

**ECE 300**  
**Signals and Systems**  
 Homework 5

**Due Date:** Thursday October 6 at 1 PM

**Reading:** K & H, pp. 145-161.

**Problems:**

1. Simplify each of the following into the form  $c_k = \alpha(k)e^{-j\beta(k)}\text{sinc}(\lambda k)$

$$\text{a) } c_k = \frac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$$

$$\text{b) } c_k = \frac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$$

$$\text{c) } c_k = \frac{e^{j5k} - e^{j2k}}{k}$$

*Scrambled Answers*  $c_k = 3\pi e^{-\frac{7\pi k}{2}} \text{sinc}\left(\frac{3k}{2}\right), c_k = 3e^{j\left(\frac{7}{2}k + \frac{\pi}{2}\right)} \text{sinc}\left(\frac{3k}{2}\right),$

$$c_k = 9e^{j\frac{5}{2}k\pi} \text{sinc}\left(k\frac{9}{2}\right)$$

2. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of  $\omega_0$  and the  $c_k$ . Hint: Draw the signal, and then use the sifting property to calculate the  $c_k$ . *Hint: If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t-3p)$$

3. For the periodic square wave  $x(t)$  with period  $T_o = 0.5$  and

$$x(t) \begin{cases} 1 & 0 \leq t < 0.25 \\ -1 & 0.25 \leq t < 0.5 \end{cases}$$

show that the Fourier series coefficients are given by

$$c_k = \begin{cases} \frac{-2j}{k\pi} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

where  $x(t) = \sum_k c_k e^{jk4\pi t}$

4. Assume periodic function  $x(t)$  has the Fourier series representation

$$x(t) = 1 + \sum_{k=-\infty}^{k=\infty} \frac{2}{1-jk} e^{j3kt}$$

a) What is the period  $T_o$  of  $x(t)$ ?

b) What is the average (DC) value of  $x(t)$ ?

5. K & H, Problem 4.9. For part **c** you should get  $c_k^v = c_{k-1}^x$ , use Euler's identity for part **d**.

6. K & H, Problem 4.12 parts **a** and **b** only. Write the integral as the sum of two integrals (with zero as the midpoint). Change variables to make the limits on the integrals the same.

7. Show that any function  $x(t)$  can be written in terms of an even function and an odd function, i.e.  $x(t) = x_e(t) + x_o(t)$ , where  $x_e(t)$  is an even function, and  $x_o(t)$  is an odd function. Determine expressions for  $x_e(t)$  and  $x_o(t)$  in terms of  $x(t)$  (if you can do this then you have shown that  $x(t) = x_e(t) + x_o(t)$ ).