Name Solutions CM

ECE 300
Signals and Systems

Exam 3
10 November, 2009

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam. ***<u>You must show your work to receive credit for a problem.</u>***

Problem 1	________ / 35
Problem 2	________/30
Problem 3	________/ 20
Problem 4	________/ 15

Exam 3 Total Score: _______ / 100

90-100 3
80-89 13
70-79 6
60-69 8
<60 8
median = 71

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1. (35 points) Assume $x(t) = 4\,\text{sinc}\left[\frac{1}{\pi}(t-2)\right]\cos(4(t-2))$ is the input to an LTI system with transfer function

$$H(\omega) = \begin{cases} \frac{1}{\pi}e^{-j\omega 3} & |\omega| > 4 \\ 0 & else \end{cases}$$

a) Determine the Fourier transform $X(\omega)$ of $x(t)$

b) Accurately sketch the magnitude and phase of $X(\omega)$

c) Determine the energy in $x(t)$

d) Sketch the magnitude and phase of $Y(\omega)$

e) Determine the system output $y(t)$

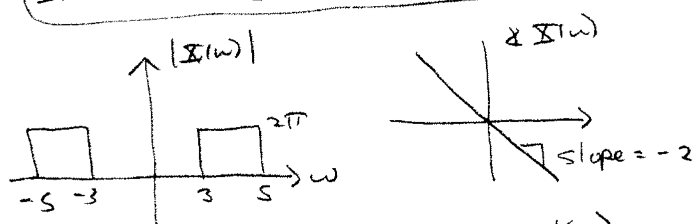
a) for $x_1(t) = 4\,\text{sinc}\left(\frac{t}{\pi}\right) \longleftrightarrow X_1(\omega) = 4\pi\,\text{rect}\left(\frac{\omega}{2}\right)$

$W = 1/\pi$

for $x_2(t) = x_1(t)\cos(4t) \longleftrightarrow X_2(\omega) = 2\pi\,\text{rect}\left(\frac{\omega+4}{2}\right) + 2\pi\,\text{rect}\left(\frac{\omega-4}{2}\right)$

$$\boxed{X(\omega) = 2\pi\left[\text{rect}\left(\frac{\omega+4}{2}\right) + \text{rect}\left(\frac{\omega-4}{2}\right)\right]e^{-j2\omega}}$$

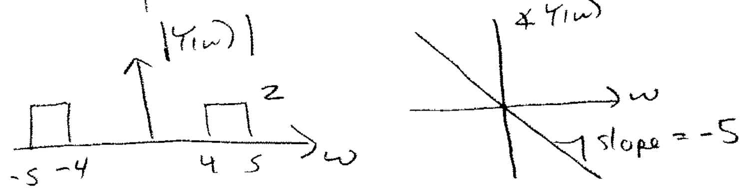
b)



c) $E = 2\left[\frac{1}{2\pi}\int_3^5 (2\pi)^2 d\omega\right]$

$= 2\,\frac{1}{2\pi}(2\pi)^2\cdot 2 = \boxed{8\pi = E}$

d)



e) $Y(\omega) = \left[2\,\text{rect}(\omega+4.5) + 2\,\text{rect}(\omega-4.5)\right]e^{-j5\omega}$

$W\,\text{sinc}(Wt) \Longleftrightarrow \text{rect}\left(\frac{\omega}{2\pi W}\right)$ $W = 1/2\pi$ so $\frac{1}{2\pi}\text{sinc}\left(\frac{t}{2\pi}\right) \Longleftrightarrow \text{rect}(\omega)$

$\frac{1}{2\pi}\text{sinc}\left(\frac{t}{2\pi}\right)\cos(4.5t) \Longleftrightarrow \frac{1}{2}\text{rect}(\omega+4.5) + \frac{1}{2}\text{rect}(\omega-4.5)$

$\frac{2}{\pi}\text{sinc}\left(\frac{t}{2\pi}\right)\cos(4.5t) \Longleftrightarrow 2\,\text{rect}(\omega+4.5) + 2\,\text{rect}(\omega-4.5)$

$$\boxed{Y(\omega) = \frac{2}{\pi}\text{sinc}\left(\frac{t-5}{2\pi}\right)\cos\left(4.5(t-5)\right)}$$

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2. (30 points) Fill in the following table, show all your work

	$x(t)$	$X(\omega)$
(a)		$\left[rect\left(\frac{\omega+2}{3}\right)+rect\left(\frac{\omega-2}{3}\right)\right]e^{j\omega 3}$
(b)		$\frac{3}{2+j\left(\frac{\omega}{5}+2\right)}$
(c)	$\frac{4}{4+(2t-4)^2}$	

(a) $rect\left(\frac{\omega}{2\pi W}\right) \Longleftrightarrow W\,rect(Wt) \qquad 2\pi W = 3 \qquad W = \frac{3}{2\pi}$

for $x_1(t) = \frac{3}{2\pi} rect\left(\frac{3}{2\pi}t\right) \Longleftrightarrow X_1(\omega) = rect\left(\frac{\omega}{3}\right)$

for $x_2(t) = x_1(t)\cos(2t) \Longleftrightarrow X_2(\omega) = \frac{1}{2} rect\left(\frac{\omega+2}{3}\right) + \frac{1}{2} rect\left(\frac{\omega-2}{3}\right)$

$\boxed{x(t) = \frac{3}{2\pi} rect\left(\frac{3}{2\pi}(t+3)\right)\cos(2(t+3))}$

(b) for $x_1(t) = 3e^{-2t}u(t) \Longleftrightarrow X_1(\omega) = \frac{3}{2+j\omega}$

for $x_2(t) = x_1(t)e^{-j2t} \Longleftrightarrow X_2(\omega) = X_1(\omega+2) = \frac{3}{2+j(\omega+2)}$

for $X_3(\omega) = X_2\left(\frac{\omega}{5}\right) = \frac{3}{2+j\left(\frac{\omega}{5}+2\right)} \Longleftrightarrow x_3(t) = 5x_2(5t) = \boxed{15e^{-10t}e^{-j10t}u(t) = x(t)}$

(c) for $x_1(t) = e^{-2|t|} \Longleftrightarrow X_1(\omega) = \frac{4}{4+\omega^2}$

for $x_2(t) = X_1(t) = \frac{4}{4+t^2} \Longleftrightarrow X_2(\omega) = 2\pi x_1(-\omega) = 2\pi e^{-2|\omega|}$

for $x_3(t) = x_2(2t) = \frac{4}{4+(2t)^2} \Longleftrightarrow X_3(\omega) = \frac{1}{2}X_2\left(\frac{\omega}{2}\right) = \pi e^{-|\omega|}$

for $x_4(t) = x_3(t-2) = \frac{4}{4+(2(t-2))^2} \Longleftrightarrow X_4(\omega) = X_3(\omega)e^{-j2\omega} = \boxed{\pi e^{-|\omega|}e^{-j2\omega} = X(\omega)}$

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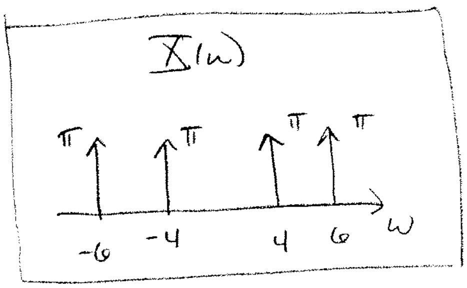
3. (20 points) Consider the signal $x(t) = \cos(4t) + \cos(6t)$

a) Sketch the spectrum of $X(\omega)$

b) $x(t)$ is the input to an ideal sampler sampling at rate $f_s = \frac{5}{2\pi} = \frac{1}{T}$ seconds. Sketch the spectrum of the sampled signal $X_s(\omega)$.

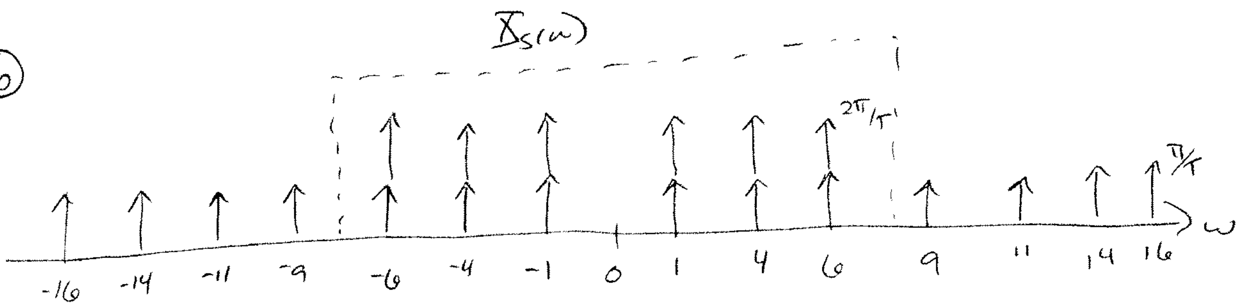
c) Assume $x_s(t)$ is the input to an ideal lowpass filter with a cutoff frequency of 7 rad/sec and passband gain of T. Determine the output signal $x_r(t)$ and write it in terms of the original signal $x(t)$ plus any aliased terms.

a



$$\omega_0 = \frac{2\pi}{T} = 2\pi\left(\frac{5}{2\pi}\right) = \boxed{5 \text{ rad/sec} = \omega_0}$$

b



c $x_r(t) = 2\cos(6t) + 2\cos(4t) + 2\cos(t)$

$$\boxed{x_r(t) = x(t) + \underbrace{\left[2\cos(t) + \cos(4t) + \cos(6t)\right]}_{\text{aliased term}}}$$

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4) (15 points) The periodic signal $x(t)$ has the Fourier series representation

$$x(t) = 2 + \sum_{k=-\infty}^{k=\infty} \frac{1}{1+kj} e^{jk3t}$$

$x(t)$ is the input to an LTI system (a high pass filter filter) with the transfer function

$$H(j\omega) = \begin{cases} 0 & |\omega| < 2 \\ 4e^{-j2\omega} & |\omega| > 2 \end{cases}$$

The steady state output of the system can be written as

$$y(t) = ax(t-b) + c + d\cos(e(t-b) + f).$$

Determine the output, writing it in as simple a form (like that above) as you can.

only DC term passes through filter

$C_0^x = 3$

$y(t) = 4(x(t-2) - 3) = \boxed{4x(t-2) - 12 = y(t)}$