ECE 300
Signals and Systems
Homework 1

Due Date: Tuesday September 11, 2007 *at the beginning of class*

Reading: ZTF pages 1-28 and your course notes.

Problems

1) ZTF, Problem 1.8.

2) ZTF, Problem 1-10.

3) ZTF, Problem 1-14.

4) ZTF, Problem 1-30.

5) ZTF, Problem 1-32.

6) Simplify the following as much as possible, giving numerical answers where possible

a) $\int_{-\infty}^{\infty} e^{-t}u(t-5)dt$

b) $\int_{-\infty}^{\infty} t^2\left[u(t-6)-u(t-5)\right]dt$

c) $\int_{-\infty}^{\infty} t^2\delta(t-2)dt$

d) $\int_{5}^{\infty} t^2\delta(t-2)dt$

e) $\int_{0}^{\infty} \sin(ty\pi)\delta(t-2)dt$

f) $\sin(ty\pi)\delta(t-2)$

g) $\int_{-\infty}^{\infty} \delta(t-3)\delta(t-4)dt$

h) $\int_{-\infty}^{\infty} u(t-3)\delta(t-4)dt$

i) $\int_{-\infty}^{\infty} u(t-x+5)\delta(t-4)dt$

j) $\int_{-\infty}^{3} u(t-x+5)\delta(t-4)dt$

k) $t\delta(t-2)+t^3\delta(t-1)$

l) $H(\omega)\delta(\omega-1)+A(\omega-x+1)\delta(\omega)$

m) $\int_{-9}^{10} u(t+3)u(t-2)dt$

7) For each of the following signals, determine if the signal is periodic and, if so, the fundamental period.

a) $x(t)=\sin(2t)+\cos(3t+30^o)$
b) $x(t)=\cos(2t)+\cos(\pi t)$
c) $x(t)=e^{-t}\cos(t)$
d) $x(t)=2e^{j2t}+3e^{j(3t+2)}$

Matlab Problems

8) Using Matlab, plot each signal from Problem 7 for three *fundamental periods* if the signal is periodic, or three times the longest period in the signal if the signal is not periodic. Be sure there are at least 50 samples per period for each waveform and your graphs are neatly labeled. **Notes:** (1) Matlab works in radians, so all angles must be converted to radians, (2) use **exp** in Matlab to get an exponential, (3) **j** is Matlab's way of indicating the square root of -1, and if you want $x(t)=e^{j2t}$ you should type something like ***x = exp(j*2*t)***, and (4) if the waveform is complex, plot the real and imaginary parts separately. The Matlab commands **real** and **imag** are very useful for this. Turn in your plots.

For the following two problems, save the files **stp_fn.m** and **rmp_fn.m** from the course website to the directory in which you will be using MATLAB. This directory is called the "working directory" in Matlab. If you do this correctly, you can use the functions **stp_fn** and **rmp_fn** just as you would any other built-in matlab function. Use these supplied MATLAB functions to generate the function

$$x(t)=3u(t-2)+4r(t-3)$$

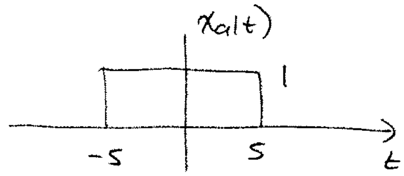
from t = 0 to 10, you might type the following in Matlab

```
t = linspace(0,10,1000);
x = 3*stp_fn(t-2)+4*rmp_fn(t-3);
```

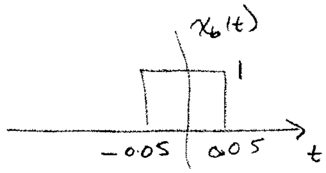
9) ZTF, Computer Exercise 1-1.
10) ZTF, Computer Exercise 1-2 (only plot parts b and c). Use subplot to plot both functions on one page.

#1 ZTM, Problem 1-8

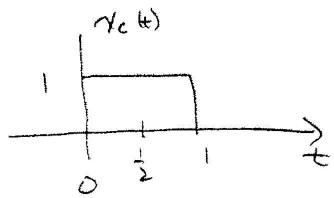
a) $x_a(t) = \Pi(0.1t) = \Pi\left(\frac{t}{10}\right)$ width = 10, centered at 0, amplitude = 1



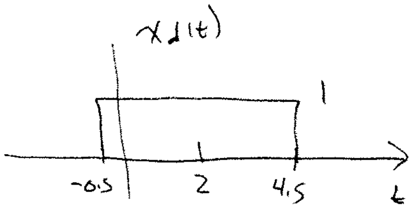
b) $x_b(t) = \Pi(10t) = \Pi\left(t/\tfrac{1}{10}\right)$ width = 1/10, centered at 0, amplitude = 1



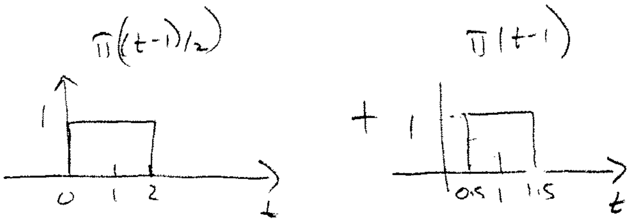
c) $x_c(t) = \Pi(t - \tfrac{1}{2})$ width = 1, centered at ½, amplitude = 1

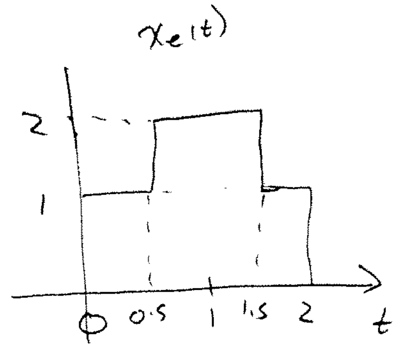


d) $x_d(t) = \Pi((t-2)/5)$ width = 5, centered at 2, amplitude = 1



e) $x_e(t) = \Pi((t-1)/2) + \Pi(t-1)$





#2

ZTF, Problem 1-10

$A = 3 + j3 \quad B = 10e^{j\pi/3}$

(a) $A = \sqrt{3^2+3^2}\, e^{j\tan^{-1}(3/3)} = \boxed{4.2426e^{j\pi/4} = A}$

$B = 10\cos(\pi/3) + j10\sin(\pi/3) = \boxed{5.0000 + j8.6603 = B}$

(b) $A+B = (3+j3)+(5+j8.6603) = \boxed{8 + j11.6603}$

(c) $A-B = (3+j3) - (5+j8.6603) = \boxed{-2 - j5.6603}$

(d) $AB = (4.242e^{j\pi/4})(10e^{j\pi/3}) = 42.42e^{j7\pi/12}$

$= \boxed{-10.98 + j40.97}$

$AB = (3+j3)(5+j8.660)$

$= (15-25.98) + j(15+25.98) = -10.98 + j40.98$ ✓

(e) $A/B = \dfrac{4.2426e^{j\pi/4}}{10e^{j\pi/3}} = 0.42426e^{-j\pi/12}$

$= \boxed{0.4098 - j0.10980}$

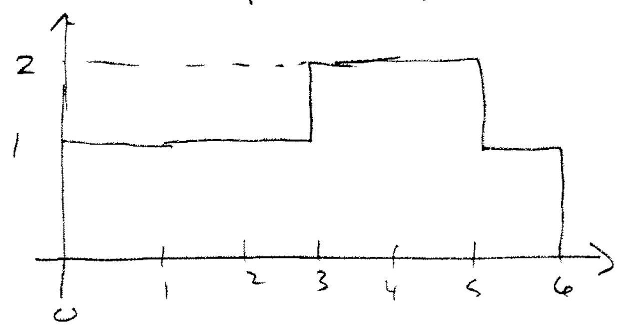
$A/B = \dfrac{3+j3}{5+j8.660} \cdot \dfrac{5-j8.660}{5-j8.660} = \dfrac{(15+25.98)+j(15-25.98)}{25+75}$

$= \dfrac{40.98 - j10.98}{100} = 0.4098 - j0.1098$ ✓

(#3)

ZTM, Problem 1-14

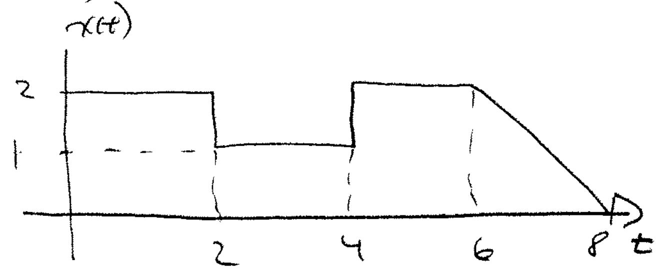
a) $x_a(t) = \Pi\left((t-3)/6\right) + \Pi\left((t-4)/2\right)$



$$\boxed{x_a(t) = u(t) + u(t-3) - u(t-5) - u(t-6)}$$

b) $$\boxed{\dot{x}_a(t) = \delta(t) + \delta(t-3) - \delta(t-5) - \delta(t-6)}$$

#4 ZTM, Problem 1-30

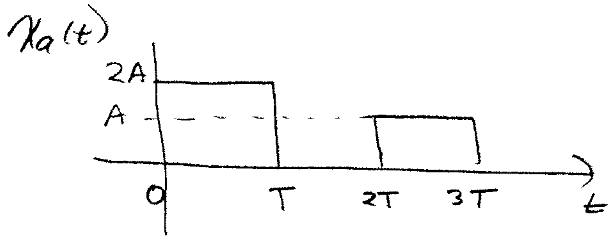


$$x(t) = 2u(t) - u(t-2) + u(t-4) - 2u(t-6)$$
$$+ [u(t-6) - u(t-8)][2 - (t-6)]$$

$$= 2u(t) - u(t-2) + u(t-4) + u(t-6)[-2 + 2 - (t-6)]$$
$$- u(t-8)[2 - t + 6]$$
$$= 2u(t) - u(t-2) + u(t-4) - (t-6)u(t-6) + (t-8)u(t-8)$$

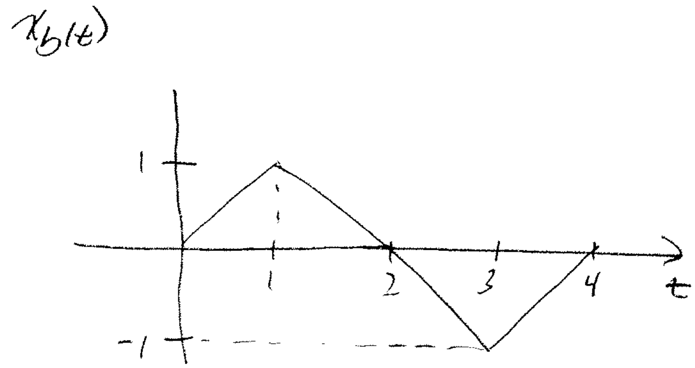
$$\boxed{x(t) = 2u(t) - u(t-2) + u(t-4) - r(t-6) + r(t-8)}$$

#5 ZTM, Problem 1-32



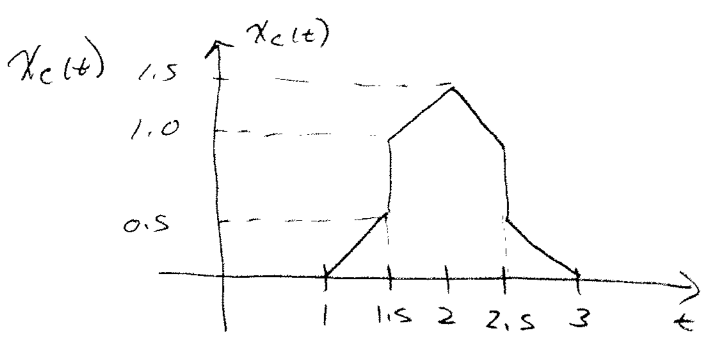
$$x_a(t) = 2A\,u(t) - 2A\,u(t-T) + A\,u(t-2T) - A\,u(t-3T)$$

$$\boxed{x_a(t) = 2A\left[u(t) - u(t-T)\right] + A\left[u(t-2T) - u(t-3T)\right]}$$



$$x_b(t) = \left[r(t) - r(t-1)\right] + \left[-r(t-1) + r(t-3)\right] + \left[r(t-3) - r(t-4)\right]$$

$$\boxed{x_b(t) = r(t) - 2r(t-1) + 2r(t-3) - r(t-4)}$$



$$x_c(t) = r(t-1) + u(t-1.5) - u(t-2.5) - 2r(t-2) + r(t-3)$$

$$\boxed{x_c(t) = r(t-1) - 2r(t-2) + r(t-3) + \left[u(t-1.5) - u(t-2.5)\right]}$$

#6

(a) $\int_{-\infty}^{\infty} e^{-t} u(t-5)\,dt = \int_5^{\infty} e^{-t}\,dt = -e^{-t}\Big|_5^{\infty} = e^{-5} = \boxed{0.00674}$

(b) $\int_{-\infty}^{\infty} t^2 [u(t-6) - u(t-5)]\,dt = -\int_{-\infty}^{\infty} t^2 [u(t-5) - u(t-6)]\,dt$

$= -\int_5^6 t^2\,dt = -\frac{t^3}{3}\Big|_5^6 = -\left[\frac{6^3 - 5^3}{3}\right] = \boxed{-30.33}$

(c) $\int_{-\infty}^{\infty} t^2 \delta(t-2)\,dt = \boxed{4}$

(d) $\int_5^{\infty} t^2 \delta(t-2)\,dt = \boxed{0}$

(e) $\int_0^{\infty} \sin(ty\pi)\,\delta(t-2)\,dt = \boxed{\sin(2y\pi)}$

(f) $\sin(t\pi y)\,\delta(t-2) = \boxed{\sin(2y\pi)\,\delta(t-2)}$

(g) $\int_{-\infty}^{\infty} \delta(t-3)\,\delta(t-4)\,dt = \boxed{0}$

(h) $\int_{-\infty}^{\infty} u(t-3)\,\delta(t-4)\,dt = u(t-3)\Big|_{t=4} = \boxed{1}$

(i) $\int_{-\infty}^{\infty} u(t-x+5)\,\delta(t-4)\,dt = \boxed{u(9-x)}$

(j) $\int_{-\infty}^{3} u(t-x+5)\,\delta(t-4) = \boxed{0}$

(k) $t\,\delta(t-2) + t^3\,\delta(t-1) = \boxed{2\,\delta(t-2) + \delta(t-1)}$

(l) $H(w)\,\delta(w-1) + A(w-x+1)\,\delta(w) = \boxed{H(1)\,\delta(w-1) + A(1-x)\,\delta(w)}$

(m) $\int_{-9}^{10} u(t+3)\,u(t-2)\,dt$

$u(t+3) = 1$ for $t > -3$
$u(t-2) = 1$ for $t > 2$

$= \int_2^{10} 1\,dt = \boxed{8}$

#7

(a) $x(t) = \sin(2t) + \cos(3t+30°)$

$x(t+T_0) = \sin(2t+2T_0) + \cos(3t+3T_0+30°)$

$= x(t)$ if $2T_0 = q(2\pi)$ $3T_0 = r(2\pi)$ q, r <u>integers</u>

$T_0 = q\pi = r\frac{2}{3}\pi$ $q = 2$ $r = 3$ works

$\boxed{\text{periodic, } T_0 = 2\pi}$

(b) $x(t) = \cos(2t) + \cos(\pi t)$

$x(t+T_0) = \cos(2t+2T_0) + \cos(\pi t + \pi T_0)$

$= x(t)$ if $2T_0 = q(2\pi)$ $\pi T_0 = r(2\pi)$ q, r <u>integers</u>

$T_0 = q\pi = 2r$ no q, r integers will solve

$\boxed{\text{not periodic}}$

(c) $x(t) = e^{-t}\cos(t)$

$x(t+T_0) = e^{-t}e^{-T_0}\cos(t+T_0)$

$= x(t)$ if $e^{-T_0} = 1$ $T_0 = (2\pi)q$ q <u>integer</u>

The only solution is $T_0 = 0$ $\boxed{\text{not periodic}}$

(d) $x(t) = 2e^{j2t} + 3e^{j(3t+2)}$

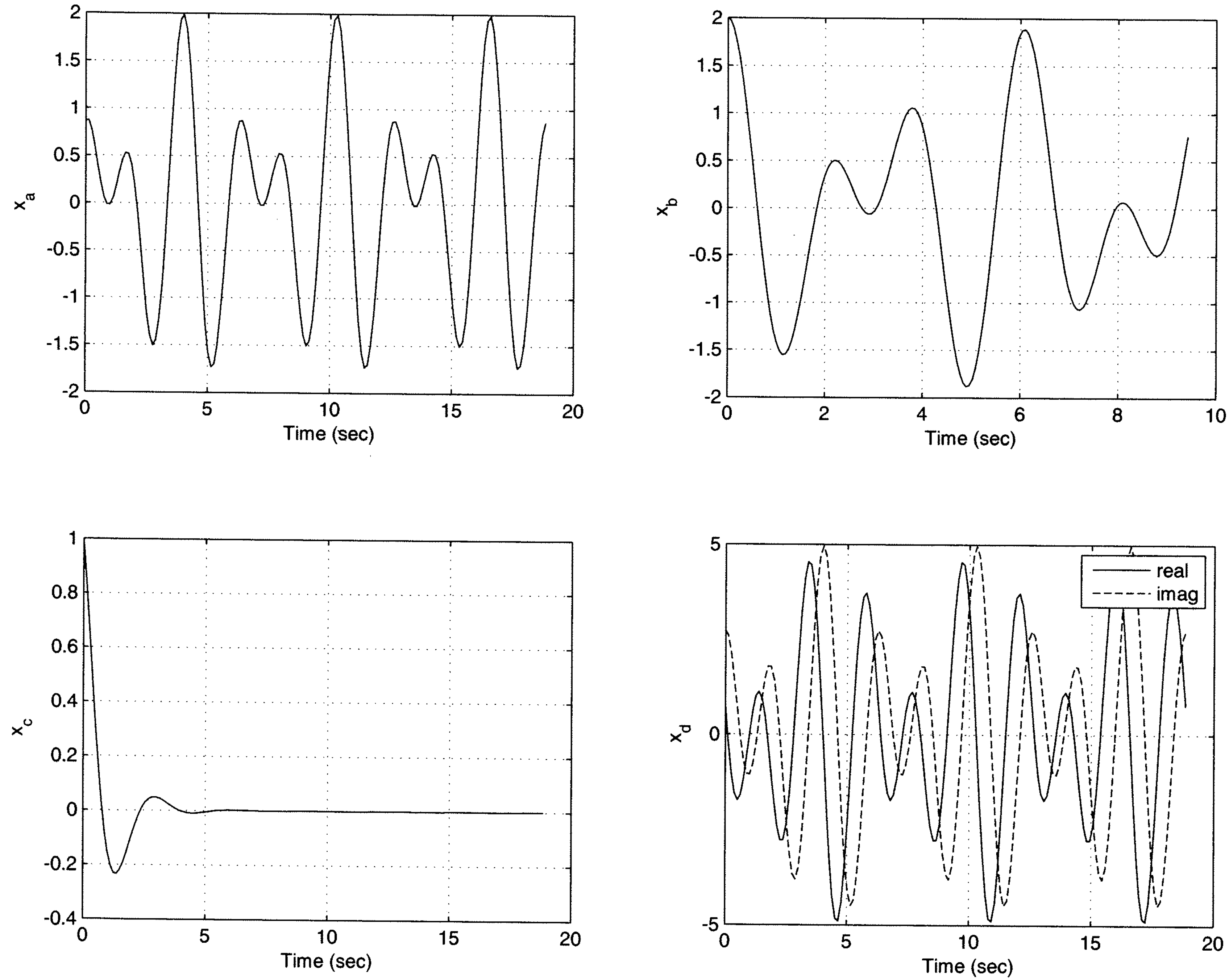
$x(t+T_0) = 2e^{j(2t+2T_0)} + 3e^{j(3t+3T_0+2)}$

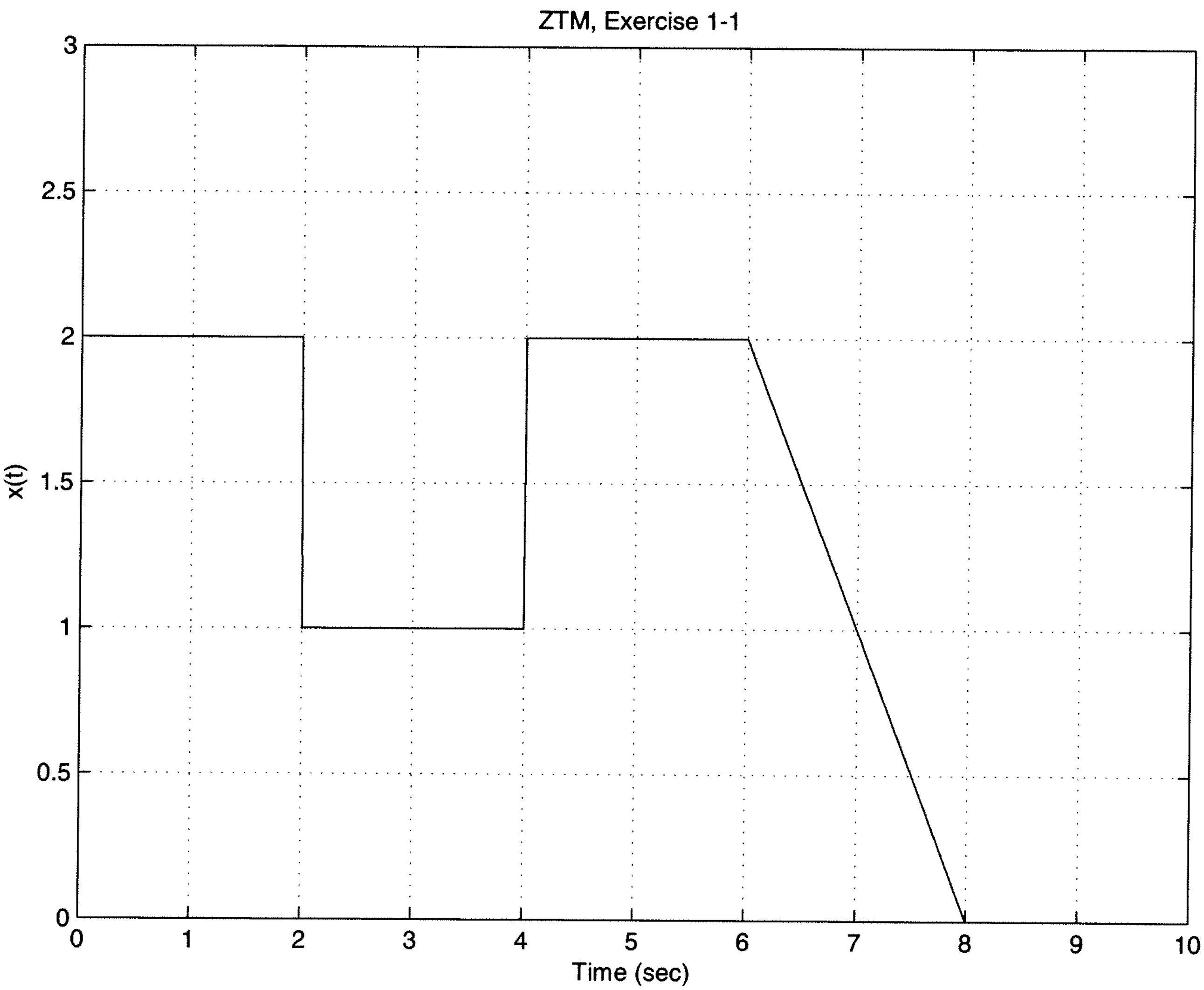
$= 2e^{j2t}e^{j2T_0} + 3e^{j(3t+2)}e^{j3T_0}$

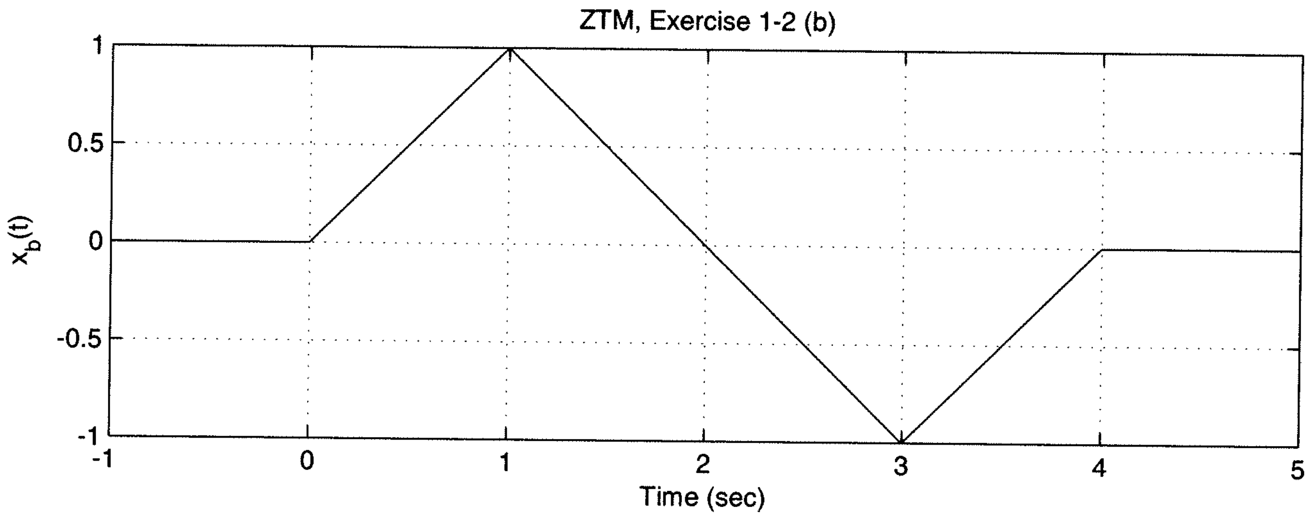
$= x(t)$ if $2T_0 = q(2\pi)$ $3T_0 = r(2\pi)$ q, r <u>integers</u>

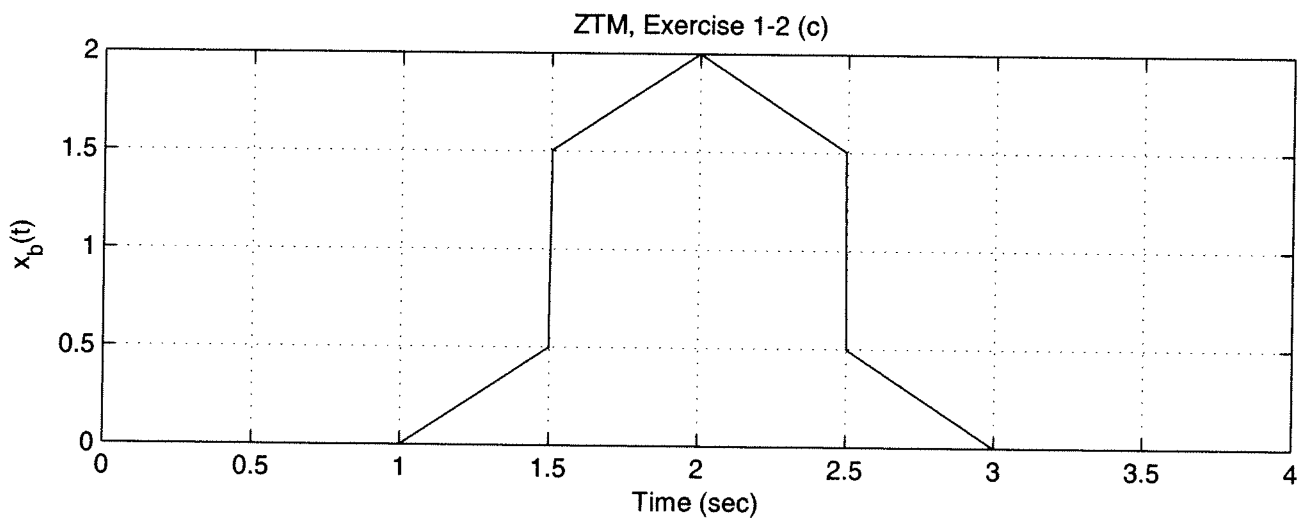
$T_0 = q\pi = \frac{2}{3}r\pi$ $q = 2$ $r = 3$ works

$\boxed{\text{periodic } T_0 = 2\pi}$


x_a
Time (sec)
x_b
Time (sec)
x_c
Time (sec)
x_d
real
imag
Time (sec)


ZTM, Exercise 1-1
x(t)
Time (sec)
0
0.5
1
1.5
2
2.5
3
0
1
2
3
4
5
6
7
8
9
10


ZTM, Exercise 1-2 (b)
1
0.5
0
-0.5
-1
$x_b(t)$
-1
0
1
2
3
4
5
Time (sec)


ZTM, Exercise 1-2 (c)
2
1.5
1
0.5
0
$x_b(t)$
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (sec)