

Name _____ CM _____

ECE 300
Signals and Systems

Exam 2
18 October 2007

NAME _____

This exam is closed-book in nature. You may use a calculator for simple calculations, but not for things like integrals. Credit will not be given if your work is not shown!

Problem 1 _____ / 25
Problem 2 _____ / 25
Problem 3 _____ / 25
Problem 4 _____ / 25

Exam 2 Total Score: _____ / 100

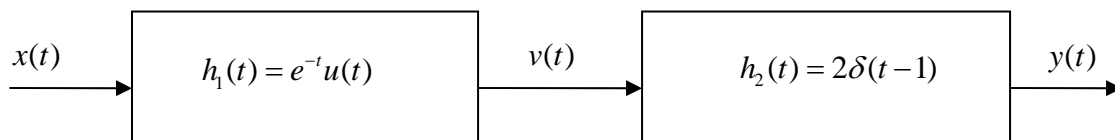
1) Short Answer Questions (5 points each):

a) Is the system with impulse response $h(t) = e^t u(t)$ BIBO stable? Why or why not?

b) Is the system $y(t) = \cos\left(\frac{1}{x(t)}\right)$ BIBO stable? Why or why not?

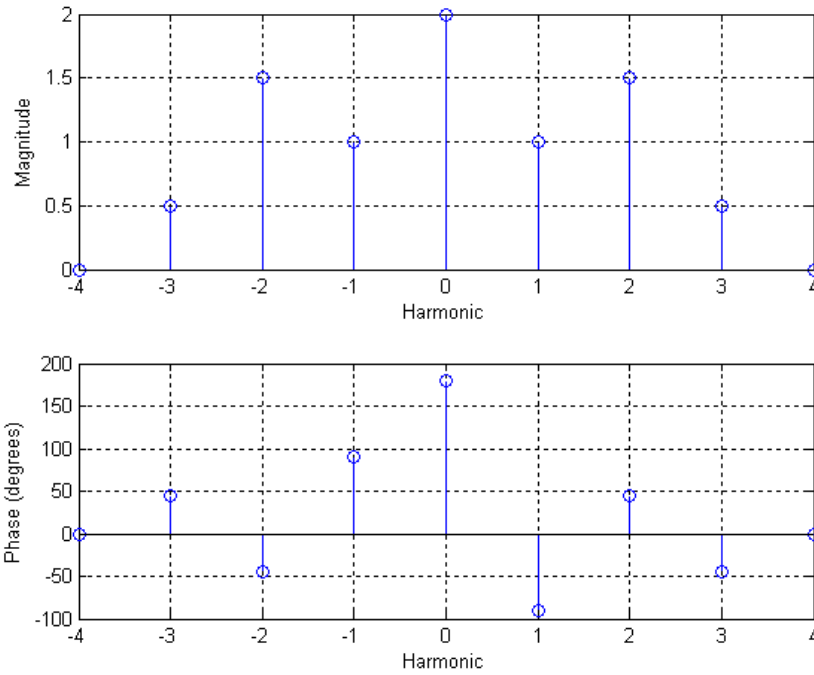
c) What is the impulse response for the system $y(t) = e^{-t} \int_{-\infty}^{t-1} e^{\lambda} x(\lambda - 2) d\lambda$? Be sure to include appropriate unit step functions.

d) Consider the two LTI systems shown below, with impulse responses shown. What is the impulse response between $x(t)$ and $y(t)$?



e) Is the function $x(t) = \cos(4\pi t + \frac{\pi}{2}) + \sin(6\pi t)$ periodic? If yes, determine the fundamental period.

2) Assume periodic signal $x(t)$ has the spectrum shown below and a fundamental frequency $\omega_0 = 3$ rad/sec. Assume all angles are multiples of 45 degrees.



a) Determine the **average value** and **average power** in $x(t)$.

b) Write an expression for $x(t)$ in terms of cosines (and/or sines).

c) Sketch the **single sided power spectrum** for $x(t)$ on the graph below. Be sure to accurately label all axes.



3) Assume periodic signal $x(t)$ has Fourier series representation

$$x(t) = \sum_{k=-\infty}^{k=\infty} \frac{jk}{1+jk} e^{jkt}$$

$x(t)$ is the input to an LTI system with transfer function given by

$$H(j\omega) = \begin{cases} \frac{1}{1+j\omega} & 1.5 < |\omega| < 2.5 \\ 0 & \text{otherwise} \end{cases}$$

Determine the steady state output of the system, $y(t)$. For full credit your answer must be written in terms of cosines (and/or sines). Clearly indicate whether you are writing your phase in degrees or in radians.

4) Assume $x(t)$ is a periodic signal with period $T_0 = 3$. $x(t)$ is defined over one period as

$$x(t) = \begin{cases} 1 & -1 < t \leq 0 \\ 0 & 0 < t \leq 2 \end{cases}$$

- a) Determine the fundamental frequency ω_0 .
- b) Determine the average value of $x(t)$.
- c) Determine the average power in the DC component of $x(t)$.
- d) Determine an expression for the expansion coefficients, X_k , where $x(t) = \sum X_k e^{jk\omega_0 t}$. You must write your expression in terms of the **sinc** function, and possibly a leading exponential term.

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Some Potentially Useful Relationships

$$E_{\infty} = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$e^{jx} = \cos(x) + j \sin(x) \quad j = \sqrt{-1}$$

$$\cos(x) = \frac{1}{2} [e^{jx} + e^{-jx}] \quad \sin(x) = \frac{1}{2j} [e^{jx} - e^{-jx}]$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x) \quad \sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$$

$$\text{rect}\left(\frac{t-t_0}{T}\right) = u\left(t-t_0 + \frac{T}{2}\right) - u\left(t-t_0 - \frac{T}{2}\right)$$