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ECE 300
Signals and Systems

Exam 2
26 October, 2009

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam. ***You must show your work to receive credit for a problem.***

Problem 1	________ / 30
Problem 2	________ /25
Problem 3	________ / 20
Problem 4	________ / 10
Problem 5	________ / 15
Exam 2 Total Score:	_______ / 100

90-100 4
80-89 8
70-79 8
60-69 8
<60 11

median = 70

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1. Impulse Response (30 points)
For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions. For full credit, simplify your answers as much as possible.*

$t-2>2 \quad t>4$

a) $y(t) = \int_{-\infty}^{t-2} e^{-(t-\lambda)}x(\lambda-2)d\lambda + e^{-t}x(t)$

$h(t) = e^{-(t-2)}u(t-4) + \cancel{e^{-t}}\delta(t)$

$\boxed{h(t) = e^{-(t-2)}u(t-4) + \delta(t)}$

b) $2\dot{y}(t) + y(t) = x(t-1)$

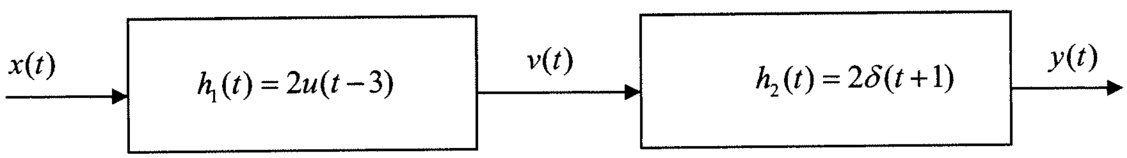
$\dot{h} + \frac{1}{2}h = \frac{1}{2}\delta(t-1)$

$\frac{d}{dt}\left(h e^{t/2}\right) = \frac{1}{2}e^{t/2}\delta(t-1) = \frac{1}{2}e^{1/2}\delta(t-1)$

$h(t)e^{t/2} = \frac{1}{2}e^{1/2}\int_{-\infty}^{t}\delta(\lambda-1)d\lambda = \frac{1}{2}e^{1/2}u(t-1)$

$\boxed{h(t) = \frac{1}{2}e^{1/2}e^{-t/2}u(t-1)}$

c) Determine the impulse response for the following system



$h(t) = h_1(t)*h_2(t) = \int_{-\infty}^{\infty} 2u(\lambda-3)\,2\delta(t-\lambda+1)d\lambda$

$= \boxed{4u(t-2) = h(t)}$

d) If the response of a system to a step of amplitude A is given by

$$s(t) = A[1+e^{-t/\tau}]u(t)$$

determine the **unit** impulse response of the system. (*Do not just guess the answer, you will probably be wrong, and besides, you need to show your work!*)

unit $\Rightarrow A=1$

$h(t) = \frac{d}{dt}s(t) = \left(\frac{d}{dt}[1+e^{-t/\tau}]\right)u(t) + [1+e^{-t/\tau}]\left(\frac{d}{dt}u(t)\right)$

$= -\frac{1}{\tau}e^{-t/\tau}u(t) + [1+e^{-t/\tau}]\delta(t)$

$\boxed{h(t) = -\frac{1}{\tau}e^{-t/\tau}u(t) + 2\delta(t)}$

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2. Fourier Series (25 points)

The periodic function $x(t)$ is defined over one period ($T_0 = 5$ seconds) as

$$x(t) = \begin{cases} 2 & -2 \le t \le 1 \\ 0 & 1 \le t \le 3 \end{cases}$$

Determine the complex Fourier series coefficients, c_k by evaluating the appropriate integral.

Be sure to simplify your answer as much as possible and use a <u>*sinc*</u> *function if appropriate.*

$\omega_0 = \frac{2\pi}{5}$

$$c_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t} dt = \frac{1}{T_0}\int_{-2}^{1} 2e^{-jk\omega_0 t} dt = \frac{2}{T_0} \left.\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right|_{-2}^{1}$$

$$= \frac{2}{\underbrace{\omega_0 T_0}_{2\pi}(-jk)}\left[e^{-jk\omega_0} - e^{jk\omega_0 2}\right] = \frac{2}{k\pi}\left[\frac{e^{jk\omega_0 2} - e^{-jk\omega_0}}{2j}\right]$$

$$= \frac{2}{k\pi} e^{jk\frac{\omega_0}{2}}\left[\frac{e^{jk\omega_0 3/2} - e^{-jk\omega_0 3/2}}{2j}\right] = \frac{2}{k\pi} e^{jk\frac{\omega_0}{2}} \sin\left(k\omega_0 3/2\right)$$

$$= \frac{2}{k\pi} e^{jk\pi/5} \sin\left(k\frac{3\pi}{5}\right) = 2e^{jk\pi/5} \frac{\sin\left(\pi \cdot \frac{3k}{5}\right)}{\pi \cdot k \frac{3}{3}\frac{5}{5}}$$

$$= 2e^{jk\pi/5} \frac{\text{sinc}\left(\frac{3k}{5}\right)}{5/3} = \boxed{\frac{6}{5} e^{jk\pi/5}\, \text{sinc}\left(\frac{3k}{5}\right) = c_k}$$

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3. (20 points) A periodic signal has the Fourier series representation $x(t) = \sum_{k=-\infty}^{k=\infty} c_k^x e^{jk2t}$.

This signal is the input to an LTI system, and the (steady state) output of the system is

$$y(t) = 4 + 4\cos(4t + 30^o) + 6\cos(6t + 30^o)$$

Fill in the following table:

k	$\|c_k^x\|$	$\|H(jk\omega_0)\|$	$\measuredangle c_k^x$	$\measuredangle H(jk\omega_0)$
0	2	2	180^o	180^0
1	3	0	-45^o	
2	1	2	45^o	-15^0
3	0.5	6	-30^o	60^0

$\omega_0 = 2$

If you cannot determine a necessary value, leave the table entry blank.

$c_0^y = 4 = c_0^x H(0) = (-2)(-2) \qquad H(0) = 2 \measuredangle 180^0$

$c_1^y = 0 = c_1^x H(j2) \Rightarrow |H(j2)| = 0$

$c_2^y = 2 \measuredangle 30^0 = c_2^x H(j4) = (1 \measuredangle 45^0)(2 \measuredangle -15^0)$

$c_3^y = 3 \measuredangle 30^0 = c_3^x H(j6) = (0.5 \measuredangle -30^0)(6 \measuredangle 60^0)$

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4. (10 points) Assuming the system input $x(t) = \sum_{k=-\infty}^{k=\infty} c_k^x e^{jk\omega_0 t}$ and output $y(t) = \sum_{k=-\infty}^{k=\infty} c_k^y e^{jk\omega_0 t}$ are related through the LTI system $\dot{y}(t) + 2y(t-2) = 6x(t-3)$

a) Determine the relationship between c_k^x and c_k^y.

b) Determine the *continuous* transfer function $H(j\omega)$ between the input and the output.

(a) $\dot{y}(t) + 2y(t-2) = 6x(t-3)$

$$\sum c_k^y \left[jk\omega_0 e^{jk\omega_0 t} + 2e^{jk\omega_0(t-2)}\right] = \sum c_k^x 6 e^{jk\omega_0(t-3)}$$

$$\sum c_k^y \left[jk\omega_0 + 2e^{-jk\omega_0 2}\right] e^{jk\omega_0 t} = \sum c_k^x \left[6 e^{-jk\omega_0 3}\right] e^{jk\omega_0 t}$$

$$\boxed{c_k^y = c_k^x \frac{6e^{-jk\omega_0 3}}{jk\omega_0 + 2e^{-jk\omega_0 2}}}$$

(b) $H(jk\omega_0) = \dfrac{6e^{-jk\omega_0 3}}{jk\omega_0 + 2e^{-jk\omega_0 2}}$

so $\boxed{H(j\omega) = \dfrac{6e^{-j3\omega}}{j\omega + 2e^{-j\omega 2}}}$

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5) (15 points) The periodic signal $x(t)$ has the Fourier series representation

$$x(t) = 2 + \sum_{k=-\infty}^{k=\infty} \frac{1}{1+kj} e^{jk3t}$$

$x(t)$ is the input to an LTI system (a high pass filter filter) with the transfer function

$$H(j\omega) = \begin{cases} 0 & |\omega| < 5 \\ 3e^{-j2\omega} & |\omega| > 5 \end{cases}$$

The steady state output of the system can be written as

$$y(t) = ax(t-b) + c + d\cos(e(t-b) + f).$$

Determine numerical values for the parameters a, b, d, e and f

$C_0^x = 3 \quad C_1^x = \frac{1}{1+j} = \frac{1}{\sqrt{2}} \measuredangle -45°$

The filter removes $K>0$ and $K>1$ terms

$y_1(t) = x(t) - 3 - \frac{2}{\sqrt{2}}\cos(3t - 45°)$

then $y(t) = 3y_1(t-2)$

$\boxed{y(t) = 3x(t-2) - 9 - \frac{6}{\sqrt{2}}\cos(3(t-2) - 45°)}$