ECE 300 Signals and Systems   Fall 2009

Filter Design and Measurement
Lab 08
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Objectives

In this lab we will examine filters in different ways. We will first investigate three different kinds of lowpass filters and the properties they possess. These filters illustrate the types of trade-offs we must make when designing a system. Finally we will determine the order of Butterworth filter we need to use to achieve a desired rejection.

Background

A periodic signal can be represented by the complex exponential form of the Fourier series. When a periodic signal is applied to the input of a filter, each of the harmonic components of the input signal experiences an amplitude and phase change caused by the filter. At the filter output the harmonic components add together to produce the output waveform. The amplitude and phase changes experienced by each of the input components combine to make the output signal different from the input signal in a predictable way. This difference between the input and output waveform is called distortion.

Stated mathematically, suppose $x(t)$ is a periodic input signal with period T_0, H(ω) is the frequency response of the filter (i.e. the Fourier transform of its input response h(t)), and $y(t)$ is the filter output. Then the input $x(t)$ can be written

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \text{ where } \omega_0 = 2\pi/T_0.$$

As the input signal passes through the filter, the input coefficients a_k become altered by the filter to become the output coefficients b_k, where

$$b_k = H(k\omega_0) a_k.$$

The output $y(t)$ is then given by

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} H(k\omega_0) a_k e^{jk\omega_0 t}$$

In practice there is not usually an infinite number of components to be added to produce either $x(t)$ or $y(t)$; only components with significant amplitudes need to be included.

Part 1: Filtering Periodic Signals

This is a continuation of the work you did in **Lab 6.** Now we are going to concentrate a bit more on the filters and examine some trade-offs between different common types of lowpass filters. Much of what follows is the same as for **Lab 6**, but there are a few changes at the end when we use different types of filters.

a) Use your code from **Lab 6** to determine the Complex Fourier series representation using 20 terms of the following periodic function (defined over one period).

$$x(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

b) For the majority of the filters it uses, Matlab assumes filter has the form

$$H(s) = \frac{b_N s^N + b_{N-1} s^{N-1} + \cdots + b_2 s^2 + b_1 s + b_0}{a_N s^N + a_{N-1} s^{N-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

Hence, in order to represent any filter, Matlab just uses an array for the b coefficients and an array for the a coefficients. For example, we might have two variables (arrays) B and A to store the coefficients. These variables would be

$$B = \begin{bmatrix} b_N & ... & b_1 & b_0 \end{bmatrix}$$
$$A = \begin{bmatrix} a_N & ... & a_1 & a_0 \end{bmatrix}$$

Let's assume we want to use a 10th order Butterworth lowpass filter with a frequency cutoff of $20\,\omega_0$. Use the help command to look up the Matlab function **butter.** You should return the coefficients in two arrays, i.e. your command should be [B,A] = butter(…..); Note that we are constructing an **analog** filter here, so read **all** of the description for the **butter** command.

c) We now again need to find the variable H0 = $H(0)$ and the variable (array) $H = \begin{bmatrix} H(j\omega_0) & H(j2\omega_0) & ... & H(jN\omega_0) \end{bmatrix}$. To find $H(0)$ we use the fact that

$$H(0) = \frac{b_0}{a_0}$$

If the b and a coefficients are stored in arrays B and A, we can write

H0 = B(end)/A(end);

Where **end** tells Matlab to get the last element of the array. The command **freqs** will be helpful for finding the variable (array) H.

d) Plot the Fourier series representation for both the input signal $x(t)$ and the output signal $y(t)$ on the same graph for N=25 terms, *using different line types and a legend.* The title of your graph should indicate that you are using a Butterworth filter. If you have done everything correctly your graph should look like that in Figure 1. *Print out this graph and attach it to the worksheet at the end of this lab.*
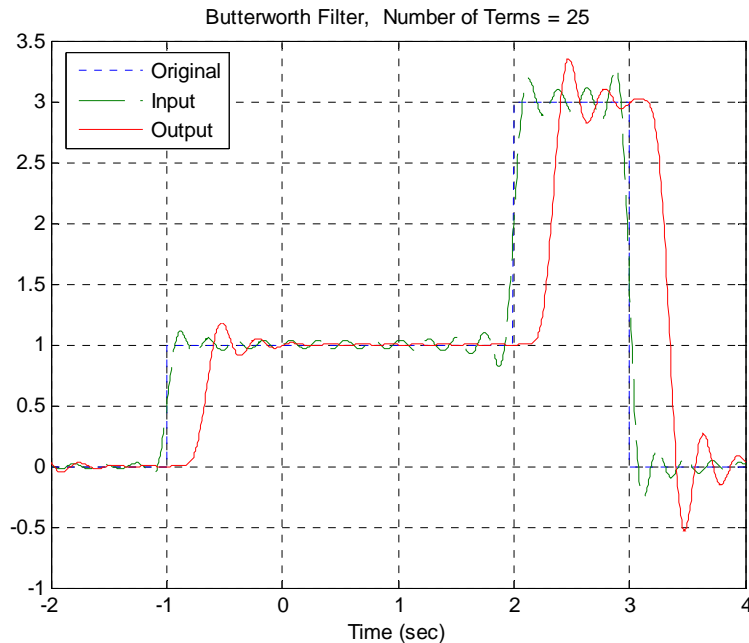

Figure 1. Input and Fourier series representation of the input and output using a 10th order Butterworth filter.

It will be useful to look at a frequency response plot of the filter you are using. You will need to make three plots on one page using the **subplot** command. Type **orient tall** before any of the plotting so you can use more of the page.

e) The first plot is the magnitude of the transfer function as a function of frequency. You will need to use the **abs** command. You should also put a **grid** on your figure.

f) The second plot is the phase of the transfer function (in degrees) as a function of frequency. In order to see the phenomena we want to see use the sequence of commands **unwrap(angle(H))*180/pi**. The most important command here is **unwrap**, which allows angles of more than 180 degrees

g) The third plot is the magnitude of the transfer function (in dB) as a function of frequency. Here you need to use the commands **semilogx** and **log10**.

Be sure to include the type of filter you are using in your title. If you have done everything correctly, you should get a plot like that shown in Figure 2. *Print out this graph and attach it to the worksheet at the end of the lab.*
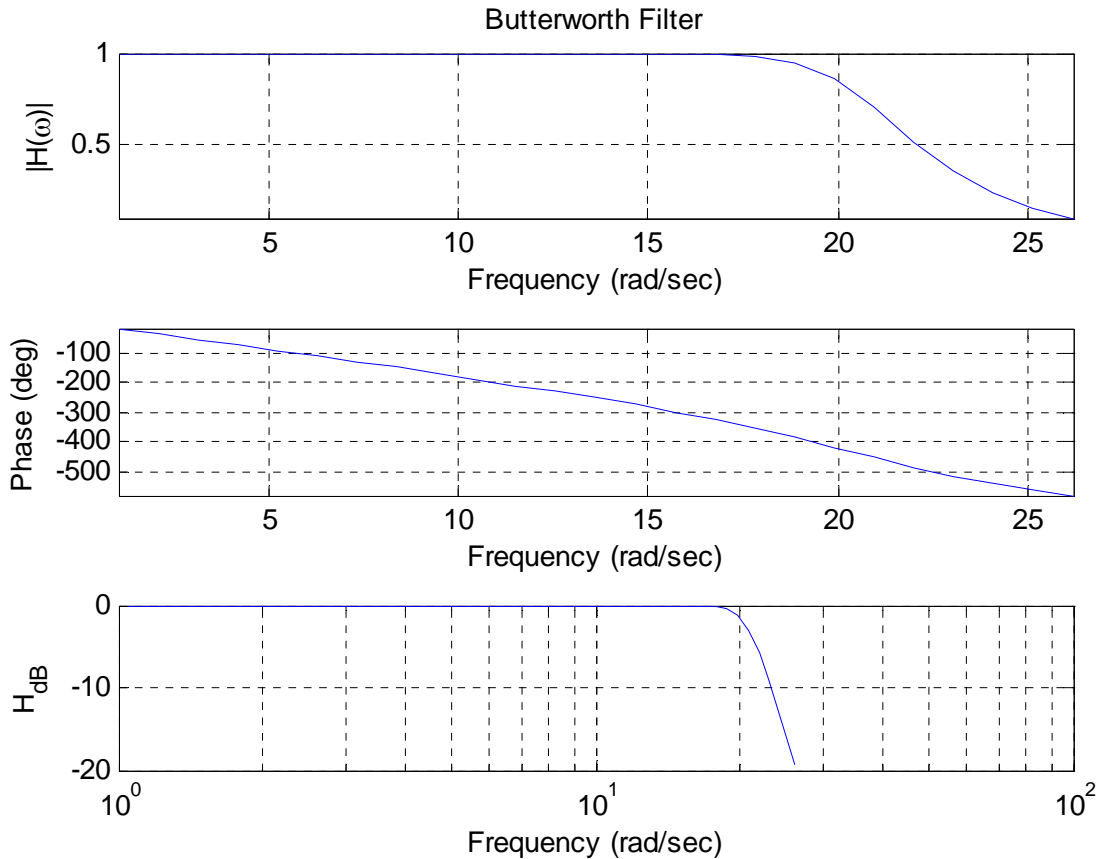
Figure 2. Frequency response of the 10th order Butterworth filter.

h) If the phase of the filter is exactly linear then we should have the relationship

$$-slope\ of\ filter\ (in\ \sec) = time\ delay$$

Using a straight edge (or ruler), draw a line on your graph between the initial point on the phase graph and the value of the phase at about $20\,\omega_0$. Estimate the delay between the input and output signal (use Matlab to zoom in on the signal and measure this accurately, don't just eyeball it!) and compare this to the predicted delay. Fill in the table on the worksheet.

i) Repeats parts **b-h** using a Bessel filter (use the command **besself**). *__Note that for this filter the frequency you specify can be considered the maximum frequency at which the phase will be linear.__* Determine, by trial and error, the frequency you need to enter in order for the cutoff frequency to be at approximately $20\,\omega_o$. For this use multiples of ω_o, such as $30\,\omega_o$, $40\,\omega_o$, etc. *Be sure to print out both graphs.*

j) Repeat parts **b-h** using a Cheyshev filter (use the command **cheby1** and the defaults if you don't know what values to use). *Be sure to print out both graphs.*

k) In the Chebyshev filter, try varying the R parameter from 0.5 (the default) to 2.0. What happens to the filter?

Part 2: Determining the Order of a Butterworth Filter

A Butterworth filter has the property that it is maximally flat in the *passband*. An nth order Butterworth filter has the magnitude squared response

$$|H(\omega)|^2 = \frac{1}{1+\left(\dfrac{\omega}{\omega_p}\right)^{2n}}$$

where ω_p is the *passband frequency*. At this frequency the power has been reduced by one half or 3 dB,

$$|H(\omega_p)|^2 = \frac{1}{1+\left(\dfrac{\omega_p}{\omega_p}\right)^{2n}} = \frac{1}{2} \quad \text{or} \quad 10\log_{10}|H(\omega_p)|^2 = 10\log_{10}\left(\frac{1}{2}\right) = -3dB$$

To determine the required order of a filter we often look at the desired *stopband frequency*, ω_s. Usually we want to indicate the minimum required power difference between the passband and the stopband, Δ. Δ is the *rejection.*Hence we have

$$\Delta = 20\log_{10}|H(0)| - 20\log_{10}|H(\omega_s)|$$

or

$$\Delta = -10\log_{10}\frac{1}{1+\left(\dfrac{\omega_s}{\omega_p}\right)^{2n}}$$

The ratio $\dfrac{\omega_s}{\omega_p}$ is called the *transition ratio.*

a) Show (on the back of the lab write-up) that we can write

$$n = \frac{\ln\left(10^{\frac{\Delta}{10}} - 1\right)}{2\ln\left(\dfrac{\omega_s}{\omega_p}\right)}$$

Note that <u>*n must be an integer*</u>, so we always round up (to the next larger integer).

b) For $\omega_p = 10$ rad/sec, $\omega_s = 20$ rad/sec, and $\Delta = 18$ dB, determine the minimum required Butterworth filter order for this filter. (Remember it must be an integer). Using Table 1, plot the Bode plot of your

Butterworth filter and verify that all frequencies $\omega > \omega_s$ have magnitude (power) less than Δ_{max} .Matlab's **tf** command and **Bode** commands will be really useful here. Note that you can click on the curve on the Bode plot to read it more accurately. Turn in your plot.

c) For $\omega_p = 15$ rad/sec, $\omega_s = 35$ rad/sec, and $\Delta = 28\,\text{dB}$, determine the required Butterworth filter order for this filter. (Remember it must be an integer). Using Table 1, plot the Bode plot of your Butterworth filter and verify that all frequencies $\omega > \omega_s$ have magnitude (power) less than Δ_{max} . Matlab's **tf** command and **Bode** commands will be really useful here. Turn in your plot.

n	denominator(s)
1	$\dfrac{s}{\omega_p}+1$
2	$\left(\dfrac{s}{\omega_p}\right)^2 +1.414\left(\dfrac{s}{\omega_p}\right)+1$
3	$\left(\dfrac{s}{\omega_p}\right)^3 +2\left(\dfrac{s}{\omega_p}\right)^2 +2\left(\dfrac{s}{\omega_p}\right)+1$
4	$\left(\dfrac{s}{\omega_p}\right)^4 +2.6131\left(\dfrac{s}{\omega_p}\right)^3 +3.4142\left(\dfrac{s}{\omega_p}\right)^2 +2.6131\left(\dfrac{s}{\omega_p}\right)+1$
5	$\left(\dfrac{s}{\omega_p}\right)^5 +3.2361\left(\dfrac{s}{\omega_p}\right)^4 +5.2361\left(\dfrac{s}{\omega_p}\right)^3 +5.2361\left(\dfrac{s}{\omega_p}\right)^2 +3.2361\left(\dfrac{s}{\omega_p}\right)+1$

Table 1: Denominators of Buterworth filter for filter orders 1-5. The numerator for the Butterworth filter is 1.

Lab 08
Instructor Verification Sheet

Name_____Date:_____

At the end of this lab you should have attached the following plots:

- *Two plots for the Butterworth filter (input/output and transfer function characteristics)*
- *Two plots for the Bessel filter*
- *Two plots for the Chebyshev filter*
- *One plot for part 2-b (Butterworth Order)*
- *One plot for part 2-c (Butterworth Order)*

Part1: Measurements of Different Filters

Filter Type	Predicted Delay (-slope of filter)	Measured Delay (time domain)
10th order Butterworth		
10th order Bessel		
10th order Chebyshev		

Question 1: From Part 1, which filter type has the most linear phase over the passband? This filter will have the smallest *phase distortion*.

Question 2: From Part 1, which filter has the flattest magnitude response over the passband? This filter will have the smallest *magnitude distortion*.

Question 3: From Part 1, which filter has the steepest rolloff after the cutoff frequency ?

Question 4: From Part 1, what is the effect of varying R in the Chebyshev filter?

Part 2-a)