Name Exam #1 Fall 2007 CM____________

Problems 1-4 are worth 4 points each.

1. Which of the following statements is the best simplification of: $\int_{-2}^{t} x(\lambda - t_0)\delta(\lambda)d\lambda$

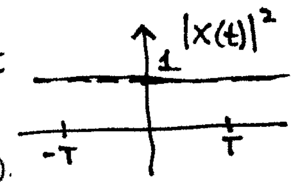
a) 0 b) $x(t-t_0)\delta(t)$ c) $x(-t_0)u(t)$ d) $x(-t_0)\delta(t)$ e) none of these

$x(\lambda-t_0)\delta(\lambda) = x(-t_0)\delta(\lambda)$

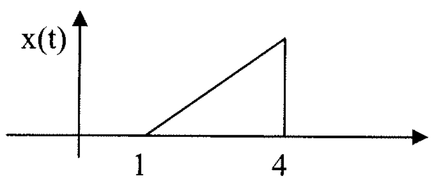
2. The average power in the signal $x(t) = u(t) - u(-t)$ is

a) 0 b) $\frac{1}{2}$ c) 1 d) ∞ e) none of these

$P_{ave} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$



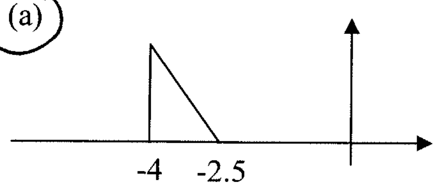
3. Given x(t) below, which of the plots labeled (a) – (d) represents $x(2(-t-2))$.



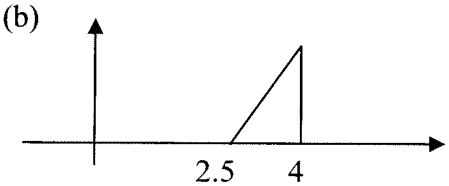
$2(-t-2) = 4 \rightarrow t = -4$

$2(-t-2) = 1 \rightarrow t = -2.5$

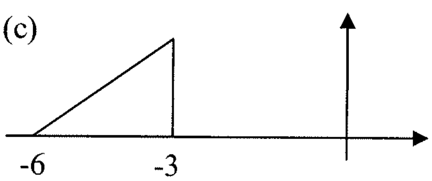
(a)



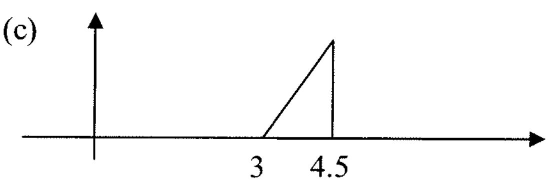
(b)



(c)



(c)



4. The signal $x(t) = \cos(4\pi t + \pi/2) + \sin(6\pi t)$ is

a) not periodic

b) periodic with fundamental period 6π seconds

c) periodic with fundamental period 1 second

d) periodic with fundamental period 3/2 seconds

e) none of the above

$4\pi T = 2\pi r$

$6\pi T = 2\pi q$

$\frac{2\pi r}{4\pi} = \frac{2\pi q}{6\pi}$

$\frac{r}{q} = \frac{2}{3}$

$T = \frac{2\pi(2)}{4\pi} = 1 = \frac{2\pi(3)}{6\pi}$

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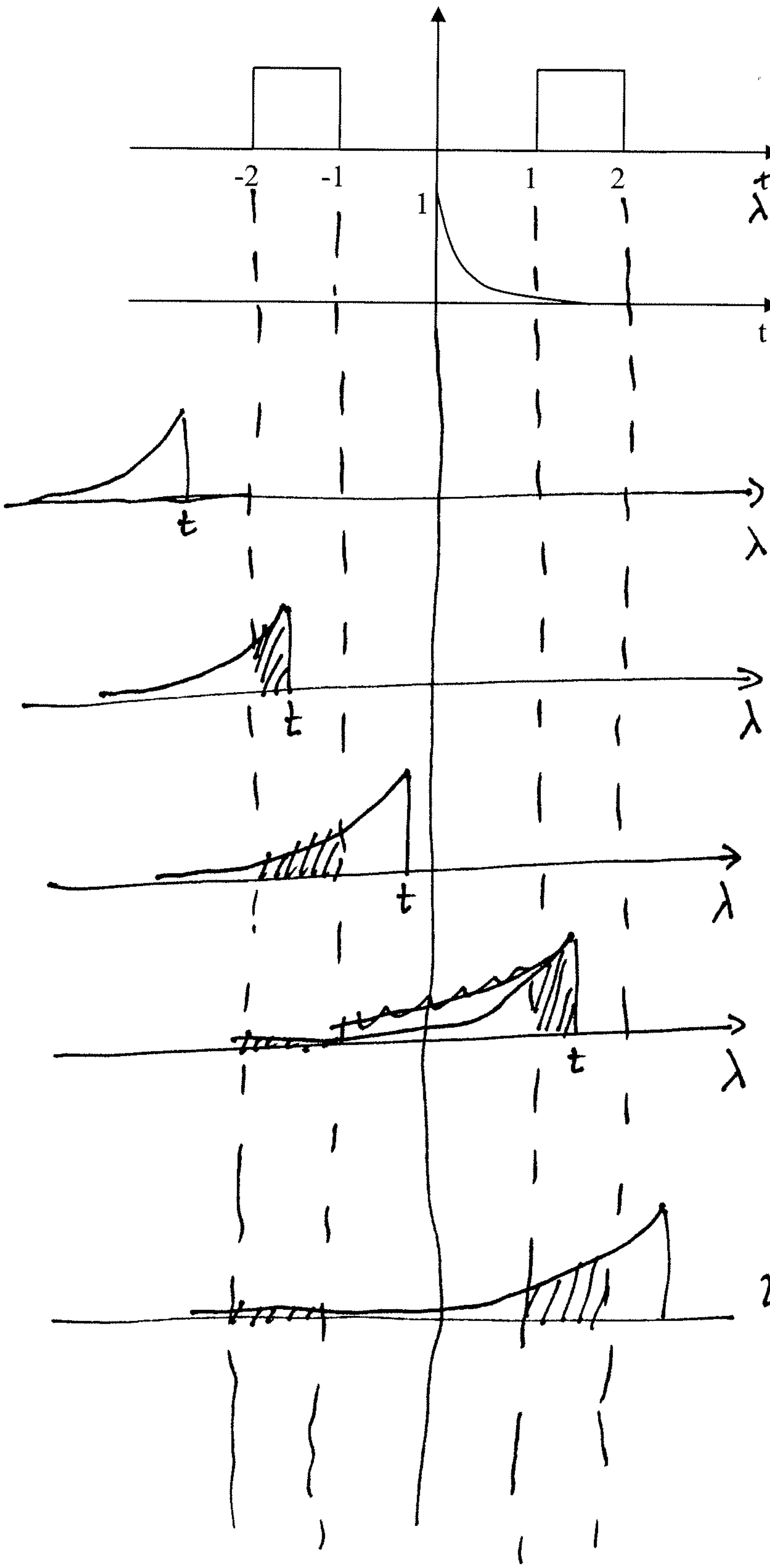
5. Graphical Convolution (29 points)
Use graphical convolution to determine the intervals of integration and their corresponding integrals $y(t) = x(t) * h(t)$ as shown in the plots below. Use x(t) as the signal to "flip and shift" (i.e. x(t-λ)) for the convolution. **DO NOT solve the integrals, just set them up.**



$h(t) = u(t+2) - u(t+1) +$

$x(t) = e^{-2t}u(t)$

$x(t-\lambda) = e^{-2(t-\lambda)}u(t-\lambda)$

$t < -2 \qquad y(t) = 0$

$-2 < t < -1 \qquad y(t) = \int_{-2}^{t} e^{-2(t-\lambda)}\, d\lambda$

$-1 < t < 1 \qquad y(t) = \int_{-2}^{-1} e^{-2(t-\lambda)}\, d\lambda$

$1 < t < 2 \qquad y(t) = \int_{-2}^{-1} e^{-2(t-\lambda)}\, d\lambda + \int_{1}^{t} e^{-2(t-\lambda)}\, d\lambda$

$2 < t \qquad y(t) = \int_{1}^{2} e^{-2(t-\lambda)}\, d\lambda + \int_{-2}^{-1} e^{-2(t-\lambda)}\, d\lambda$

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6. Impulse Response (25 points)

For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions.*

a) $y(t) = x(t) + 2x(t-2)$ $\boxed{h(t) = \delta(t) + 2\,\delta(t-2)}$

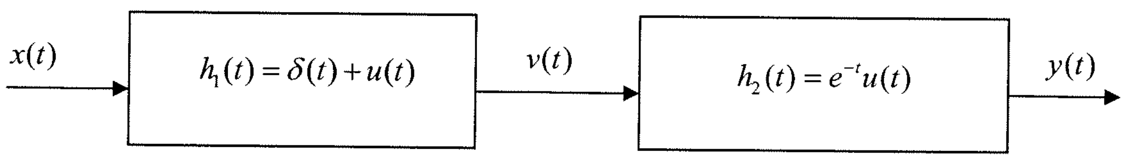
b) $\dot{y}(t) - y(t) = 2x(t)$

$$\frac{d}{dt}\left(he^{-2t}\right) = e^{-2t}\delta(t) = \delta(t)$$

$$\int_{-\infty}^{t} \frac{d}{d\lambda}\left(h(\lambda)e^{-2\lambda}\right)d\lambda = h(t)e^{-2t} = \int_{-\infty}^{t} \delta(\lambda)d\lambda = u(t)$$

$$\boxed{h(t) = e^{2t}u(t)}$$

c) For the following system, with the impulse responses of each subsystem shown,



Determine the impulse response of the **system** (relating $y(t)$ and $x(t)$).

$$h(t) = h_1(t) * h_2(t)$$

$$= \int_{-\infty}^{\infty} h_1(t-\lambda)\,h_2(\lambda)\,d\lambda$$

$$= \int_{-\infty}^{\infty} \delta(t-\lambda)e^{-\lambda}u(\lambda)\,d\lambda + \int_{-\infty}^{\infty} u(t-\lambda)\,e^{-\lambda}u(\lambda)\,d\lambda$$

$$= e^{-t}u(t) + \int_{0}^{t} e^{-\lambda}d\lambda$$

$$\boxed{\begin{aligned} h(t) &= e^{-t}u(t) + \left[1 - e^{-t}\right]u(t) \\ &= u(t) \end{aligned}}$$

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7. System Properties (25 points)

a) Fill in the following table with a Y (Yes) or N (No). Only your responses in the table will be graded, not any work. Assume $x(t)$ is the system input and $y(t)$ is the system output. Also assume we are looking at all times (positive and negative times).

System	Linear ?	Time-Invariant?	Memoryless?	Causal?
$\dot{y}(t)+t^2y(t)=x(t+1)$	Y	N	N	N
$y(t)=x\left(1-\frac{t}{2}\right)$	Y	N	N	N
$y(t)=2$	N	Y	Y	Y
$y(t)=x(2t)$	Y	N	N	N

b) For the system described below, determine the value of "c" that will make the system time-invariant. Use a formal technique such as we used in class (and on the homework) and justify your answer.

$$y(t)=e^{t}\int_{c}^{t}e^{-\lambda}x(\lambda)d\lambda$$

$$z_1 = H\{x(t-t_o)\} = e^{t}\int_{c}^{t} e^{-\lambda} x(\lambda - t_o)d\lambda$$

$$z_2 = H\{x(t)\}\Big|_{t=t-t_o} = e^{t-t_o}\int_{c}^{t-t_o} e^{-\lambda} x(\lambda)\, d\lambda$$

for time-invariance we need $z_1 = z_2$

in z_1, let $\sigma = \lambda - t_o$ or $\lambda = \sigma + t_o$, $d\lambda = d\sigma$

$$z_1 = e^{t}\int_{c-t_o}^{t-t_o} e^{-\sigma - t_o} x(\sigma)d\sigma = e^{t-t_o}\int_{c-t_o}^{t-t_o} e^{-\sigma} x(\sigma)d\sigma$$

$z_1 = z_2$ if $c = c - t_o$ this is only true for $c = \pm\infty$

we need $\boxed{c = -\infty}$