ECE 300
Signals and Systems
Homework 4

Due Date: Thursday September 29 at 1 PM ***Exam #1, Monday September 26***

Reading: K & H, pp. 145-161.

Problems

1. In this problem you will utilize the Matlab program **Fourier_Series.m** on the class website (download by ***right clicking***, ***select save target as***, and ***saving as a text document***). You are not expected to understand a lot of how this works (it used Matlab's built-in functions to numerically compute the Fourier series coefficients and evaluate the function, plus some other voo-doo to compute the Fourier series.) The arguments to this function are the initial and final times of a single period (the period starts at **Tlow** and ends at **Thigh**) and **N**, the number of terms to use (in addition to the average value term).

a. The program **Fourier_Series.m** initially determines the Fourier series representation of

$$f(t) = \begin{cases} 0.135e^{t} & 0 \le t < 2 \\ 1 & 2 \le t < 3 \\ 4 - t & 3 \le t < 4 \\ 0 & 4 \le t < 5 \end{cases}$$

The function **fcn** currently contains the representation of $f(t)$.You should examine the code to see how this function was represented in Matlab. This function has a period of 5. *You should type Fourier_Series(0,5,10) to see the Fourier series approximation to this function using 10 terms.*

Modify **Fourier_Series.m** to plot both the analytical function and the Fourier series approximation for the following functions.

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

You only need to modify the function **fcn.** Once you have determined how to represent each of these functions in Matlab, comment them out (put a % at the beginning of the line, or ***highlight*** them, ***select text***, and then ***select comment)*** before going on to the next function. We will be using these in what follows. Run the program and compare the plots of the true signals and the Fourier series representation using $N = 50$ terms to verify that everything is working Ok. **Turn in you plots.**

Note: You may want to move the legend around on the figure to get good plots. See the Matlab's **legend** function to see how to move the legend around. As an alternarive, you can often just drag the legend where you want it to be.

b. The average power in a periodic signal is defined as $P_{ave} = \frac{1}{T}\int_T |x(t)|^2 dt$ where T is the fundamental period of $x(t)$. Show that the average power in each of the periodic signals ($f_1(t)$, $f_2(t)$, and $f_3(t)$) in **a** is 0.166, 2.917, and 2.000, respectively.

c. We can also compute the **average power** in the Fourier series representation of a signal as

$$P_{ave} = |c_0|^2 + 2\sum_{k=1}^{N} |c_k|^2$$

You are to write a function (add it to the end of **Fourier_Series.m**) that computes the average power in a signal. The input to the function will be c_0 and the array $c = [c_1 c_2 ... c_N]$. The output will be the average power. Matlab's built-in functions that may be helpful are **abs**, **sum**, and **.^**. ***<u>You are not to use any loops</u>***. You need to modify the title of the graph to print out the average power. You need to use the function **num2str** in the title (as was done for printing N). ***<u>Do not hard code the value for the power</u>***. If you use N = 5 for the functions in **a**, you should get average powers of 0.158, 2.820, and 1.886, respectively. Run your programs for $N = 5$ to verify all if working correctly. You do not need to turn anything in for this part.

d. **<u>Parseval's Theorem</u>** actually tells us that the average power in a signal is the same whether we utilize a time domain representation or a frequency representation, that is

$$P_{ave} = \frac{1}{T}\int_T |x(t)|^2 \, dt = |c_0|^2 + 2\sum_{k=1}^{\infty} |c_k|^2$$

Note that we must use all of the terms in the summation for the two sides to be exact. For each of the periodic signals in **a**, utilize **Fourier_Series.m** to determine the smallest number of terms N we need to use to before

$$|c_0|^2 + 2\sum_{k=1}^{N} |c_k|^2 \geq 0.99 P_{ave}$$

and

$$|c_0|^2 + 2\sum_{k=1}^{N} |c_k|^2 \geq 0.90 P_{ave}$$

i.e., the Fourier series representation contains at least 99% and at least 90% of the average power in the signal $x(t)$. **Turn in your plots using this value of N** (two for each signal).

e. Another useful way of presenting information about the Fourier series representation of a signal is a **single sided power spectrum**, which tells us how the signal is distributed in frequency. To plot the single sided power spectrum, we just plot the power terms $|c_0|^2 \quad 2|c_1|^2 \quad 2|c_2|^2 \quad \ldots \quad 2|c_N|^2$ versus the corresponding frequency $0 \quad \omega_0 \quad 2\omega_0 \quad \ldots \quad N\omega_0$. Since the fundamental frequency ω_0 is common to all of the frequency terms, we often just plot $|c_0|^2 \quad 2|c_1|^2 \quad 2|c_2|^2 \quad \ldots \quad 2|c_N|^2$ versus $0 \quad 1 \quad 2 \quad \ldots \quad N$. You are to write a function in **Fourier_Series.m** to plot the single sided power spectrum of the signal. The arguments to the function should again be c_0 and the array $c = [c_1 c_2 \ldots c_N]$. Utilize the **stem** command in Matlab to do the plotting. You may want to use the Matlab function **length** to determine the length of c. You may need to use the **figure** function so you can plot both the Fouier series (time-domain) plot and the power spectrum plot in two different windows. Plot the single sided power spectrum for each of the signals in **a** utilizing N = 10 terms. The y-axis should be labeled *Average Power*, the x-axis labeled *Term* and the graph should be titled *One Sided Power Spectrum*. You don't need to turn anything in for this part.

f. Being the intelligent and inquisitive type, you have probably lost sleep over the fact that in parts **d** and **e** we assumed that as we added more terms the previous values of the c_k did not change. That is, if we used $N = 2$ we would compute $c_0, c_1,$ and c_2. If we use $N = 5$ we would compute $c_0, c_2, c_3, c_4,$ and c_5. However, why should we expect that $c_0, c_1,$ and c_2 are the same regardless of whether we use $N = 2$, $N = 5$, or even $N = 1000$? The fact is that for a given function the c_k do not change as we add more terms since we are expanding $f(t)$ in terms of **orthogonal functions**. As a simple illustration, plot the power spectrum for each of the function in part a for $N = 5$ and $N = 10$. Verify by comparing the power spectrum graphs that $|c_0|^2$, $2|c_1|^2 \ldots 2|c_5|^2$ are the same for $N = 5$ and $N = 10$. **Turn in your plots (two plots for each function.)**

2) (From Lathi) A signal $x(t)$ is approximated in terms of a signal $v(t)$ over the interval $[t_1, t_2]$,

$$x(t) \approx cv(t)$$

where c is chosen to minimize the squared error (the magnitude of the error signal).

a) Show that $v(t)$ and $e(t) = x(t) - cv(t)$ are orthogonal over the interval $[t_1, t_2]$.

b) Can you explain this result in terms of an analogy with vectors.

This is a very important result!!!

3) ***<u>This problem should be done in Maple</u>***. Consider the following functions:

$$v_1(t) = 0.7071$$
$$v_2(t) = 1.0754e^t - 1.2638$$
$$v_3(t) = 4.9632t + 4.9632 - 4.2232e^t$$

a) Show that these functions are (approximately) orthogonal over the interval [-1,1].

b) Assume we want to approximate $x(t) = t^4 - t$ in this interval using only $v_1(t)$, $x(t) \approx c_1 v_1(t)$. Determine c_1 and plot the approximation $c_1 v_1(t)$ and the real function $x(t)$ on the same graph.

c) Assume we want to approximate $x(t)$ using the first two functions, $x(t) \approx c_1 v_1(t) + c_2 v_2(t)$. Determine c_1 and c_2 then plot the approximation and the real function on the same graph.

d) Assume we want to approximate $x(t)$ using the last two functions, $x(t) \approx c_2 v_2(t) + c_3 v_3(t)$. Determine c_2 and c_3, then plot the approximation and the real function on the same graph.

e) Assume we want to approximate $x(t)$ using all three functions. Determine the expansion coefficients, then plot the approximation and the real function on the same graph.