ECE 300
Signals and Systems
Homework 9

 Due Date: _FRIDAY_ November 3 at 2:30 PM **Note: Exam 3 Tuesday November 7**

**Note**: Use the Fourier transform table given out in class. Also, you do not need to plot the figures in the problems where the text problem says to plot the figure.

Problems

1. Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t) = 5\operatorname{sinc}\left(\frac{t}{0.002}\right) + 5\operatorname{sinc}\left(\frac{t}{0.001}\right)$$

2. K & H, Problem 3.21 (**a,b,c** only)

3. K & H, Problem 3.24

4. K & H, Problem 5.14

5. K & H, Problem 5.16 (**a, b, c** only)

6. K & H, Problem 5.19 (For part **c** you should get $y(t) = 6x(t) - 2$)

7. (**Matlab Problem**, but you are also responsible for this material) A Butterworth filter has the property that it is maximally flat in the _passband_. An nth order Butterworth filter has the magnitude squared response

$$|H(\omega)|^2 = \frac{1}{1 + \left(\dfrac{\omega}{\omega_p}\right)^{2n}}$$

where ω_p is the _passband frequency._ At this frequency the power has been reduced by one half or 3 dB,

$$|H(\omega_p)|^2 = \frac{1}{1 + \left(\dfrac{\omega_p}{\omega_p}\right)^{2n}} = \frac{1}{2} \quad \text{or} 10\log_{10}|H(\omega_p)|^2 = 10\log_{10}\left(\frac{1}{2}\right) = -3dB$$

To determine the required order of a filter we often look at the desired _stopband frequency_, ω_s . Usually we want to indicate the minimum required power

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difference between the passband and the stopband, Δ. Δ is the *rejection.* Hence we have

$$\Delta = 20\log_{10}|H(0)| - 20\log_{10}|H(\omega_p)|$$

or

$$\Delta = -10\log_{10}\cfrac{1}{1+\left(\cfrac{\omega_s}{\omega_p}\right)^{2n}}$$

The ratio $\dfrac{\omega_s}{\omega_p}$ is called the *transition ratio.*

a) Show that we can write

$$n = \frac{\ln\left(10^{\frac{\Delta}{10}}-1\right)}{2\ln\left(\dfrac{\omega_s}{\omega_p}\right)}$$

Note that _n must be an integer_, so we always round up (to the next larger integer).

b) For $\omega_p = 10$ rad/sec, $\omega_s = 20$ rad/sec, and $\Delta = 18$ dB, determine the required Butterworth filter order for this filter. (Remember it must be an integer). Using the Table at the end of this problem, plot the Bode plot of your Butterworth filter and verify that all frequencies $\omega > \omega_s$ have   magnitude (power) less than Δ_{max}.
Matlab's **tf** command and **Bode**   commands will be really useful here. Note that you can click on the curve on the Bode plot to read it more accurately. Turn in your plot.

c) Matlab's command **r = pole(H)**, where H is the transfer function of the Butterworth filter, returns the poles of the transfer function in the array **r**. Using Matlab's commands **abs** and **angle,** relate the magnitude of the poles to ω_p, then plot the pole locations in the complex plane on a circle with radius ω_p. (Note that **angle** returns angles in radians, and you   probably want angle in degrees. You might also look at the function **pzplot**.) Note that the pole locations are all separated by an angle θ. What is this angle?

d) For $\omega_p = 15$ rad/sec, $\omega_s = 35$ rad/sec, and $\Delta = 28$ dB, determine the required Butterworth filter order for this filter. (Remember it must be an integer). Using the Table at the end of this problem, plot the Bode plot of your Butterworth filter and verify that all frequencies $\omega > \omega_s$ have magnitude (power) less than Δ_{max}.

Matlab's **tf** command and **Bode** commands will be really useful here. Turn in your plot.

e) Matlab's command **r = pole(H)**, where H is the transfer function of the Butterworth filter, returns the poles of the transfer function in the array **r**. Using Matlab's commands **abs** and **angle,** relate the magnitude of the poles to ω_p, then plot the pole locations in the complex plane on a circle with radius ω_p. (Note that **angle** returns angles in radians, and you probably want angle in degrees.) Note that the pole locations are all separated by an angle θ. What is this angle?

n	denominator(s)
1	$\dfrac{s}{\omega_p}+1$
2	$\left(\dfrac{s}{\omega_p}\right)^2+1.414\left(\dfrac{s}{\omega_p}\right)+1$
3	$\left(\dfrac{s}{\omega_p}\right)^3+2\left(\dfrac{s}{\omega_p}\right)^2+2\left(\dfrac{s}{\omega_p}\right)+1$
4	$\left(\dfrac{s}{\omega_p}\right)^4+2.6131\left(\dfrac{s}{\omega_p}\right)^3+3.4142\left(\dfrac{s}{\omega_p}\right)^2+2.6131\left(\dfrac{s}{\omega_p}\right)+1$
5	$\left(\dfrac{s}{\omega_p}\right)^5+3.2361\left(\dfrac{s}{\omega_p}\right)^4+5.2361\left(\dfrac{s}{\omega_p}\right)^3+5.2361\left(\dfrac{s}{\omega_p}\right)^2+3.2361\left(\dfrac{s}{\omega_p}\right)+1$

Table 1: Denominators of Buterworth filter for filter orders 1-5. The numerator for the Butterworth filter is 1.

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