

**ECE 300**  
**Signals and Systems**  
 Homework 3

**Due Date:** Tuesday September 23, 2008 *at the beginning of class*

**Reading** Roberts pages 54-58, 115-140

**Problems**

1) Consider the following mathematical models of systems:

**a**  $y(t) = e^{-(t+1)}x(t)$    
 **b**  $y(t) = \begin{cases} x(t) & |x(t)| \leq 10 \\ 10 & |x(t)| > 10 \end{cases}$    
 **c**  $y(t) = x\left(1 - \frac{t}{2}\right)$   
**d**  $y(t) = x\left(\frac{t}{3}\right) + 2$    
 **e**  $y(t) = \sin(x(t))$    
 **f**  $y(t) = 1 - e^{-x(t)}$   
**g**  $\frac{dy(t)}{dt} = y(t)x(t)$    
**h**  $\frac{dy(t)}{dt} = 3y(t) + 2x(t)$    
**i**  $\frac{dy(t)}{dt} = 3ty(t) + 2x(t)$   
**j**  $y(t) = \int_{-\infty}^t (t - \lambda)x(\lambda + 1)d\lambda$

Fill in the following table (Y or N for each question) for each system. You must justify your answers to receive credit. Assume  $t$  can be any possible value.

Part	Causal?	Memoryless?	Linear?	Time Invariant?
a				
b				
c				
d				
e- $x(t)$ large				
e- $x(t)$ small				
f- $x(t)$ large				
f- $x(t)$ small				
g				
h				
i				
j				

For parts **e** and **f**, look at the case when  $x(t)$  is large, and then when  $x(t)$  is assumed to be sufficiently small that you can use a small value (Taylor series)

approximation. Even though  $x(t)$  is a function, you can still use the same approximations you would use if it was just a small number.

For part **g**, you should show  $y(t) = y(t_0)e^{\int_{t_0}^t x(\lambda)d\lambda}$  in order to determine whether the system is or is not causal and has memory or is memoryless.

For part **h** you should show  $y(t) = y(t_0)e^{3(t-t_0)} + \int_{t_0}^t 2e^{3(t-\lambda)}x(\lambda)d\lambda$  in order to determine the system is or is not causal and has memory or is memoryless.

For part **i** you should solve the DE first (see handout about integrating factors) and then determine whether the system is or is not causal and has memory or is memoryless. Your solution to the DE will have a similar form to that given for part **g**.

**2)** For the system  $y(t) = e^t \int_c^t e^{-\lambda} x(\lambda) d\lambda$ , determine the value of “c” that will make the system time-invariant. Use a formal technique such as you used in problem 1 to justify your answer.

**3) (Matlab Problem)** The **average value** of a function  $x(t)$  is defined as

$$\bar{x} = \frac{1}{b-a} \int_a^b x(t) dt$$

and the **root-mean-square (rms)** value of a function is defined as

$$x_{rms} = \sqrt{\frac{1}{b-a} \int_a^b x^2(t) dt}$$

Read the **Appendix** (located at the end of this homework), then

**a)** use Matlab to find the average and rms values of the function  $x(t) = t^2$  for  $-1 < t < 1$

**b)** use Matlab to find the average and rms values of the following functions

$$\begin{aligned}
 x(t) &= \cos(t) & 0 < t < \pi \\
 x(t) &= \cos(t) & 0 < t < 2\pi \\
 x(t) &= |t| & -1 < t < 1 \\
 x(t) &= t \cos(t) & -2 < t < 4
 \end{aligned}$$

Hint: You will probably find the **sqrt** function useful. You should write a Matlab m-file for this problem, and turn it in with your homework, as well as the answers.

## Appendix

Maple is often used for symbolically integrating a function. Sometimes, though, what we really care about is the numerical value of the integral. Rather than integrating symbolically, we might want to just use numerical integration to evaluate the integral. Since we are going to be using Matlab a great deal in this course, in this appendix we will learn to use one of Matlab's built-in functions for numerical integration. In order to efficiently use this function, we will learn how to construct what are called *anonymous* functions. We will then use this information to determine the average and rms value of a function. Some of this is going to seem a bit strange at first, so just try and learn from the examples.

**Numerical Integration in Matlab** Let's assume we want to numerically integrate the following:

$$I = \int_0^{2\pi} (t^2 + 2)dt$$

In order to do numerical integration in Matlab, we will use the built-in command **quadl**. The **arguments** to quadl, e.g., the information passed to quadl, are

- A function which represents the integrand (the function which is being integrated). Let's call the integrand  $x(t)$ . This function must be written in such a way that it returns the value of  $x(t)$  at each time  $t$ . Clearly here  $x(t) = t^2 + 2$
- The lower limit of integration, here that would be 0
- The upper limit of integration, here that would be  $2\pi$

Note that an optional fourth argument is the tolerance, which defaults to  $10^{-6}$ . When the function value is very small, or the integration time is very small, you will have to change this.

**Anonymous Functions** Let's assume we wanted to use Matlab to construct the function  $x(t) = t^2 + 2$ . We can do this by creating what Matlab calls an **anonymous function**. To do this, we type into Matlab

```
x = @(t) t.*t+2;
```

If we want the value of  $x(t)$  at  $t = 2$ , we just type `x(2)`

Hence, to evaluate the integral  $I = \int_0^{2\pi} (t^2 + 2)dt$  in Matlab we would type

```
x = @(t) t.*t+2;  
I = quadl(x,0,2*pi)
```

Note that it is important to define  $x$  **before** it is used by (passed to) `quadl`

**Example 1** To numerically evaluate  $I = \int_{-1}^1 e^{-t} \cos(2t)dt$  we could type

```
x = @(t) exp(-t).*cos(2*t);  
I = quadl(x,-1,1);
```

**Example 2** To numerically evaluate  $I = \int_{-2}^1 |t| e^{-|t|}dt$  we could type

```
y = @(t) abs(t).*exp(-abs(t));  
I = quadl(y,-2,1);
```

**Integrating Products of Functions** Sometimes we are going to want to integrate the product of functions. While we could just multiply the functions together, it is usually easier to let Matlab do it for us.

Let's assume we want to evaluate the integral  $I = \int_0^1 x(t)y(t)dt$ , and let's assume that we already have anonymous functions  $x$  and  $y$ . The function **`quadl`** needs to be passed a function which is the product of  $x$  and  $y$ . To do this, we make a new anonymous function  $z$ , using the following:

```
z = @(t) x(t).*y(t);
```

and then perform the integration

```
I = quadl(z,0,1)
```

An alternative is to write

```
I = quadl(@(t) x(t).*y(t),0,1);
```

**Example 3** To numerically evaluate  $I = \int_{-1}^1 e^{-t} \cos(2t)dt$  we could type

```
x = @(t) exp(-t)
y = @(t) cos(2*t);
z = @(t) x(t).*y(t);
I = quadl(z,-1,1);
```

or

```
I = quadl(@(t) x(t).*y(t),-1,1);
```

**Example 4** To numerically evaluate  $I = \int_{-2}^1 |t| e^{-|t|} dt$  we could type

```
x = @(t) abs(t);
y = @(t) exp(-abs(t));
z = @(t) x(t).*y(t);
I = quadl(z,-2,1);
```

or

```
I = quadl(@(t) x(t).*y(t),-2,1);
```