Name ~~Exam 2 Solns~~ CM

1) Short Answer Questions (5 points each):

a) Is the system with impulse response $h(t) = e^{t}u(t)$ BIBO stable? Why or why not?

$\int_{-\infty}^{\infty} |h(t)|\,dt = \infty \quad \therefore$ not BIBO stable

b) Is the system $y(t) = \cos\left(\dfrac{1}{x(t)}\right)$ BIBO stable? Why or why not?

yes $|y(t)| \leq 1$ for all $x(t)$

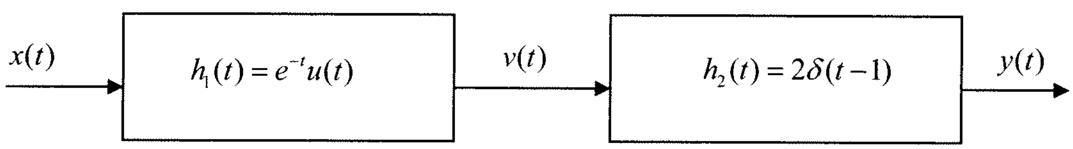
$t-1 > 2$

$t > 3$

c) What is the impulse response for the system $y(t) = e^{-t}\int_{-\infty}^{t-1} e^{\lambda}x(\lambda-2)\,d\lambda$? Be sure to include appropriate unit step functions.

$$h(t) = e^{-t}\int_{-\infty}^{t-1} e^{\lambda}\delta(\lambda-2)\,d\lambda = e^{-t}e^{2}u(t-3)$$

d) Consider the two LTI systems shown below, with impulse responses shown. What is the impulse response between $x(t)$ and $y(t)$?



$$h(t) = 2e^{-(t-1)}u(t-1)$$

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

e) Is the function $x(t) = \cos(4\pi t + \frac{\pi}{2}) + \sin(6\pi t)$ periodic? If yes, determine the fundamental period.

$4\pi T = g\,2\pi$

$6\pi T = r\,2\pi$

$T = \frac{g}{2} = \frac{r}{3}$ $\quad g = 2$, $r = 3$

$T = 1$

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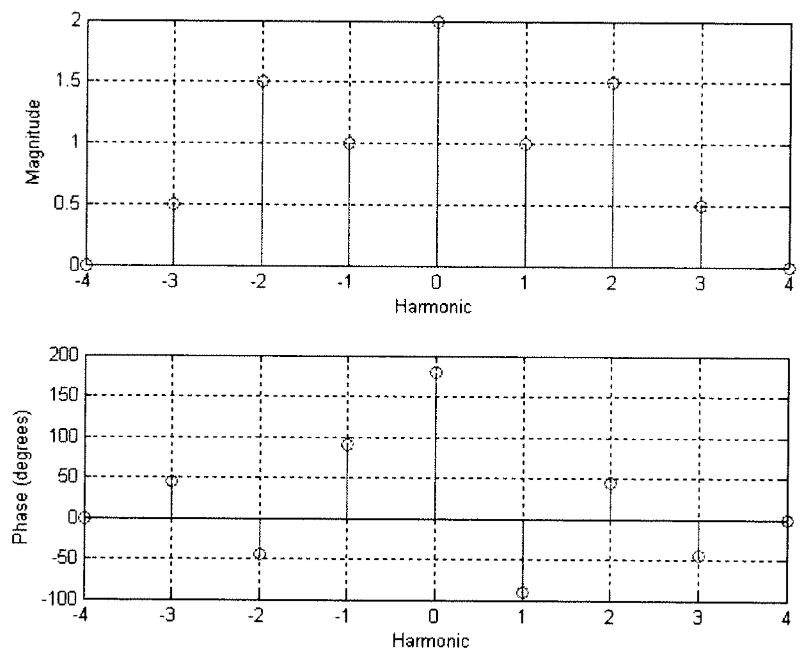
2) Assume periodic signal $x(t)$ has the spectrum shown below and a fundamental frequency $\omega_o = 3$ rad / sec . Assume all angles are multiples of 45 degrees.

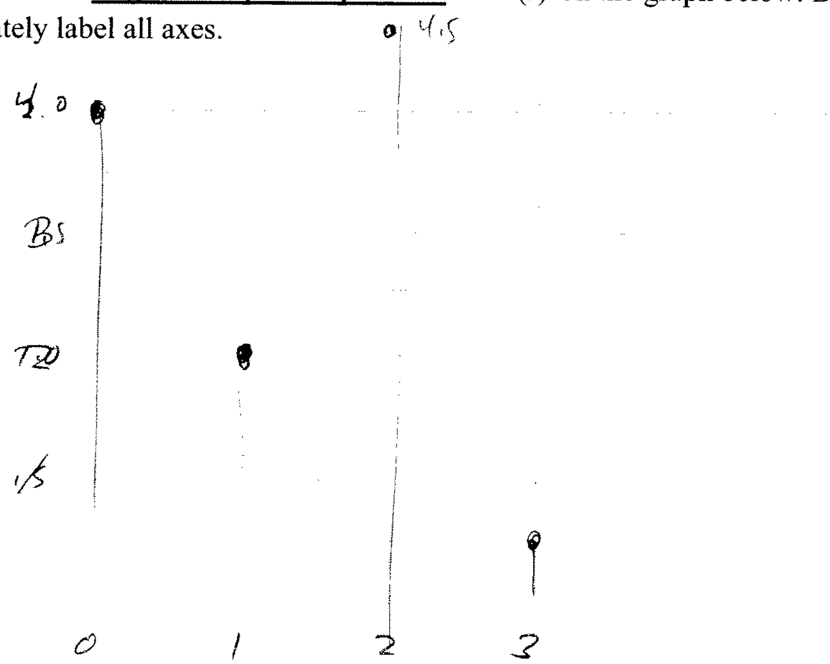


a) Determine the **average value** and **average power** in $x(t)$. $C_0 = -2$ $P_x = 0.5^2 + 1.5^2 + 1^2 + 2^2 + 1^2 + 1.5^2 + .5^2 = 11 = P_x$

c) Write and expression for $x(t)$ in terms of cosines (and/or sines).

$x(t) = -2 + 2\cos(3t - 90°) + 3\cos(6t + 45°) + \cos(9t - 45°)$

d) Sketch the ***single sided power spectrum*** for $x(t)$ on the graph below. Be sure to accurately label all axes.



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3) Assume periodic signal $x(t)$ has Fourier series representation

$$x(t) = \sum_{k=-\infty}^{k=\infty} \frac{jk}{1+jk} e^{jkt}$$

$x(t)$ is the input to an LTI system with transfer function given by

$$H(j\omega) = \begin{cases} \dfrac{1}{1+j\omega} & 1.5 < |\omega| < 2.5 \\ 0 & otherwise \end{cases}$$

only 2nd harmonic passes.

Determine the steady state output of the system, $y(t)$. For full credit your answer must be written in terms of cosines (and/or sines). Clearly indicate whether you are writing your phase in degrees or in radians (the phase of $H(j\omega)$ is in radians).

$\omega_0 = 1 \qquad K = 2$

$$X_2 = \frac{j2}{1+j2} = \frac{2\angle 90^\circ}{\sqrt{5}\angle 63^\circ} = 0.89\angle 27^\circ$$

$$H(j2) = \frac{1}{1+j2} = \frac{1\angle 0^\circ}{\sqrt{5}\angle 63^\circ} = 0.447\angle -63^\circ$$

$$Y_2 = (0.89\angle 27^\circ)(0.447\angle -63^\circ) = \cancel{0.4}\ 0.398\angle -36^\circ$$

$$\boxed{y(t) = 0.796\cos(2t - 36^\circ)}$$

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4) Assume $x(t)$ is a periodic signal with period $T_0 = 3$. $x(t)$ is defined over one period as

$$x(t) = \begin{cases} 1 & -1 < t \le 0 \\ 0 & 0 < t \le 2 \end{cases}$$

a) Determine the fundamental frequency ω_0.

b) Determine the average value of $x(t)$.

c) Determine the average power in the DC component of $x(t)$.

d) Determine an expression for the expansion coefficients,. X_k, where $x(t) = \sum_k X_k e^{jk\omega_0 t}$. You must write your expression in terms of the **sinc** function, and possibly a leading phase term.

(a) $\omega_0 = \frac{2\pi}{3}$

(b) $c_0 = \frac{1}{3}\int_{-1}^{0} 1\,dt = \frac{1}{3} \cdot 1 = \boxed{\frac{1}{3} = c_0}$

(c) $\boxed{P_0 = \frac{1}{9}}$

(d)

$$c_k = \frac{1}{3}\int_{-1}^{0} e^{-jk\omega_0 t}\,dt = \frac{1}{3}\,\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Bigg|_{-1}^{0} = \frac{1}{3}\left[\frac{1 - e^{jk\omega_0}}{-jk\omega_0}\right]$$

$$= \frac{e^{jk\frac{\omega_0}{2}}}{3}\left[\frac{e^{-jk\frac{\omega_0}{2}} - e^{jk\frac{\omega_0}{2}}}{-jk\omega_0}\right] = \frac{2}{3}\,\frac{e^{jk\frac{\omega_0}{2}}}{k\omega_0}\left[\frac{e^{jk\frac{\omega_0}{2}} - e^{-jk\frac{\omega_0}{2}}}{2j}\right]$$

$$= \frac{2}{3}\,\frac{e^{jk\frac{\omega_0}{2}}}{k\omega_0}\sin\left(\frac{k\omega_0}{2}\right) = \frac{\cancel{2}}{3}\,\frac{e^{jk\pi/3}\sin\left(k\frac{\pi}{3}\right)}{k\frac{\cancel{2}\pi}{3}}$$

$$= \frac{e^{jk\pi/3}\sin\left(\pi\frac{k}{3}\right)}{3\;\pi\frac{k}{3}} = \boxed{\frac{1}{3}e^{jk\pi/3}\,\mathrm{sinc}\left(\frac{k}{3}\right) = c_k}$$