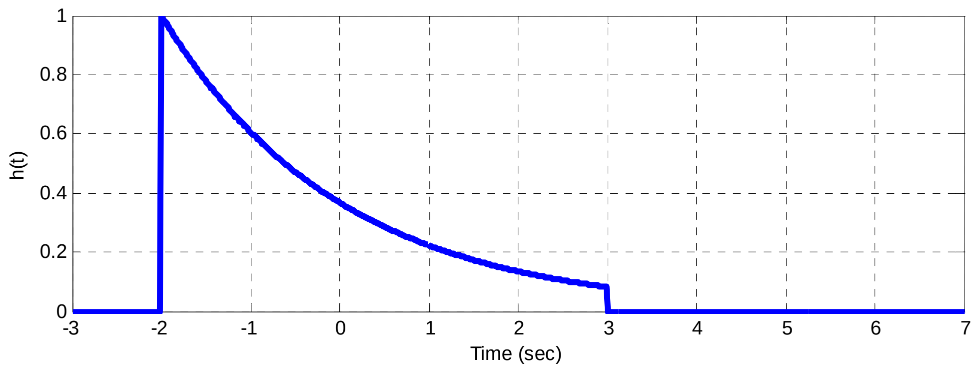
3. Graphical Convolution (30 points)

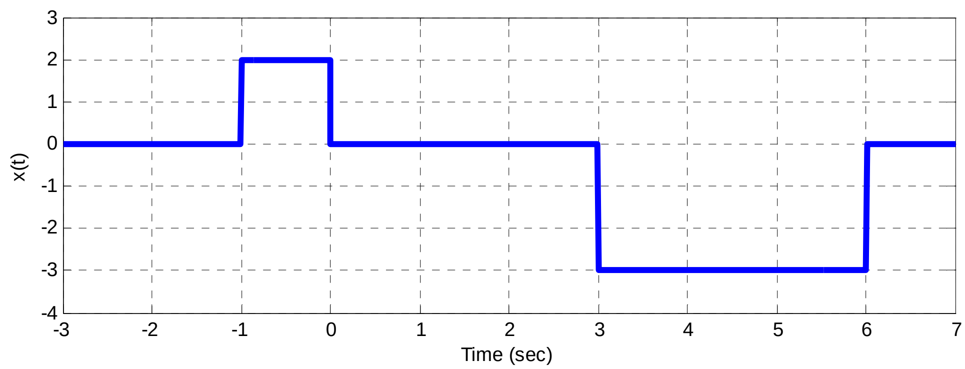
Consider a noncausal linear time invariant system with impulse response given by

$$h(t) = e^{-0.5(t+2)}[u(t+2) - u(t-3)]$$

The input to the system is given by

$$x(t) = 2[u(t+1) - u(t)] - 3[u(t-3) - u(t-6)]$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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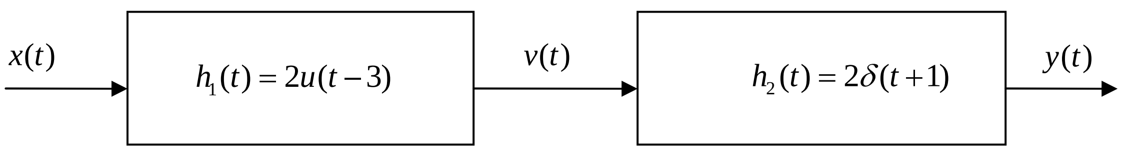
4. Impulse Response (30 points)
For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions. For full credit, simplify your answers as much as possible.*

a) $y(t) = \int_{-\infty}^{t-2} e^{-(t-\lambda)} x(\lambda - 2) d\lambda + e^{-t} x(t)$

b) $2\dot{y}(t) + y(t) = x(t-1)$

c) Determine the impulse response for the following system



d) If the response of a system to a step of amplitude A is given by

$$s(t) = A[1 + e^{-t/\tau}]u(t)$$

determine the **unit** impulse response of the system. (*Do not just guess the answer, you will probably be wrong, and besides, you need to show your work!)*

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5. System Properties (25 points)
a) Fill in the following table with a Y (Yes) or N (No). Only your responses in the table will be graded, not any work. Assume $x(t)$ is the system input and $y(t)$ is the system output. Also assume we are looking at all times (positive and negative times).

System	Linear ?	Time-Invariant?	Memoryless?	Causal?
$\dot{y}(t)+y(t)=e^{(t+1)}x(t)$				
$y(t)=x\left(-\frac{t}{2}\right)$				
$y(t)=x(t-1)-1$				
$y(t)=x^2(t)$				

b) For the system described below, use a formal technique such as we used in class (and on the homework) to determine if the system is time invatriant. *You will be graded more on your method of arriving at an answer than the answer itself!*

$$y(t)=\int_{-\infty}^{t} e^{-(t+\lambda)}x(\lambda)d\lambda$$

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Some Potentially Useful Relationships

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\mathrm{rect}\left(\frac{t-t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$