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ECE-205

Exam 3

Winter 2016

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/15

Problem 2 ________/20

Problem 3 ________/20

Problem 4 ________/23

Problem 5 ________/22

Total ____________

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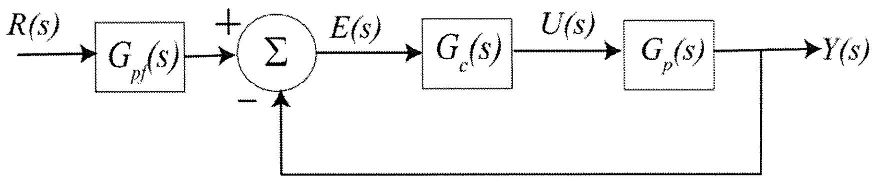
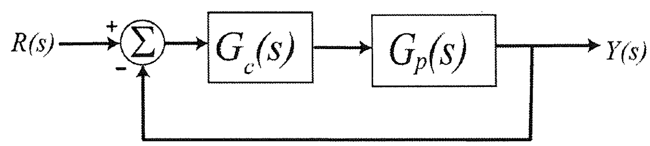
1) (15 points)

For the following problems, the closed loop transfer function for the following feedback system is

$$\frac{Y(s)}{R(s)} = G_o(s) = \frac{G_{pf}(s)G_c(s)G_p(s)}{1+G_c(s)G_p(s)}$$



Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{2}{s+3}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

$$\boxed{T_s = \frac{4}{3}}$$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer)

$$e_{ss} = 1 - \tfrac{2}{3} = \boxed{\tfrac{1}{3} = e_{ss}}$$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$G_0(s) = \frac{k_p \frac{2}{s+3}}{1 + k_p \frac{2}{s+3}} = \boxed{\frac{2k_p}{s+3+2k_p} = G_0(s)}$$

d) Determine the settling time of the closed loop system , in terms of k_p

$$\boxed{T_s = \frac{4}{3+2k_p}}$$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer)

$$e_{ss} = 1 - \frac{2k_p}{3+2k_p} = \boxed{\frac{3}{3+2k_p} = e_{ss}}$$

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2) (20 points)

a) For transfer function $H(s)=\dfrac{2}{(s+1)^2+2^2}$ and input $X(s)=\dfrac{1}{s+3}$, determine $y(t)$ where $Y(s)=H(s)X(s)$

b) For impulse response $h(t)=e^{-3(t-2)}u(t-2)$ and input $x(t)=e^{-2(t-1)}u(t-1)$, determine the output $y(t)$ using Laplace transforms. *You will not receive credit if you solve this problem in the time domain.*

a) $Y(s) = H(s)X(s) = \dfrac{2}{(s+3)\left[(s+1)^2+2^2\right]} = \dfrac{A}{s+3} + B\left[\dfrac{2}{(s+1)^2+2^2}\right] + C\left[\dfrac{s+1}{(s+2)^2+2^2}\right]$

$A = \dfrac{2}{2^2+2^2} = \dfrac{2}{8} = \dfrac{1}{4} = A$ $\quad \times s$, let $s \to \infty$ $\quad 0 = A + C$ $\quad C = -1/4$

let $s=-1$ $\quad \dfrac{2}{2\cdot 2^2} = \dfrac{A}{2} + \dfrac{B}{2}$ $\quad \dfrac{1}{4} = \dfrac{1}{8} + \dfrac{B}{2}$ $\quad B = 1/4$

$$\boxed{y(t) = \left[\frac{1}{4} + \frac{1}{4}e^{-t}\sin(2t) - \frac{1}{4}e^{-t}\cos(2t)\right]u(t)}$$

b) $H(s) = \dfrac{e^{-2s}}{s+3}$ $\quad X(s) = \dfrac{e^{-s}}{s+2}$ $\quad Y(s) = H(s)X(s) = \dfrac{e^{-3s}}{(s+2)(s+3)}$

$= e^{-3s}G(s)$

$G(s) = \dfrac{1}{(s+2)(s+3)} = \dfrac{A}{s+2} + \dfrac{B}{s+3}$ $\quad A = 1$ $\quad B = -1$

$g(t) = \left[e^{-2t} - e^{-3t}\right]u(t)$

$y(t) = g(t-3) = \boxed{\left[e^{-2(t-3)} - e^{-3(t-3)}\right]u(t-3) = y(t)}$

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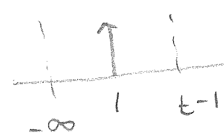
3) (20 points) Simplify the following as much as possible. Be sure to include any necessary step functions. Note that $*$ denotes convolution integral.

a) $\int_{-\infty}^{\infty}\delta(\lambda+2)u(\lambda)d\lambda = \int_{-\infty}^{\infty}\delta(\lambda+2)u(-2)\,d\lambda = u(-2)\int_{-\infty}^{\infty}\delta(\lambda+2)d\lambda = u(-2) = \boxed{0}$

b) $\delta(t-1)*\delta(t-3) = \int_{-\infty}^{\infty}\delta(t-\lambda-1)\delta(\lambda-3)d\lambda = \int_{-\infty}^{\infty}\delta(t-4)\,\delta(\lambda-3)d\lambda$

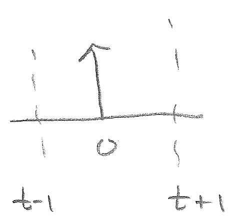
$= \delta(t-4)\int_{-\infty}^{\infty}\delta(\lambda-3)d\lambda = \boxed{\delta(t-4)}$

c) $\int_{-\infty}^{t-1}u(\lambda+1)\delta(\lambda-1)d\lambda = \int_{-\infty}^{t-1}u(2)\,\delta(\lambda-1)d\lambda = \int_{-\infty}^{t-1}\delta(\lambda-1)d\lambda = \boxed{u(t-2)}$



$t-1>1$ $\quad t-2>0$

d) $\int_{t-1}^{t+1}\delta(\lambda)d\lambda$



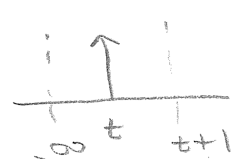
$t-1<0$ and $t+1>0$

$t<1$ and $t>-1$

$\boxed{u(t+1)-u(t-1) = u(t+1)u(1-t)}$

e) $\int_{-\infty}^{t+1}u(\lambda+t)\delta(t-\lambda)d\lambda = \int_{-\infty}^{t+1}u(2t)\,\delta(t-\lambda)d\lambda = u(2t)\int_{-\infty}^{t+1}\delta(t-\lambda)d\lambda$

$= \boxed{u(2t)}$



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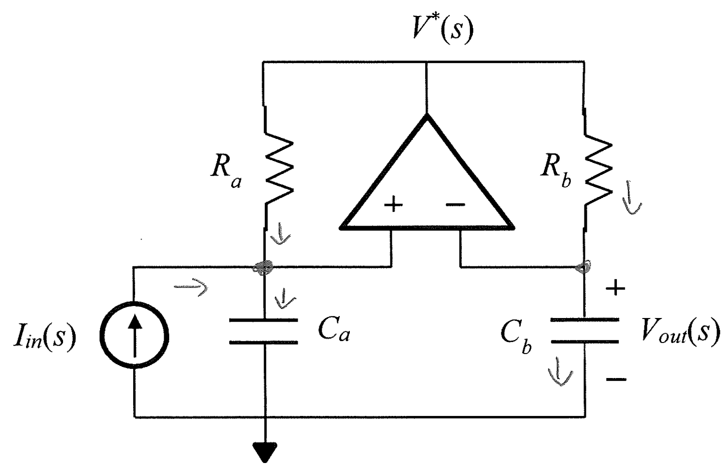
4) (23 points) Determine the transfer function $H(s) = \dfrac{V_{out}(s)}{I_{in}(s)}$ of the following circuit.



Hint: Define the node voltage $V^*(s)$ at the output of the op-amp as an intermediate variable.

V^+ terminal

$$I_{in}(s) + \frac{V^*(s) - V^+(s)}{R_a} = \frac{V^+(s)}{1/C_a s} \qquad V^+(s) = V_{out}(s)$$

$$I_{in}(s) R_a + V^*(s) - V_{out}(s) = V_{out}(s) R_a C_a s$$

$$V^*(s) = V_{out}(s)\left[1 + R_a C_a s\right] - I_{in}(s) R_a$$

V^- terminal

$$\frac{V^*(s) - V_{out}(s)}{R_b} = \frac{V_{out}(s)}{1/C_b s}$$

$$V^*(s) = V_{out}(s)\left[1 + R_b C_b s\right] = V_{out}(s)\left[1 + R_a C_a s\right] - I_{in}(s) R_a$$

$$\left[R_b C_b s - R_a C_a s\right] V_{out}(s) = -R_a I_{in}(s)$$

$$\boxed{\frac{V_{out}(s)}{I_{in}(s)} = \frac{R_a}{(R_a C_a - R_b C_b)\, s}}$$

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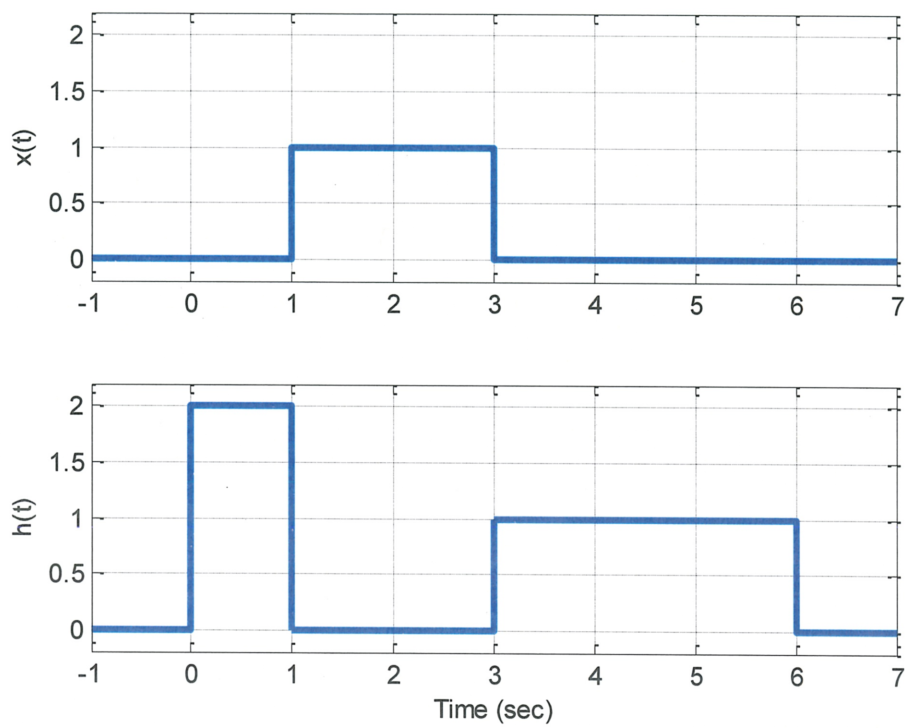
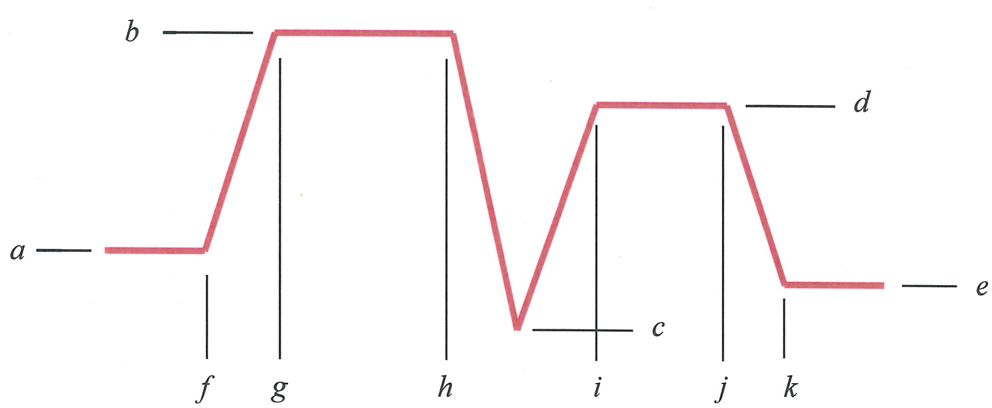
5) (22 points)

Below are functions for $x(t)$ and $h(t)$ for a system and the result of the convolution. Provide values for the specified artifacts in the convolution. Note that *a-e* are values while *f-k* are times. *The convolution diagram is a rough sketch and not to scale!*





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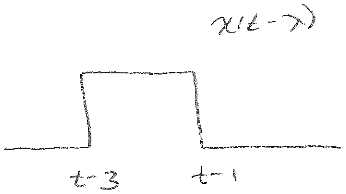
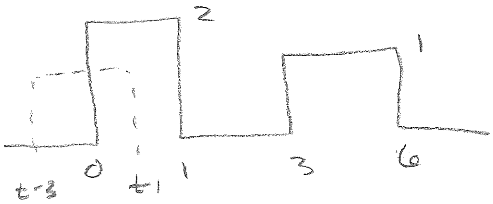


$y(t) = \boxed{0 = a}$ $t-1 \le 0$ or $t \le 1$ $f = 1$



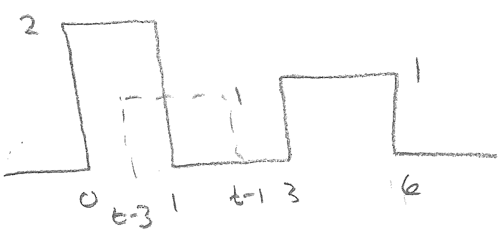
$y(t) = (1)(2)(1) = 2 = b$ for $0 \le t-1 \le 1$

or $1 \le t \le 2$

$g = 2$

$y(t)$ starts to decrease when $t-3 = 0$

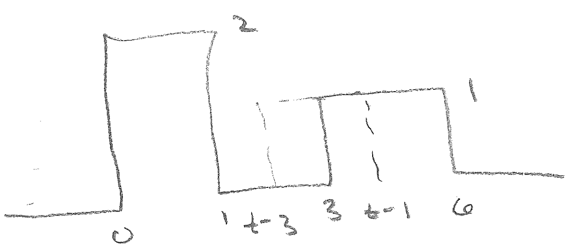
or $t = 3$ $h = 3$



$y(t) = 0$ when $t-1 = 3$ or $t-3 = 1$

$t = 4$ $t = 4$

$c = 0$ when $t = 4$

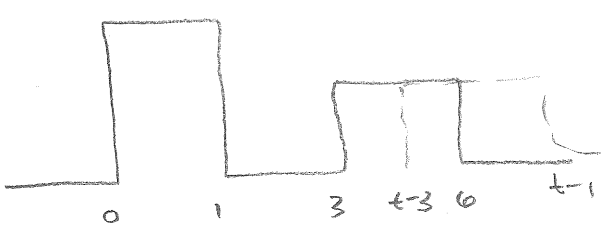


$y(t) = (1)(1)(2) = 2 = d$

when $t-3 \ge 3$ and $t-1 \le 6$

$t \ge 6$ $t \le 7$

$i = 6$ $j = 7$



$y(t) = 0$ when $t-3 \ge 6$

$t \ge 9$ $k = 9$