Panel 1

ECE 300 Review Exam 3 Fall 07-08

2. Using the ***duality property***, find the corresponding Fourier transform for the following:

a) $g(t) = \text{sinc}^2(Bt)$ ← where does g(t) fit into temp.

b) $g(t) = \text{sinc}(Wt)$

c) $g(t) = \delta(t)$

d) $g(t) = \cos(\omega_0 t)$

Duality

$$* \quad x(t) \xrightarrow{\mathcal{F}} X(\omega)$$

Know Pair

$$* \quad X(t) \xrightarrow{\mathcal{F}} 2\pi x(-\omega)$$

start finish

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Panel 2

$$X(t) = \text{sinc}^2(Bt)$$

t 1

1. $X(\omega) = \text{sinc}^2(B\omega)$

2. $\mathcal{F}^{-1}\{X(\omega)\} = \frac{2}{4\pi B}\Lambda\left(\frac{t}{4\pi B}\right) = \frac{1}{2\pi B}\Lambda\left(\frac{t}{4\pi B}\right)$

$$\frac{2}{W}\Lambda\left(\frac{t}{W}\right) \Longleftrightarrow \frac{W}{2}\frac{2}{W}\text{sinc}^2\left(\frac{W}{4\pi}\omega\right)$$

$$\frac{W}{4\pi} = B$$

$$W = 4\pi B$$

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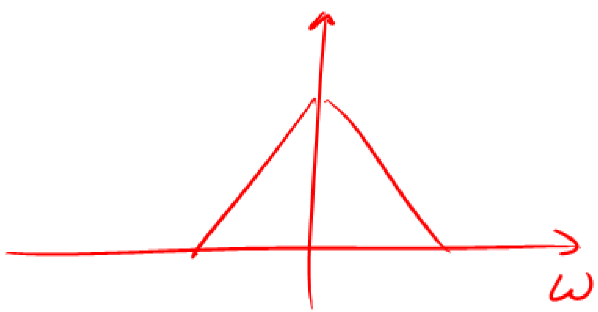
Panel 3

(3) $x(t) = \frac{1}{2\pi B} \Lambda\left(\frac{t}{4\pi B}\right)$

$$2\pi\, x(-\omega) = \frac{1}{B} \Lambda\left(\frac{-\omega}{4\pi B}\right) = \frac{1}{B} \Lambda\left(\frac{\omega}{4\pi B}\right)$$



$$\mathcal{F}\{\operatorname{sinc}^2(Bt)\} = \frac{1}{B} \Lambda\left(\frac{\omega}{4\pi B}\right)$$

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Panel 4

$$g(\omega) = \text{sinc}^2(B\omega)$$

$$\begin{tikzcd}
x(t) \arrow[r] \arrow[r, bend left, "2"] & X(\omega) \arrow[dl, "3"] \\
\underset{\text{finish}}{X(t)} \arrow[r] & \underset{\text{Start}}{2\pi\, x(-\omega)} \arrow[ul, "1"]
\end{tikzcd}$$

1. $\text{sinc}^2(B\omega) = 2\pi\, x(-\omega)$

$$x(-\omega) = \frac{1}{2\pi}\,\text{sinc}^2(B\omega)\Big|_{\omega=-t}$$

$$x(t) = \frac{1}{2\pi}\,\text{sinc}^2(-Bt) = \frac{1}{2\pi}\,\text{sinc}^2(Bt)$$

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Panel 5

② $x(t) = \frac{1}{2\pi} \operatorname{sinc}^2(Bt)$

$X(\omega) = \frac{1}{2\pi B} \Lambda\left(\frac{\omega}{4\pi B}\right)$

③ $X(t) = \frac{1}{2\pi B} \Lambda\left(\frac{t}{4\pi B}\right)$

$\mathcal{F}^{-1}\{\operatorname{sinc}^2(B\omega)\} = \frac{1}{2\pi B} \Lambda\left(\frac{t}{4\pi B}\right)$

Panel 6

4. Consider a linear time invariant system with impulse response given by $h(t)=\frac{1}{2\pi}\operatorname{sinc}\left(\frac{t-2}{2\pi}\right)$ with input $x(t)=\frac{4}{\pi}\operatorname{sinc}\left(\frac{2t}{\pi}\right)\cos(t)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Determine the energy in $x(t)$

d) Determine $H(\omega)$.

e) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

f) Determine $y(t)$, the output of the system.

g) Determine the energy in $y(t)$.

a) $x(t) = g_1(t)\cos(t) \longrightarrow X(\omega) = \frac{1}{2}\left[G_1(\omega-1) + G_1(\omega+1)\right]$

$$G_1(\omega) = \mathcal{F}\left\{\frac{4}{\pi}\operatorname{sinc}\left(\frac{2t}{\pi}\right)\right\} = \frac{4}{\cancel{\pi}}\cdot\frac{\cancel{\pi}}{2}\operatorname{rect}\left(\frac{\omega}{2\pi\left(\frac{2}{\pi}\right)}\right)$$

$$W = \frac{2}{\pi} \qquad = 2\operatorname{rect}\left(\frac{\omega}{4}\right)$$

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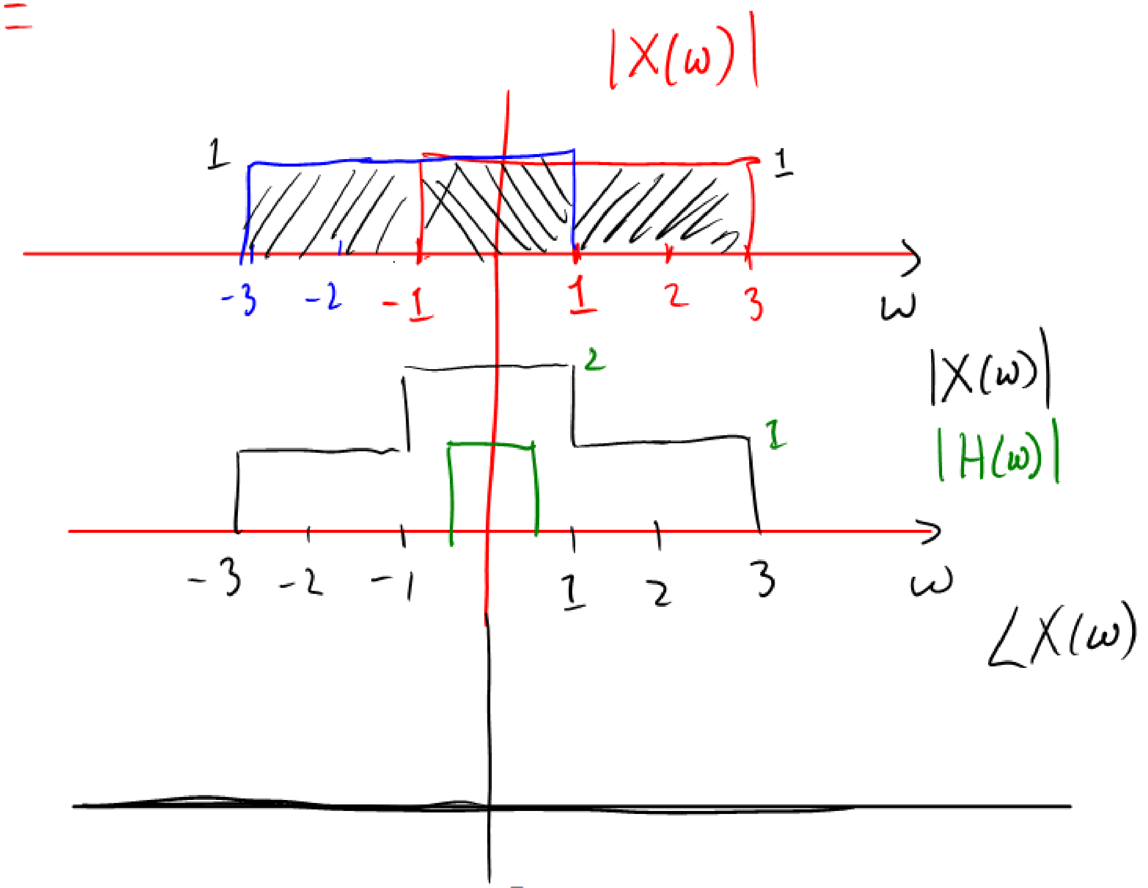
Panel 7

$$X(\omega) = \frac{1}{\cancel{2}}\left[\cancel{2}\,\mathrm{rect}\left(\frac{\omega-\overset{\text{center}}{1}}{\underset{\text{width}}{4}}\right) + \cancel{2}\,\mathrm{rect}\left(\frac{\omega+1}{4}\right)\right] e^{j0}$$

$=$

(b)



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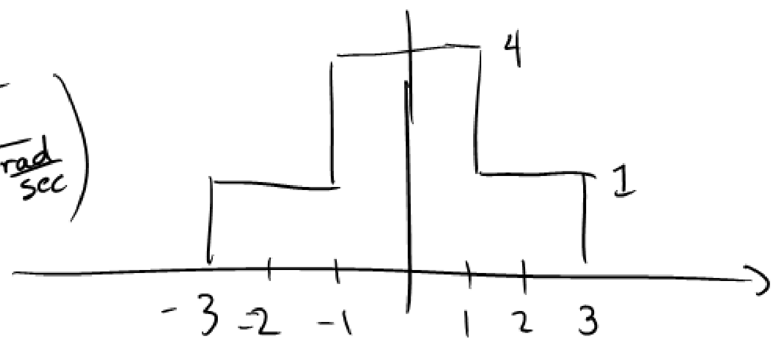
Panel 8

(c) find energy in $x(t)$

$$E_\infty = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, \underline{d\omega} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \frac{1}{2\pi}\left[2(1)(2) + 4(2)\right] = \frac{12}{2\pi} = \frac{6}{\pi} \, (J)$$

$|X(\omega)|^2 \left(\frac{J}{1 \frac{rad}{sec}}\right)$



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Panel 9

(d) $h(t) = \frac{1}{2\pi} \operatorname{sinc}\left(\frac{t-2}{2\pi}\right)$

$g_1(t) = \frac{1}{2\pi} \operatorname{sinc}\left(\frac{t}{2\pi}\right) \longrightarrow G_1(\omega) = \frac{1}{2\pi} \cdot 2\pi \operatorname{rect}\left(\frac{\omega}{2\pi\left(\frac{1}{2\pi}\right)}\right)$

$W = \frac{1}{2\pi}$

$G_1(\omega) = \operatorname{rect}(\omega)$

$g_2(t) = g_1(t-2) \longrightarrow G_1(\omega)\, e^{-j2\omega}$

$H(\omega) = \operatorname{rect}(\omega)\, e^{-j2\omega}$

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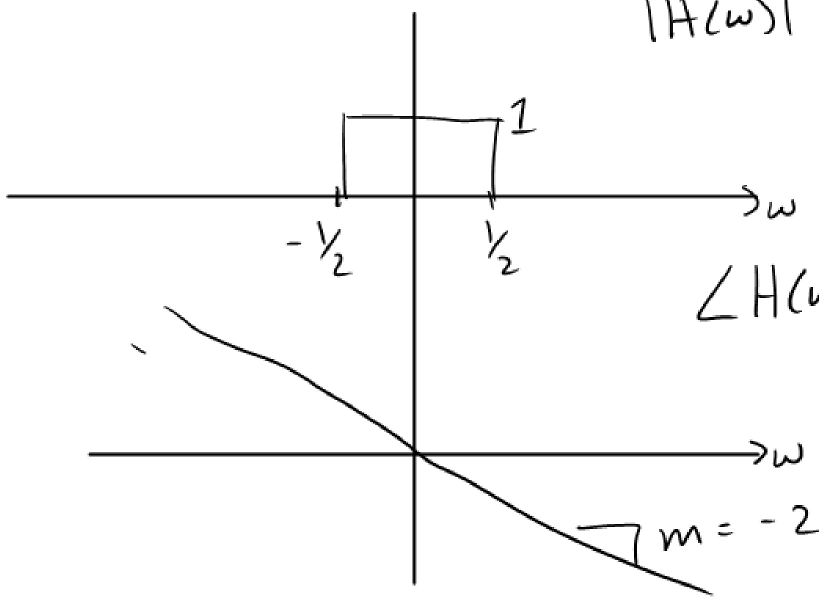
e) $H(\omega) = \text{rect}(\omega)\, e^{-j2\omega}$

$|H(\omega)|$



$\angle H(\omega) = -2\omega$

(f) $y(t) = \mathcal{F}^{-1}\{X(\omega) H(\omega)\} = \mathcal{F}^{-1}\{2\, H(\omega)\}$

$= 2\, h(t) = \underline{\frac{1}{\pi}\, \text{sinc}\left(\frac{t-2}{2\pi}\right)}$

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Panel 11

(g) find Energy in $y(t)$

$$Y(\omega) = 2H(\omega)$$

$$|Y(\omega)|^2 = 4|H(\omega)|^2$$

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega = \frac{1}{2\pi}\left[4(1)(1)\right] = \frac{2}{\pi} \text{ J}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left[\frac{1}{\pi} \operatorname{sinc}\left(\frac{t-2}{2\pi}\right)\right]^2 dt = 0$$

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Panel 13

WS

5. Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t) = 5\,\mathrm{sinc}\left(\frac{t}{0.002}\right) + 5\,\mathrm{sinc}\left(\frac{t}{0.001}\right) \quad \textit{Answer}\ \ 56\%$$

$$E_\infty = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

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Panel 14

Sol

5. Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t) = 5\,\text{sinc}\left(\frac{t}{0.002}\right) + 5\,\text{sinc}\left(\frac{t}{0.001}\right) \quad \textit{Answer: } 56\%$$

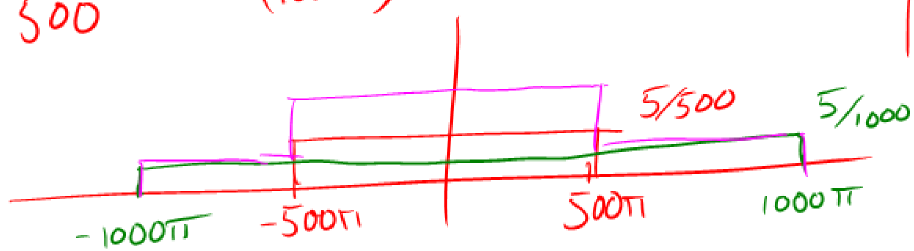
$$E_\infty = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$x(t) = g_1(t) + g_2(t)$$

$$W_1 = 500 \qquad W_2 = 1000$$

$$X(\omega) = \frac{5}{500}\,\text{rect}\left(\frac{\omega}{1000\pi}\right) + \frac{5}{1000}\,\text{rect}\left(\frac{\omega}{2000\pi}\right)$$

$|X(\omega)|$



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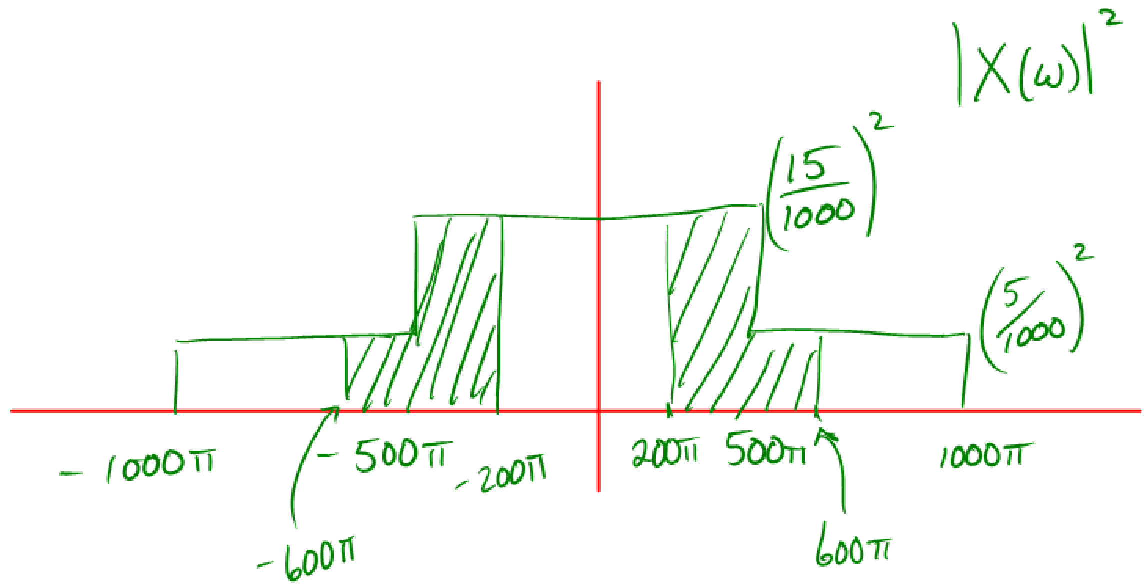
$$\frac{E_{100-300}}{E_{TOT}} = \frac{\frac{1}{\pi}\int_{200\pi}^{600\pi} |X(\omega)|^2 d\omega}{\frac{1}{2\pi}\int_{-1000\pi}^{1000\pi} |X(\omega)|^2 d\omega}$$



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Panel 16

Exam 2 # 1 (c)

given
$$y(t) = e^{-t} \int_{-\infty}^{t-1} e^{\lambda} x(\lambda - 2)\, d\lambda$$

find $h(t) = y(t)$ when $x(t) = \delta(t)$

$$h(t) = e^{-t} \int_{-\infty}^{t-1} \underbrace{e^{\lambda} \delta(\lambda - 2)}_{e^{2} \delta(\lambda - 2)}\, d\lambda$$

$$= e^{-t} e^{2} \underbrace{\left[\int_{-\infty}^{t-1} \delta(\lambda - 2)\, d\lambda \right]}_{} = \begin{cases} 1 & t-1 \geq 2 \\ 0 & \text{else} \end{cases}$$

$t \geq 3$

$$= e^{2-t}\, u(t-3)$$

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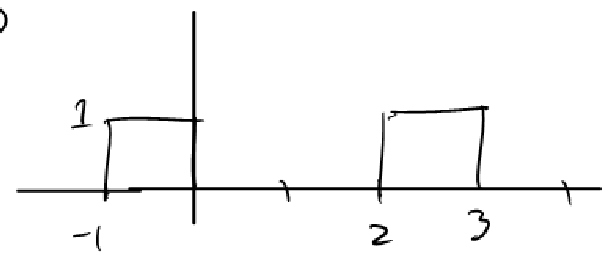
Panel 17

Exam 2 4 (d)

$x(t)$ periodic w/ $T_0 = 3$

$$x(t) = \begin{cases} 1 & -1 \le t < 0 \\ 0 & 0 < t \le 2 \end{cases}$$



Find Fourier Series Coeff X_k where $x(t) = \sum X_k e^{jk\omega_0 t}$ as sinc fcn

$$X_k = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{T_0/2} x(t) e^{-jk\omega_0 t}\, dt = \frac{1}{3}\int_{-1}^{0} e^{-jk\omega_0 t}\, dt$$

$$= \frac{-1}{3jk\omega_0}\left[e^{-jk\omega_0 t}\right]\Bigg|_{t=-1}^{t=0} = \frac{-1}{3jk\omega_0}\left[1 - e^{-jk\omega_0 t}\right]$$

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$$= \frac{-1}{3jk\omega_0}\left[e^{-jk\omega_0 t}\right]\Bigg|_{t=-1}^{t=0} = \frac{-1}{3jk\omega_0}\left[1 - e^{-jk\omega_0 t}\right]$$

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} = \frac{\frac{1}{2j}\left[e^{j(\pi x)} - e^{-j(\pi x)}\right]}{\pi x}$$

$$(1 - x^2) = \frac{x}{2}\left(\frac{2}{x} - 2x\right)$$

$$(1 - e^{jb}) = e^{ja} - e^{jb} = e^{j\frac{a+b}{2}}\left(e^{j\frac{a-b}{2}} - e^{-j\frac{a-b}{2}}\right)$$

$a = 0 \quad b = -k\omega_0 t$

$$(1 - e^{-jk\omega_0 t}) = \underline{e^{-j\frac{k\omega_0 t}{2}}}\left(e^{j\frac{k\omega_0 t}{2}} - e^{-j\frac{k\omega_0 t}{2}}\right)$$

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$$\underbrace{\frac{-1}{3jk\omega_0}} e^{-j\frac{k\omega_0 t}{2}} \underbrace{\left[e^{j\frac{k\omega_0 t}{2}} - e^{-j\frac{k\omega_0 t}{2}} \right]}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3}$$

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$= \frac{-1}{3jk\frac{2\pi}{3}} \left[\qquad \right]$$

$$= \frac{-1}{jk\pi 2} \left[\qquad \right]$$

$$= -\frac{1}{k\pi} e^{-j\frac{k\omega_0 t}{2}} \underline{\sin\left(\frac{k\omega_0 t}{2}\right)} = \frac{-1}{k\pi} e^{-j\frac{k\omega_0 t}{2}} \sin\left(\frac{k2\pi t}{2T_0}\right)$$

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Panel 20

Quiz 3 #6

$$x(t) \longrightarrow \boxed{h_1(t) = e^{-t}u(t)} \xrightarrow{z(t)} \boxed{h_2(t) = 2\delta(t-1)} \longrightarrow y(t)$$

find $h(t)$ of overall system $x(t) \to y(t)$

$$z(t) = x(t) * h_1(t)$$

$$y(t) = z(t) * h_2(t) = x(t) * h_1(t) * h_2(t)$$

$$h(t) = h_1(t) * h_2(t) = e^{-t}u(t) * 2\delta(t-1)$$

$$= 2e^{-(t-1)}u(t-1)$$

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Panel 21

$$e^{-t}u(t) * 2\delta(t-1)$$

$$= \int_{-\infty}^{\infty} \underbrace{2e^{-(t-\lambda)}\,u(t-\lambda)\,\delta(\lambda-1)}_{\lambda = 1}\,d\lambda$$

$$= \int_{-\infty}^{\infty} \underbrace{2e^{-(t-1)}\,u(t-1)}\,\delta(\lambda-1)\,d\lambda$$

$$= 2e^{-(t-1)}\,u(t-1)\left[\cancel{\int_{-\infty}^{\infty}\delta(\lambda-1)\,d\lambda}\right] \to 1$$

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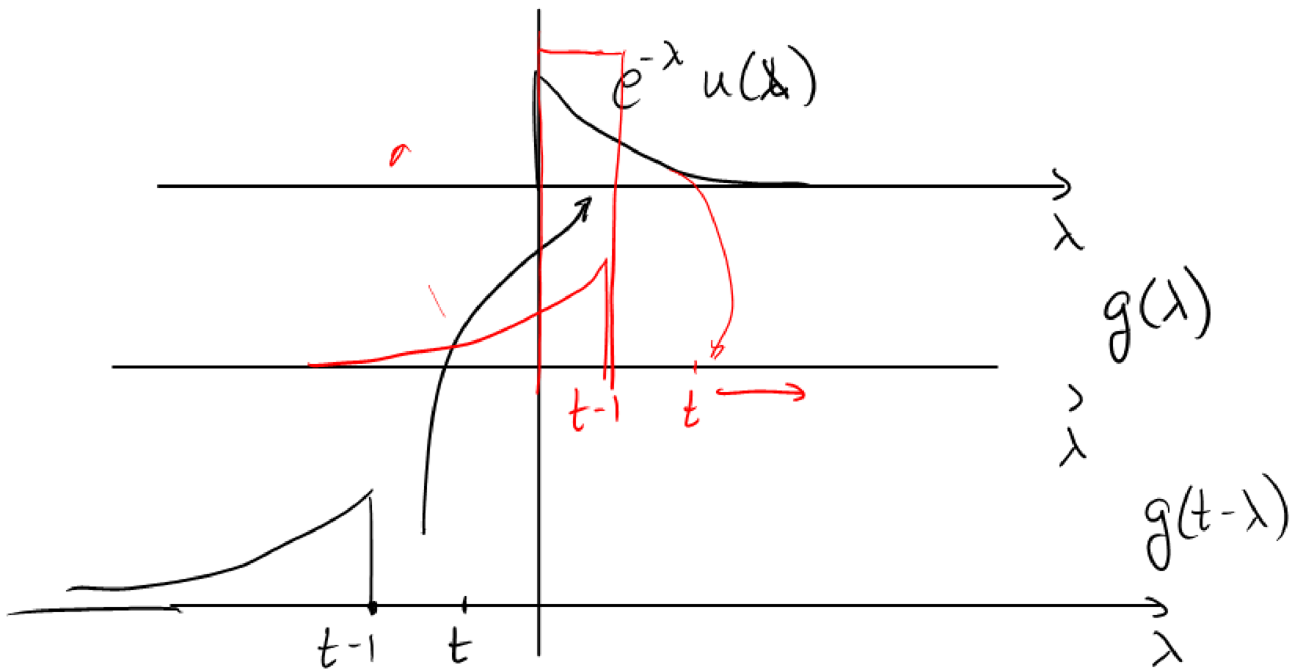
Panel 22

$$e^{-t}u(t) * e^{-(t-1)}u(t-1)$$



$t < 1 \qquad y(t) = 0$

$t \geq 1 \qquad y(t) = \int_0^{t-1} e^{-\lambda} e^{-(t-\lambda-1)}\, d\lambda$

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$$= \int_0^{t-1} e^{-\lambda} e^{-(t-\lambda-1)} \, d\lambda$$

$$= \int_0^{t-1} e^{-(t-\lambda-1)-\lambda} \, d\lambda = \int_0^{t-1} e^{-t+\lambda+1-\lambda} \, d\lambda$$

$$= \int_0^{t-1} e^{-t+1} \, d\lambda = e^{-t+1} \left[\lambda \right]_0^{t-1}$$

$$= e^{-t+1} \left[t-1 \right] \text{ only for } t \geq 1$$

$$= e^{-t+1} (t-1) \, u(t-1) = y(t)$$

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Panel 24

Quiz 6

$$X(\omega) = \frac{j\omega}{2-j\omega} \xleftrightarrow{\mathcal{F}^{-1}} x(t)$$

$$G_1(\omega) = \frac{1}{|-1|} G_2\left(\frac{\omega}{-1}\right) \longrightarrow g_1(t) = g_2(-t)$$

exp pulse + time scaling w/ $a = -1$

$$G_2(\omega) = \frac{1}{2+j\omega} \longrightarrow g_2(t) = e^{-2t}u(t)$$

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Panel 25

$$G_1(\omega) = G_2(-\omega) \longrightarrow g_2(-t) = e^{2t}u(-t)$$

$$G_2(\omega) = \frac{1}{2+j\omega} \longrightarrow g_2(t) = e^{-2t}u(t)$$

$$\frac{1}{|a|}X\left(\frac{\omega}{a}\right) \longrightarrow x(at)$$

$$a = -1$$

$$\frac{1}{|-1|}X\left(\frac{\omega}{-1}\right) \longrightarrow x(-t)$$

$$X(-\omega) \longrightarrow x(-t)$$

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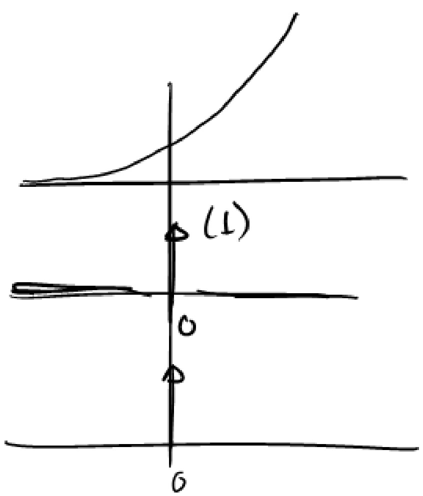
$$\mathcal{F}^{-1}\left\{\frac{1}{2-j\omega}\right\} = e^{2t}u(-t)$$

$$\mathcal{F}^{-1}\left\{j\omega\,\frac{1}{2-j\omega}\right\} = \frac{d}{dt}e^{2t}u(-t)$$



$$= 2e^{2t}u(-t) + e^{2t}\cancelto{-\delta(t)}{\frac{du(-t)}{dt}}$$

$$= 2e^{2t}u(-t) - e^{2t}\delta(t)$$

$$= 2e^{2t}u(-t) - \delta(t)$$

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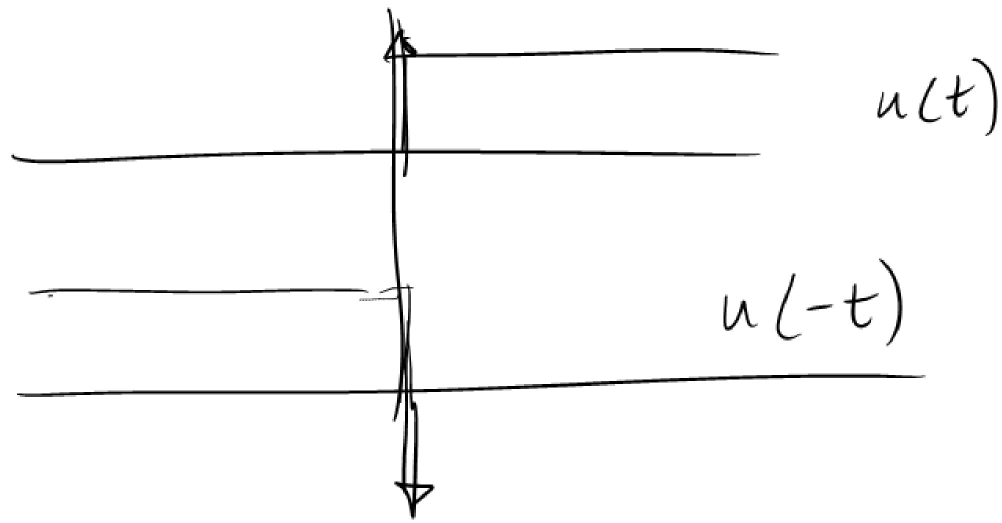
Panel 27



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