ROSE-HULMAN INSTITUTE OF TECHNOLOGY
Department of Electrical and Computer Engineering

ECE 300 Signals and Systems   Fall 2007

Measurement of Fourier Coefficients
Lab 05
by Bruce A. Black with some tweaking by others

Objectives

- To measure the Fourier coefficients of several waveforms and compare the measured values with theoretical values.

- To become acquainted with the Agilent E4402B Spectrum Analyzer.

Equipment
Agilent E4402B Spectrum Analyzer   Oscilloscope
Agilent Function Generator   BNC T-Connector

Background
Recently we learned to calculate the spectrum of a periodic signal by using the Fourier series. We have in our lab *spectrum analyzers* that can display the spectrum of a signal in pseudo- real time. The Agilent E4402B Spectrum Analyzer (SA) can be used to view the *power spectrum* of any signal of frequency up to 3 GHz. The SA displays a "one-sided spectrum" in decibels (dBs) versus frequency. In lab we will observe the spectra of sinusoids, square and triangle waves, and pulse trains, but first we must learn how to convert the Fourier series coefficients that we calculate to the dB values displayed by the spectrum analyzer.

Recall that any periodic signal $x(t)$ can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}, \text{ where } f_0 = 1/T_0 \ .$$

Writing out a few terms gives

$$
\begin{aligned}
x(t) &= \cdots + a_{-2}e^{-j2\pi 2 f_0 t} + a_{-1}e^{-j2\pi f_0 t} + a_0 + a_1 e^{j2\pi f_0 t} + a_2 e^{j2\pi 2 f_0 t} + \cdots \\
&= \cdots + |a_2|e^{-j\angle a_2}e^{-j2\pi 2 f_0 t} + |a_1|e^{-j\angle a_1}e^{-j2\pi f_0 t} + a_0 + |a_1|e^{j\angle a_1}e^{j2\pi f_0 t} + |a_2|e^{j\angle a_2}e^{j2\pi 2 f_0 t} + \cdots
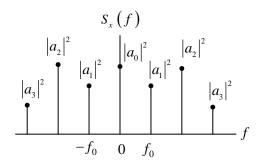\end{aligned}
\tag{1}
$$

where we have used the fact that $a_{-k} = a_k^*$ whenever $x(t)$ is real-valued. Notice that, aside from a_0, the terms come in pairs (actually, complex conjugate pairs). We can combine each positive-frequency term with its matching negative frequency term to obtain

$$x(t) = a_0 + 2|a_1|\cos(2\pi f_0 t + \angle a_1) + 2|a_2|\cos(2\pi 2 f_0 t + \angle a_2) + \cdots . \tag{2}$$

A power spectrum for $x(t)$ based on the Fourier series is shown in Fig. 1. This is a *two-sided* spectrum, in which the power associated with the complex exponential at frequency $k f_0$ is seen

to be $\left|a_k\right|^2$ (and corresponds most directly with equation (1)). The corresponding *one-sided* power spectrum is shown in Fig. 2. To make the one-sided spectrum, the powers associated with complex exponentials at frequencies kf_0 and $-kf_0$ are added. The result, representing the average power in the sinusoid $2\left|a_k\right|\cos\left(2\pi f_k t + \measuredangle a_k\right)$, is shown at frequency kf_0 (and corresponds most directly with equation (2)).
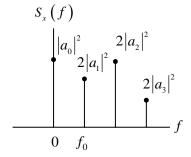

Figure 1: Two-Sided Power Spectrum   Figure 2: One-Sided Power Spectrum

The SA displays a one-sided spectrum as shown in Fig. 2, but instead of showing the value of $2\left|a_k\right|^2$ at each frequency, the spectrum analyzer shows average power in decibels with respect to a one millivolt RMS reference. For the sinusoid at frequency kf_0, the average power in decibels is given by

$$P_k\big|_{dB} = 10\log_{10}\frac{P_k}{P_{ref}},$$

where the power P_k represents the power spectrum coefficient $2\left|a_k\right|^2$, and the power P_{ref} is the average power delivered to a one-ohm resistor by a one millivolt RMS sinusoid. We have

$$P_k\big|_{dBmV} = 10\log\frac{2\left|a_k\right|^2}{\left(0.001\right)^2}\ \text{dBmV}.$$

The units "dBmV" indicate that the reference for the decibels is a one millivolt RMS sinusoid.*

<u>Note:</u> The spectrum analyzer will not display the DC term $\left|a_0\right|^2$ even when one is present in the signal. Instead it displays a large spike at zero frequency allowing for easy location of DC on the display. Also, because it is showing a power spectrum, the spectrum analyzer does not measure or display the phase angles $\measuredangle a_k$.

* Further information on working with dBs is available in the document called "Guide to dBs" available on the class webpage. It is suggested you read this before lab.
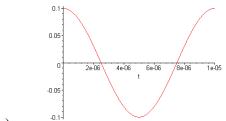
Procedure

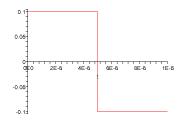Read the documents "SA_hints_E4402B" and "Reading_SA_Display_E4402B", both available on the class webpage.

The waveforms we will be analyzing, each having zero DC offset, are:

a) $x_1(t) = 0.1\cos(2\pi 100 \times 10^3 t)$ V

b) $x_2(t)$ is a square wave of period 10 μs and peak-to-peak amplitude 0.2 V.

c) $x_3(t)$ is a triangle wave of period 10 μs and peak-to-peak amplitude 0.2 V.

Just to be sure there is no confusion regarding the waveforms, they are displayed below.

a)

b)

c)

Getting Ready

Before connecting any input to the spectrum analyzer, be sure that there is no large DC offset on the signal – in our case, no DC offset should be contained in the signal. The only sure way to know this is to properly examine the signal on the oscilloscope. Be sure you understand the implications of the different input impedance of the scope and the spectrum analyzer before you begin this lab.** Summarize the proper measurement procedure on the final page of the lab.

Calibrate the Spectrum Analyzer

Calibrate the spectrum analyzer as described in the document **SA_hints_E4402B**.

Measuring the Spectrum

1. Use the function generator to generate a sinusoid of frequency 100 kHz and (open circuit) amplitude 0.1 V (waveform a). Use the oscilloscope to verify the amplitude**. Now observe the signal power spectrum on the spectrum analyzer. Measure the power level and frequency. Record your measurements in the table on the final page of this lab.

2.   Informally vary the frequency and the amplitude of the sinusoid and observe how the spectrum analyzer display changes.

3. Use the function generator to generate a square wave of period 10 μs and peak-to-peak amplitude 0.2 V (waveform b). Using the spectrum analyzer, measure the level of the first nine harmonics. Record your measurements in the table on the final page of this lab.

4. Use the function generator to generate a triangle wave of period 10 μs and peak-to-peak amplitude 0.2 V (waveform c). Use the spectrum analyzer to measure the level of the first nine harmonics. Record your measurements in the table on the final page of this lab.

** **Note** concerning Agilent FG amplitude readings.

The Agilent function generators have an interesting feature built into their displays. The FG is a 50 Ω output impedance device, designed to deliver maximum power to a 50 Ω load. The FG by default will display the voltage amplitude delivered to a 50 Ω load, independent of what is actually connected. By changing the display setting to "High-Z", the FG display will display the voltage delivered to a high impedance load, assuming a high impedance load is connected. However, the FG remains a 50 Ω output impedance device. Since the scope and spectrum analyzer have different input impedances, caution must be used in reading the FG display.

Measurement of Fourier Coefficients-Instructor Verification Sheet

Name _____ Date of Lab: _____

Record your procedure for using the spectrum analyzer in the space below. How should the output of the signal generator be set? Is the signal measured on the oscilloscope the size predicted by the signal generator?

Summary of Results for Waveform 1

Harmonic	Prelab Predicted Power (dBmV)	Measured Power (dBmV)	Error (dBmV)	Frequency (kHz)
1	37.0			

Summary of Results for Waveform 2

Harmonic	Prelab Predicted Power (dBmV)	Measured Power (dBmV)	Error (dBmV)	Frequency (kHz)
1	39.1			
2	---			
3	29.5			
4	---			
5	25.1			
6	---			
7	22.2			
8	---			
9	20.0			

Summary of Results for Waveform 3

Harmonic	Prelab Predicted Power (dBmV)	Measured Power (dBmV)	Error (dBmV)	Frequency (kHz)
1	35.1			
2	---			
3	16.1			
4	---			
5	7.2			
6	---			
7	1.4			
8	---			
9	-2.9			