Name ____ CM___________

ECE 300
Signals and Systems

Exam 3
6 November 2007

NAME ____

This exam is closed-book in nature. You may use the provided table of common Fourier Transform pairs and propterties. You may use a calculator for simple calculations, but not for things like integrals. You must show your work to receive credit!

Problem 1	________	/ 25
Problem 2	________	/ 30
Problem 3	________	/ 25
Problem 4	________	/ 20

Exam 3 Total Score: _______ / 100

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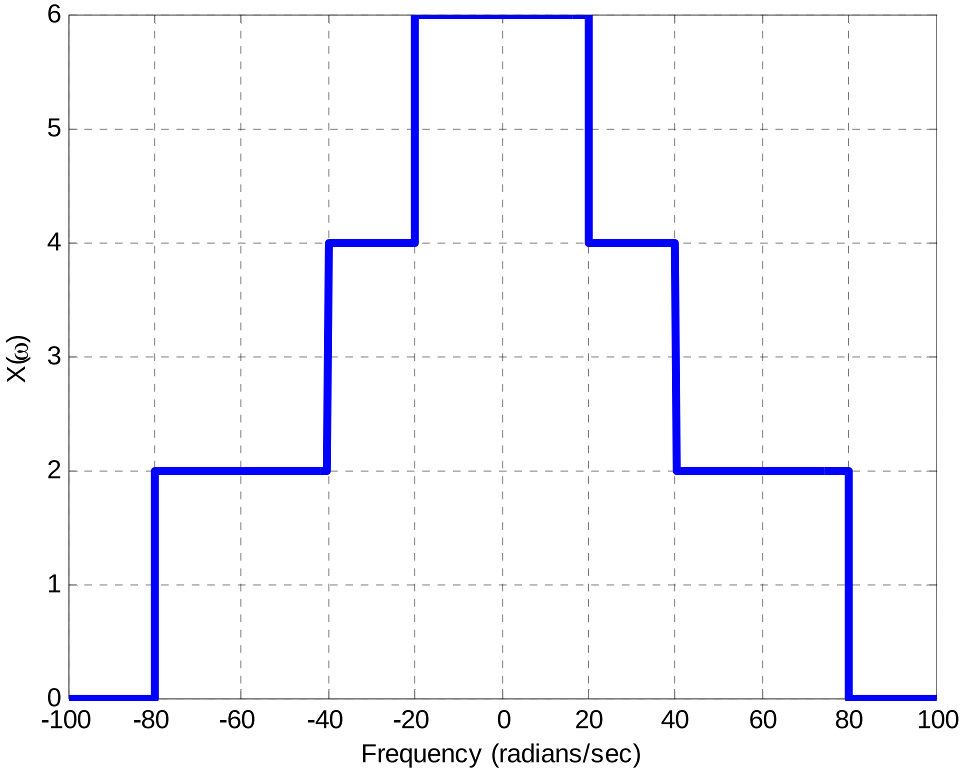
1. (**25 points**) **Finding the energy in a given bandwidth**

For the signal $x(t)$ with spectrum shown below:



Determine the percentage of the total energy in $x(t)$ between 20 and 60 radians/sec.

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2. (**30 points**) **System analysis with the Fourier Transform**

Consider a linear time invariant system with impulse response given by

$$h(t) = \frac{3}{2\pi}\text{sinc}\left(\frac{\text{t-2}}{2\pi}\right)$$

with input

$$x(t) = \frac{2}{\pi}\text{sinc}^2\left(\frac{\text{t-3}}{\pi}\right)\cos(t-3)$$

The output of the system is $y(t)$. Show all of your work and draw a ***BOX*** around your final answer.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Determine $H(\omega)$.

d) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

e) Determine $y(t)$, the output of the system.

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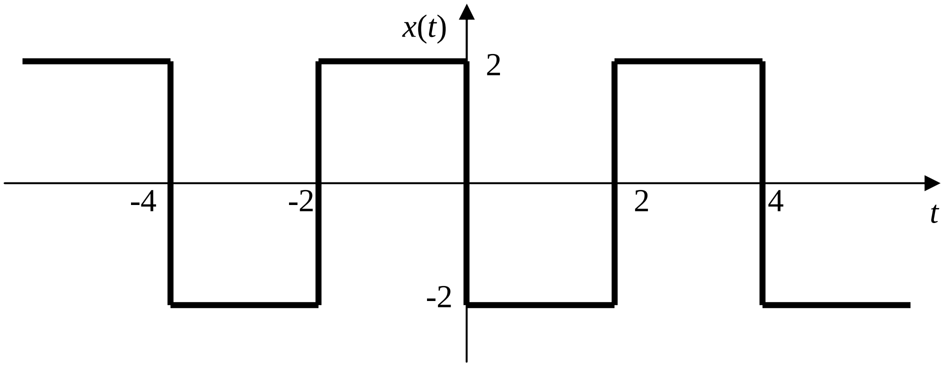
3. (**25 points**) **Fourier Series of a Periodic Signal**
The following set of questions refer to the signal below



(a) What is the fundamental frequency of x(t) in (rad/s)?

(b) Find an expression for the Fourier Series Coefficients, c_k, of x(t). Simplify your answer as much as possible.

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4. (**20 points**) **Properties of the Fourier Transform**

Show that if a signal, x(t), is real and even, then the Fourier Transform of the signal, X(ω), is also real and even.

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Some Potentially Useful Relationships

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j\sin(x) \qquad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\text{rect}\left(\frac{t-t_0}{T}\right) = u\left(t - t_0 + \frac{T}{2}\right) - u\left(t - t_0 - \frac{T}{2}\right)$$