ECE 300
Signals and Systems
Homework 5

<u>Due Date:</u> Tuesday October 3 at 2:30 PM

<u>Problems:</u>

1. Simplify each of the following into the form $c_k = \alpha(k) e^{-j\beta(k)} \mathrm{sinc}(\lambda k)$

a) $c_k = \dfrac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

b) $c_k = \dfrac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

c) $c_k = \dfrac{e^{j5k} - e^{j2k}}{k}$

Scrambled Answers $c_k = 3\pi e^{-j\frac{7\pi k}{2}} \mathrm{sinc}\left(\frac{3\mathrm{k}}{2}\right)$, $c_k = 3e^{j(\frac{7}{2}k + \frac{\pi}{2})} \mathrm{sinc}\left(\frac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi} \mathrm{sinc}\left(k\frac{9}{2}\right)$

2. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of ω_0 and the c_k. *Hints:* (1) *Draw the signal, and then use the sifting property to calculate the* c_k. (2) *If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - 3p)$$

3. For the periodic square wave $x(t)$ with period $T_o = 0.5$ and

$$x(t) \begin{cases} 1 & 0 \le t < 0.25 \\ -1 & 0.25 \le t < 0.5 \end{cases}$$

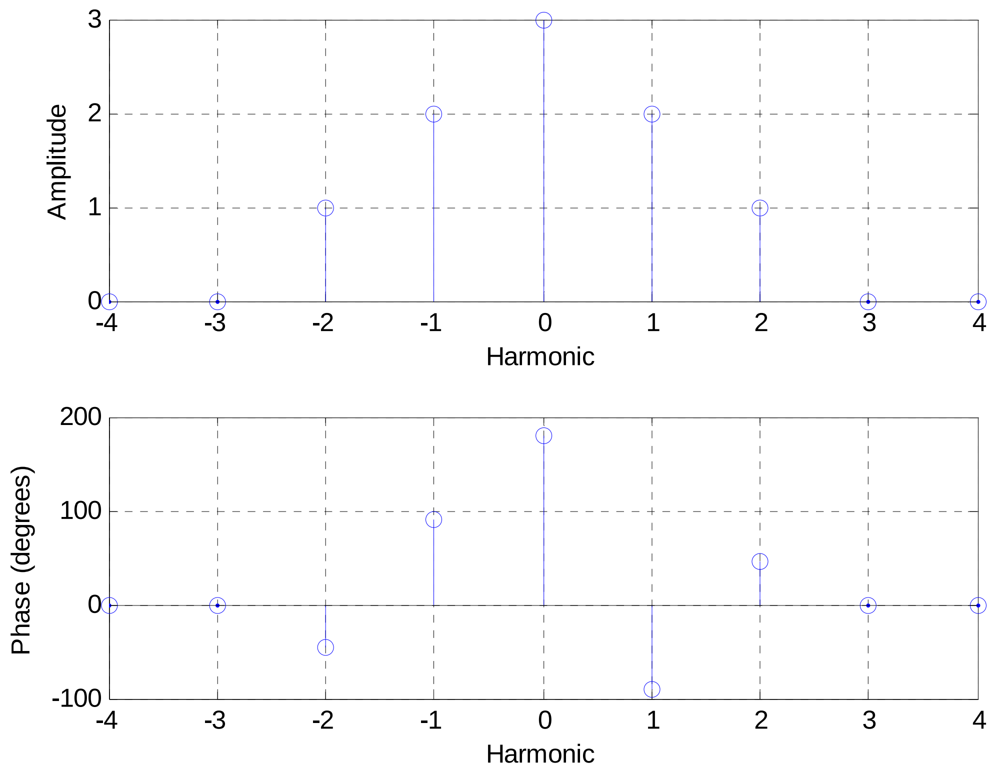
show that the Fourier series coefficients are given by

$$c_k = \begin{cases} \dfrac{-2j}{k\pi} & k \quad odd \\ 0 & k \quad even \end{cases}$$

where $x(t) = \sum_k c_k e^{jk4\pi t}$

4 K & H, Problem 3.13. For part **c** you should get $c_k^v = c_{k-1}^x$, use Euler's identity for part **d**.

5 A signal $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45 degrees)

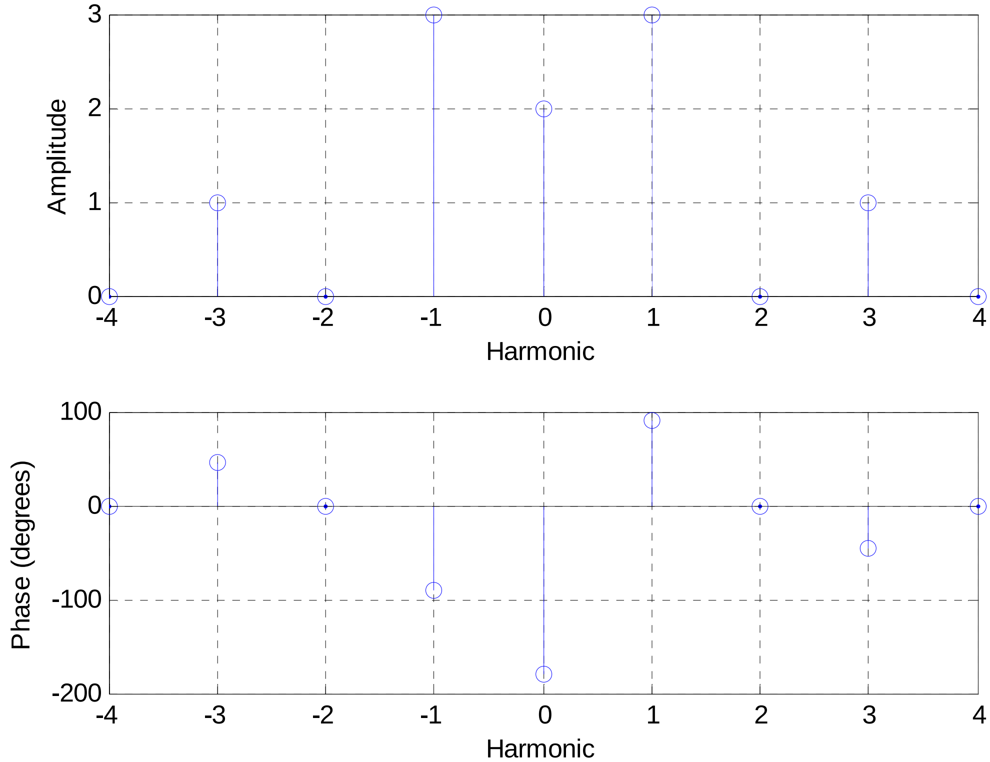


a) What is $x(t)$? Your expression must be real.

b) What is the average value of $x(t)$?

c) What is the average power in $x(t)$? (See the Appendix)

6. A signal $x(t)$, which has a fundamental period of 3 seconds, has the following spectrum (all phases are multiples of 45 degrees)



a) What is $x(t)$? Your expression must be real.

b) What is the average value of $x(t)$?

c) What is the average power in $x(t)$? (See the Appendix)

Special Note: We will be using the code you write in the next part for the next few homeworks and labs, so be sure you do this and understand what is going on!

7. **Pre-Lab/Matlab Exercises (to be done by all students. Turn this in with your homework and bring a copy of this with you to lab! Yes, it counts towards both grades!)**

Read the **Appendix** and then do the following:

a) Copy the file **Trigonometric_Fourier_Series.m** to file **Complex_Fourier_Series.m**.

b) Modify **Complex_Fourier_Series.m** so it computes the average value c_o

c) Modify **Complex_Fourier_Series.m** so it also computes c_k for $k=1$ to $k=N$

d) Modify **Complex_Fourier_Series.m** so it also computes the Fourier series estimate using the formula

$$x(t) \approx c_o + \sum_{k=1}^{N} 2\,|c_k|\cos(k\omega_o t + \measuredangle c_k)$$

You will probably need to use the Matlab functions **abs** and **angle** for this.

e) Using the code you wrote in part **d,** find the complex Fourier series representation for the following functions (defined over a single period)

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

These are the same functions you used for the trigonometric Fourier series. Use N = 10 and turn in your plots for each of these functions. Also, turn in your Matlab program for one of these. *Note that the values of **low** and **high** will be different for each of these functions!*

f) Show that the average power in each of the periodic signals ($f_1(t)$, $f_2(t)$, and $f_3(t)$) in **e** is 0.166, 2.917, and 2.000, respectively.

g) Compute the average power in a signal using the Fourier series terms (c_0 and the array $c = [c_1 \quad c_2 \quad \ldots \quad c_N]$). Matlab's built-in functions that may be helpful are **abs**, **sum**, and **.^**. ***<u>You are not to use any loops for this part</u>***. Modify the title of the graph in your code to print out the average power, based on the Fourier series terms. You need to use the function **num2str** in the title (as is done in the code for printing N). ***<u>Do not hard code the value for the power</u>***. If you use N = 5 for the functions in **e**, you should get average powers of 0.158, 2.820, and 1.886, respectively. Run your programs for $N=5$ for each of the three functions to verify you code is working correctly. Turn in your plots and your code. Note that as the number of terms increases, the average power in the Fourier series representation approaches that of the time domain representation of the function.

Appendix

In the majority of this course we will be using the complex (or exponential) form of the Fourier series, since it is really easier to do various mathematical things with it once you get used to it.

__Exponential Fourier Series__ If $x(t)$ is a periodic function with fundamental period T, then we can represent $x(t)$ as a Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}$$

where $\omega_o = \frac{2\pi}{T}$ is the fundamental period, $\mathrm{c_o}$ is the average (or DC, i.e. zero frequency) value, and

$$\mathrm{c_o} = \frac{1}{T}\int_0^T x(t)dt$$

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt$$

If $x(t)$ is a real function, then we have the relationships $|c_k| = |c_{-k}|$ (the magnitude is even) and $\measuredangle c_{-k} = -\measuredangle c_k$ (the phase is odd). Using these relationships we can then write

$$x(t) = c_o + \sum_{k=1}^{\infty} 2|c_k|\cos(k\omega_o t + \measuredangle c_k)$$

This is usually a much easier form to deal with, since it lends itself easily to thinking of a phasor representation of $x(t)$. This will be particularly useful when we starting filtering periodic signals.

Average Power of a Periodic Signal The average power in a periodic signal is defined as $P_{ave} = \frac{1}{T}\int_T |x(t)|^2 dt$ where T is the fundamental period of $x(t)$. Parseval's Theorem actually tells us that the average power in a signal is the same whether we utilize a time domain representation or a frequency representation, that is

$$P_{ave} = \frac{1}{T}\int_T |x(t)|^2 \, dt = |c_0|^2 + 2\sum_{k=1}^{\infty}|c_k|^2$$

Note that we must use all of the terms in the summation for the two sides to be exactly equal.